KINETIC PROPERTIES OF THE GRIBOV–ZWANZIGER PLASMA*

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The idea of the Gribov–Zwanziger plasma is introduced and used to calculate the bulk and shear viscosities of the system of gluons at high temperature.

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1. Introduction

In this proceedings contribution, I discuss recent results obtained in collaboration with Ryblewski, Su, and Tywoniuk [1, 2] that describe the bulk and shear viscosity coefficients of the Gribov–Zwanziger (GZ) plasma. The idea of such a system follows from the use of the Gribov dispersion relation for gluons [3],

$$E(\boldsymbol{k}) = \sqrt{\boldsymbol{k}^2 + \frac{\gamma_{\rm G}^4}{\boldsymbol{k}^2}}\,, \tag{1}$$

to describe hot and dense gluonic matter. In Eq. (1), $\boldsymbol{k}$ is the three-momentum of gluons and E denotes their energy. The parameter $\gamma_{\rm G}$ appears during the quantisation of the Yang–Mills theory. It takes into account the expected behavior of gluons in the infrared region [3–7].

Thermodynamic properties of the GZ plasma have been studied in [8, 9]. The main results are obtained with the dispersion relation (1) used in the Bose–Einstein distribution function

$$f_{\rm GZ} = \frac{1}{\exp(E(\boldsymbol{k})/T) - 1}\,, \tag{2}$$

where T is the system's temperature. This approach provides good agreement with the lattice results [10]. Encouraged by the success connected with the use of Eq. (1) in equilibrium, in Refs. [1, 2] an attempt has been made

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to construct a simple kinetic theory that incorporates (1) and may describe transport phenomena. In particular, in [1], we obtained the formula for the bulk viscosity coefficient. The latter attracts more and more attention in the last years [11–15].

2. Lorentz covariance and boost-invariance

Equation (1) is derived in the Coulomb gauge that violates Lorentz invariance. In order to regain a covariant framework for fluid dynamics, one has to make assumptions about the Lorentz transformation properties of the quantities that appear in (1). The strategy to recover covariant description is not straightforward and this issue is discussed in more detail in [2]. In our approach, formula (1) is transformed into the covariant expression of the form [1, 2]

$$E(k\cdot u) = \sqrt{(k\cdot u)^2 + \frac{\gamma_{\rm G}^4}{(k\cdot u)^2}}\,, \tag{3}$$

where u is the four-velocity of the fluid element. The Coulomb gauge as well as the in-medium value of the Gribov parameter $\gamma_{\rm G}$ are fixed in the local rest frame where $u^\mu = (1,0,0,0)$. One introduces $k^0 \equiv |\boldsymbol{k}|$, which is the magnitude of the three-vector $\boldsymbol{k} \equiv (k_x, k_y, k_\|)$, and $k_\perp = \sqrt{k_x^2 + k_y^2}$ such that $k^\mu = (k^0, \boldsymbol{k})$ satisfies the condition $k^2 = k^\mu k_\mu = 0$. The four-vector k may be interpreted as the four-momentum of a perturbative, non-interacting gluon.

A convenient way to obtain viscosity coefficients is to analyze a (0+1)-dimensional, boost-invariant and transversally homogeneous system. In contrast to the problem of Lorentz covariance, which is the fundamental issue connected with the use of the Coulomb gauge, imposing boost-invariance is just a technical method that facilitates our manipulations.

The boost-invariant (Bjorken) flow has the form of $u^\mu = (t/\tau, 0, 0, z/\tau)$ [16]. Moreover, all scalar functions of space and time depend, in this case, only on the proper time $\tau = \sqrt{t^2 - z^2}$. One may furthermore introduce the boost-invariant variables $v = k^0 t - k_\| z$ and $w = k_\| t - k^0 z$ [17]. In this case, the gluon energy is

$$E\left(\tau, w, k_\perp\right) = \sqrt{\frac{w^2}{\tau^2} + k_\perp^2 + \frac{\gamma_{\rm G}^4}{\frac{w^2}{\tau^2} + k_\perp^2}}\,. \tag{4}$$

The covariant momentum integration measure can be written as

$$\int {\rm d}K(\dots) = \frac{g_0}{(2\pi)^3} \int\limits_{-\infty}^{\infty} \frac{{\rm d}w}{\tau} \int {\rm d}^2 k_\perp (\dots)\,, \tag{5}$$

where g_0 is the number of internal degrees of freedom. We note that the phase-space distribution function f, which is also a Lorentz scalar, may depend in our case only on τ, w and $k_\perp$, namely $f = f(\tau, w, k_\perp)$ [17].

3. Kinetic equation

The arguments presented in [1, 2] suggest that one can use the standard kinetic equation in the relaxation-time approximation (RTA) [18–20]

$$\frac{\partial f\left(\tau, w, k_\perp\right)}{\partial \tau} = \frac{f_{\rm GZ}\left(\tau, w, k_\perp\right) - f\left(\tau, w, k_\perp\right)}{\tau_{\rm rel}(\tau)}\,, \tag{6}$$

where $\tau_{\rm rel}$ is the relaxation time and the effective temperature T that defines the equilibrium distribution function $f_{\rm GZ}$ is determined from the condition

$$\int {\rm d}K\, E\left(\tau, w, k_\perp\right)\, f_{\rm GZ}\left(\tau, w, k_\perp\right) = \int {\rm d}K\, E\left(\tau, w, k_\perp\right)\, f\left(\tau, w, k_\perp\right)\,. \tag{7}$$

Equation (7) is known as the Landau matching condition for the energy. The formal solution of Eq. (6) is [21–23]

$$f\left(\tau, w, k_\perp\right) = f_0\left(w, k_\perp\right) D(\tau, \tau_0) + \int\limits_{\tau_0}^{\tau} \frac{{\rm d}\tau'}{\tau_{\rm rel}(\tau')} D(\tau, \tau') f_{\rm GZ}\left(\tau', w, k_\perp\right)\,, \tag{8}$$

where the damping function $D(\tau_2, \tau_1)$ has the form of

$$D(\tau_2, \tau_1) = \exp\left[-\int\limits_{\tau_1}^{\tau_2} \frac{{\rm d}\tau}{\tau_{\rm rel}(\tau)}\right]\,. \tag{9}$$

Inserting the formal solution (8) into the Landau matching condition (7), one finds the time dependence of the system's temperature, $T(\tau)$. This, in turn, when used in (8), allows us to find the time dependence of various system's characteristics such as the energy density and transverse and longitudinal pressures.

Besides the exact treatment of the kinetic equation that has been sketched above, we may use the linear response method. In this case, we seek the solution of Eq. (6) in the form of

$$f \approx f_{\rm GZ} + \delta f + \ldots\,, \tag{10}$$

where $\delta f = -\tau_{\rm rel}{\rm d}f_{\rm GZ}/{\rm d}\tau$. This leads directly to the expression

$$\delta f = -\frac{E\,\tau_{\rm rel}}{T\tau}\left\{\frac{w^2}{E^2\tau^2}\left[1 - \frac{\gamma_{\rm G}^4}{\left(\frac{w^2}{\tau^2} + k_\perp^2\right)^2}\right] + c_{\rm s}^2\right\} f_{\rm GZ}\left(1 + f_{\rm GZ}\right)\,. \tag{11}$$

The speed of sound squared, $c_{\rm s}^2$, can be calculated from the equation of state. The correction to the distribution function (11) can be used further to obtain the shear tensor $\pi^{\mu\nu}$ and the bulk pressure Π. These quantities define directly the bulk and shear viscosities if the Navier–Stokes limit is considered. Straightforward calculations lead to the following expression for the bulk viscosity [1]

$$\zeta = \frac{g_0 \gamma_{\rm G}^5}{3\pi^2} \frac{\tau_{\rm rel}}{T} \int\limits_0^\infty {\rm d}y \left[c_{\rm s}^2 - \frac{1}{3} \frac{y^4-1}{y^4+1} \right] f_{\rm GZ} \left(1+f_{\rm GZ}\right) , \tag{12}$$

where $f_{\rm GZ} = \{\exp[\gamma_{\rm G}\sqrt{y^2+y^{-2}}/T]-1\}^{-1}$. Similarly, one obtains the shear viscosity [1,2]

$$\eta = \frac{g_0 \gamma_{\rm G}^5}{30\pi^2} \frac{\tau_{\rm rel}}{T} \int\limits_0^\infty {\rm d}y \, \frac{\left(y^4-1\right)^2}{y^4+1} f_{\rm GZ}(1+f_{\rm GZ}) \, . \tag{13}$$

4. Numerical results

Let us now turn back to the discussion of the exact solutions of the kinetic equation. Knowing the time dependence of the effective temperature T, we find the longitudinal and transverse pressures from the equations

$$P_\parallel = \int {\rm d}K \, \frac{w^2}{\tau^2 E(\tau,w,k_\perp)} \left[1 - \frac{\gamma_{\rm G}^4}{\left(w^2/\tau^2+k_\perp^2\right)^2} \right] f \, , \tag{14}$$

$$P_\perp = \int {\rm d}K \, \frac{k_\perp^2}{2\,E(\tau,w,k_\perp)} \left[1 - \frac{\gamma_{\rm G}^4}{\left(w^2/\tau^2+k_\perp^2\right)^2} \right] f \, , \tag{15}$$

and the total pressure is obtained as $P = \frac{1}{3}(P_\parallel + 2P_\perp)$ (note that the parallel pressure acts in the direction of the beam axis, while the transverse pressure acts in the transverse direction to the beam). Given $P_\parallel$ and $P_\perp$, one finds the shear and bulk viscous pressures from the expressions

$$\pi = \tfrac{2}{3}\left(P_\perp - P_\parallel\right) \, , \qquad \Pi = P - P_{\rm GZ} \, . \tag{16}$$

The last two equations allow us to define the effective shear and bulk viscosities

$$\pi = \frac{4}{3}\frac{\eta_{\rm eff}}{\tau} \, , \qquad \Pi = -\frac{\zeta_{\rm eff}}{\tau} \, . \tag{17}$$

The effective coefficients should agree with the standard coefficients when the system is close to equilibrium. This is demonstrated in Figs. 1 and 2.

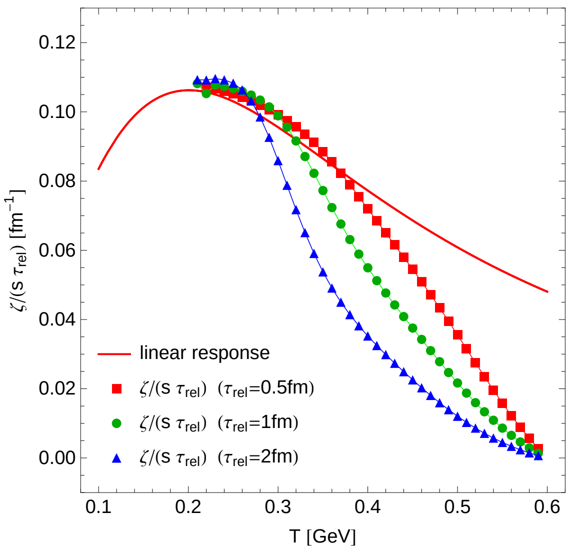

Fig. 1. Temperature dependence of the bulk viscosity scaled by the relaxation time and entropy density. Triangles, circles and squares show the effective bulk viscosity for three different values of the relaxation time (defined in the figure), whereas the solid line shows the result of the linear-response formula.

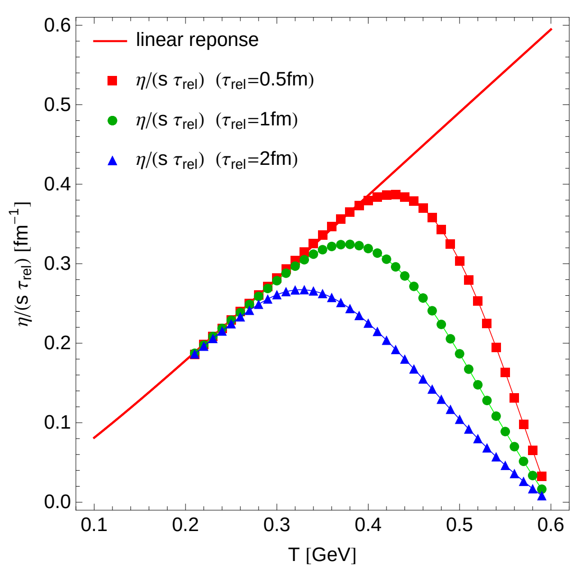

Fig. 2. The same as Fig. 1 but for the shear viscosity.

5. Conclusions

It has been shown before that the Gribov–Zwanziger quantisation of the Yang–Mills theory leads to the dispersion relation that can be successfully used in the studies of gluon thermodynamics. In this contribution, I have presented an extension of this approach to non-equilibrium situations. The results for the bulk and shear viscosities of the Gribov–Zwanziger plasma have been presented, which may be useful for future phenomenological applications in ultrarelativistic heavy-ion collisions.

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