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INITIAL STATE CORRELATIONS AND THE RIDGE*

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We point out that Bose enhancement in a hadronic wave function generically leads to correlations between produced particles. We show explicitly, by calculating the projectile density matrix in the Color Glass Condensate approach to high-energy hadronic collisions, that the Bose enhancement of gluons in the projectile leads to azimuthal collimation of long range rapidity correlations of the produced particles, the so-called ridge correlations.

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1. Introduction

The ridge structure observed in high multiplicity p–p and p–Pb collisions at the Large Hadron Collider (LHC) has triggered an intense activity aimed at understanding the possible physical origin of correlations between emitted particles. One of the ideas that is suggested is that the final state correlations carry the imprint of the partonic correlations that exist in the initial state. "Glasma graph" contributions [1] is one of the proposed models within the Color Glass Condensate (CGC) approach to explain ridge structure. Even though the numerical calculations based on the glasma graph approach have

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been very successful in reproducing the systematics of ridge correlations, the physics behind this approach was not completely clear. Recently, it has been shown that the ridge correlations that are observed in the final state originate from the Bose enhancement of the gluons in the incoming hadronic wave function [2].

2. Gluon production and Bose enhancement

2.1. Basics of Bose enhancement

In order to understand the basic idea behind the Bose enhancement, let us consider a state with fixed occupation numbers of N species of bosons at different momenta. This state can be written as

$$\left|\{n^i(p)\}\right\rangle \equiv \prod_{i,p} \frac{1}{\sqrt{n^i(p)!}} \left(\frac{a_i^\dagger(p)}{\sqrt{V}}\right)^{n^i(p)} |0\rangle \tag{1}$$

with a finite volume V and periodic boundary conditions so that momenta are discrete. The state is translationally invariant with mean particle density

$$n \equiv \left\langle\{n^i(p)\}\right| a^{\dagger i}(x) a^i(x) \left|\{n^i(p)\}\right\rangle = \sum_{i,p} n^i(p)\,. \tag{2}$$

Hereafter, we take $\sum_p \approx \int \mathrm{d}^3p/(2\pi)^3$. The 2-particle correlator in coordinate space is

$$D(x,y) \equiv \langle\{n(p)\}|a^{\dagger i}(x)a^{\dagger j}(y)a^i(x)a^j(y)|\{n(p)\}\rangle\,. \tag{3}$$

This is calculated by going to momentum space, where the operator averages are simple,

$$\begin{aligned}&\langle\{n(p)\}|a^{\dagger i}(p)a^{\dagger j}(q)a^i(l)a^j(m)|\{n(p)\}\rangle\\ &\approx \delta(p-l)\delta(q-m)\sum_i n^i(p)\sum_j n^j(q) + \delta(p-m)\delta(q-l)\sum_i n^i(p)n^i(q)\,,\end{aligned} \tag{4}$$

where we have neglected the terms where all momenta are equal, which are suppressed by a phase-space factor. Using this, the result for $D(x,y)$ reads

$$D(x,y) = n^2 + \sum_i \left|\int \frac{\mathrm{d}^3p}{(2\pi)^3}\, e^{ip(x-y)} n^i(p)\right|^2\,. \tag{5}$$

The last term expresses the Bose enhancement. It vanishes when the points are very far away, and gives $\mathcal{O}(1/N)$ enhancement when the points coincide.

The $\mathcal{O}(1/N)$ suppression of the second term relative to the first one is due to the fact that the second term contains a single sum over the species index. The physics is that only bosons of the same species are correlated with each other. Technically, the origin of this additional contribution is the "wrong contraction" term in Eq. (4).

The Bose enhancement is a generic phenomenon, and is not tied specifically to the state with fixed number of particles. An overwhelming majority of pure states or quantum density matrices exhibit Bose enhancement at some degree. There is, however, one type of states that do not exhibit such behavior, notably classical-like coherent states. Consider a coherent state

$$|b(x)\rangle \equiv \exp\left\{ i \int \mathrm{d}^3x\, b^i(x) \left(a^i(x) + a^{\dagger i}(x) \right) \right\} |0\rangle \,. \tag{6}$$

A trivial calculation in this state gives

$$\begin{aligned} \langle b(x)|a^{\dagger i}(x)a^i(x)|b(x)\rangle \;&=\; b^i(x)b^i(x)\,, \\ \langle b(x)|a^{\dagger i}(x)a^{\dagger j}(y)a^i(x)a^j(y)|b(x)\rangle \;&=\; b^i(x)b^i(x)b^j(y)b^j(y)\,, \end{aligned} \tag{7}$$

so $D(x, y) = n(x)n(y)$. Thus, in order to exhibit Bose enhancement, a state has to be nonclassical.

2.2. Gluon production within glasma graph approximation

Our aim is to show that the angular collimation arising from the glasma graph calculation is due to the Bose enhancement in the projectile wave function. Following [1], we consider the calculation of inclusive two particle production.

The glasma graphs that contribute to this observable come in three varieties, see Fig. 1. Type A graphs contribute to the case where two gluons from the incoming projectile wave function scatter independently on the target. The incoming gluons have transverse momenta k_1 and k_2 respectively. While propagating through the target, the first particle picks up transverse momentum $p - k_1$ and the second particle picks up transverse momentum $q - k_2$, so that the outgoing particles have momenta p and q. Type B and C graphs from the projectile point of view are "interference graphs", in the sense that the final state gluon with momentum p comes from the projectile gluons with different momenta in the amplitude and complex conjugate amplitude. Type B and C diagrams contain leading contributions that can be reinterpreted as Type A but with gluons originating from the target rather than from the projectile and, additionally, subleading contributions, including those that lead to HBT correlations [3, 4]. Therefore, in the following, we will only discuss those of Type A, keeping in mind this complementary interpretation of the leading pieces of Type B and C.

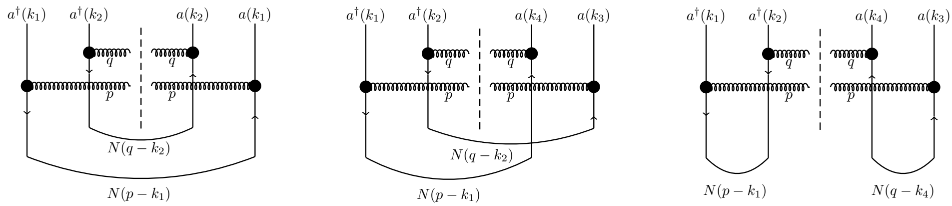

Fig. 1. Glasma graphs for two gluon inclusive production before averaging over the projectile color charge density ρ.

The Type A contribution to double inclusive gluon production can be written as

$$C\int\limits_{k_1,k_2}\langle \mathrm{in}|a^{\dagger i}_a(k_1)a^{\dagger j}_b(k_2)a^k_a(k_1)a^l_b(k_2)|\mathrm{in}\rangle\left[\delta^{ik}-\frac{k_1^ik_1^k}{p^2}\right]\left[\delta^{jl}-\frac{k_2^jk_2^l}{q^2}\right]$$
$$\times N(p-k_1)N(q-k_2)\,, \tag{8}$$

where $|\mathrm{in}\rangle$ is the wave function of the incoming projectile, C is a constant, $N(p-k)$ is the probability that the incoming gluon with transverse momentum k acquires transverse momentum p after scattering and, hereafter, we use the notation $\int_k \equiv \int\frac{\mathrm{d}^2k}{(2\pi)^2}$. This scattering probability is, of course, determined by the distribution of target fields (within the glasma graph calculation, the scattering of the two gluons is independent).

We have to understand what is the nature of the projectile state $|\mathrm{in}\rangle$, and, in particular, we need to calculate

$$D(k_1,k_2)\equiv\langle \mathrm{in}|a^{\dagger i}_a(k_1)a^{\dagger j}_b(k_2)a^k_a(k_1)a^l_b(k_2)|\mathrm{in}\rangle\,. \tag{9}$$

Averaging over the projectile state in the standard CGC approach involves two elements. One needs to calculate the average over the soft degrees of freedom, as well as that over the valence color charge density. Conventionally, this is done in the spirit of the Born–Oppenheimer approximation, namely first one averages over the soft gluon degrees of freedom at fixed valence color charge density ρ, and subsequently, averages over the valence density distribution.

The wave function of the soft fields for fixed valence color charge density for a dilute projectile is a simple coherent state

$$|\mathrm{in}\rangle_\rho=\exp\left\{i\int\limits_k b^i_a(k)\left[a^{\dagger i}_a(k)+a^i_a(-k)\right]\right\}|0\rangle\,, \tag{10}$$

with the Weizsäcker–Williams field $b^i_a(k)=g\rho_a(k)\frac{ik^i}{k^2}$.

The averaging over the soft degrees of freedom leads to the well-known expression for the observable in terms of the charge density

$$D(k_1, k_2)_\rho = b^i_a(k_1) b^j_b(k_2) b^k_a(-k_1) b^l_b(-k_2)\,. \tag{11}$$

Since at fixed ρ the soft gluon state is a coherent state, this expression does not seem to exhibit Bose enhancement. This is, however, misleading, since averaging over ρ is part of the quantum averaging over the initial state wave function $|\mathrm{in}\rangle$. It is, therefore, instructive to reverse the conventional order of averaging, and average over the valence degrees of freedom first. The result of such a procedure is a density matrix on the soft gluon Hilbert space. The subsequent averaging over this density matrix is a direct way to find out whether the projectile wave function exhibits Bose enhancement.

2.3. The soft gluon density matrix

The soft gluon density matrix depends, of course, on the weight for the valence color charge density. For illustrative purposes, we choose the same Gaussian weight used in the glasma graph calculation, the McLerran–Venugopalan model [5],

$$\langle \ldots \rangle_\rho = \mathcal{N} \int D[\rho]\, \ldots\, e^{-\int\limits_k \frac{1}{2\mu^2(k)}\, \rho_a(k)\rho_a(-k)}\,, \tag{12}$$

where $\mathcal{N}$ is the normalization factor. Thus, the density matrix of the soft gluons is given by

$$\hat{\rho} = \mathcal{N} \int D[\rho]\; e^{-\int_k \frac{1}{2\mu^2(k)} \rho_a(k)\rho_a(-k)} e^{i\int_q b^i_b(q)\phi^i_b(-q)} |0\rangle\langle 0|\; e^{-i\int_p b^j_c(p)\phi^j_c(-p)}\,, \tag{13}$$

where we have defined $\phi^i_a(k) = a^i_a(k) + a^{\dagger i}_a(-k)$. The integral over ρ can be performed with the result

$$\begin{aligned} \hat{\rho} \;=\; & e^{-\int_k \frac{g^2\mu^2(k)}{2k^4} k^i k^j\, \phi^i_b(k)\, \phi^j_b(-k)} \left\{ \sum_{n=0}^{+\infty} \frac{1}{n!} \left[\prod_{m=1}^{n} \int_{p_m} \frac{g^2\mu^2(p_m)}{p^4_m}\, p^{i_m}_m\, \phi^{i_m}_{a_m}(p_m) \right] |0\rangle \right. \\ & \left. \times \langle 0| \left[\prod_{m=1}^{n} p^{j_m}_m \phi^{j_m}_{a_m}(-p_m) \right] \right\} e^{-\int_{k'} \frac{g^2\mu^2(k')}{2k'^4} k'^{i'} k'^{j'}\, \phi^{i'}_c(k')\, \phi^{j'}_c(-k')}\,. \end{aligned} \tag{14}$$

The interesting correlator is given by

$$D(k_1, k_2) = \mathrm{tr}\left[\hat{\rho} a^{\dagger i}_a(k_1) a^{\dagger j}_b(k_2) a^k_a(k_1) a^l_b(k_2) \right] \tag{15}$$

which can be calculated explicitly thanks to Eq. (14). The result reads

$$D(k_1,k_2) = S^2\left(N_c^2-1\right)^2 \frac{k_1^i k_1^k k_2^j k_2^l}{k_1^2 k_2^2}\frac{g^4\mu^2(k_1)\mu^2(k_2)}{k_1^2k_2^2} \times\left\{1+\frac{1}{S\left(N_c^2-1\right)}\left[\delta^{(2)}(k_1-k_2)+\delta^{(2)}(k_1+k_2)\right]\right\}. \quad (16)$$

3. Discussion

The first term in Eq. (16) is the "classical" term equal to the square of the number of particles. The second term is the typical Bose enhancement term, suppressed with respect to the first "classical" term by the total number of degrees of freedom (color and area). The third term is specific to the density matrix at hand and it appears due to reality of the gluon field scattering amplitude. This establishes our point that the soft glue density matrix exhibits Bose enhancement, so that the likelihood of finding two gluons with the same transverse momentum is higher than average. Note that this effect is naturally subleading in N_c as the enhancement is only effective if both gluons are in the same color state.

As a typical Bose enhancement contribution, the second term in Eq. (16) is nonvanishing only when the momenta of the two gluons are equal. Note, however, that k_1 and k_2 are *not* the momenta of observed gluons, but rather the momenta of gluons in the wave function of the incoming projectile. The two gluons then scatter on the target and acquire momenta p and q with the probability $N(p-k_1)N(q-k_2)$, as indicated in Eq. (8).

REFERENCES

[1] A. Dumitru, F. Gelis, L. McLerran, R. Venugopalan, *Nucl. Phys. A* **810**, 91 (2008) [`arXiv:0804.3858 [hep-ph]`].

[2] T. Altinoluk *et al.*, *Phys. Lett. B* **751**, 448 (2015) [`arXiv:1503.07126 [hep-ph]`].

[3] T. Altinoluk *et al.*, *Phys. Lett. B* **752**, 113 (2016) [`arXiv:1509.03223 [hep-ph]`].

[4] Y.V. Kovchegov, D.E. Wertepny, *Nucl. Phys. A* **906**, 50 (2013) [`arXiv:1212.1195 [hep-ph]`].

[5] L.D. McLerran, R. Venugopalan, *Phys. Rev. D* **49**, 2233 (1994) [`arXiv:hep-ph/9309289`].