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THERMODYNAMICS OF THE VECTOR MESON EXTENDED LINEAR SIGMA MODEL*

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We use a χ^2 minimization procedure to parameterize the Polyakov loop and (axial-)vector meson extended $N_{\rm f} = 2+1$ flavor linear sigma model (LσM) based on tree-level decay widths and vacuum scalar and pseudoscalar curvature masses which includes the contribution of the constituent quarks. Using a quark improved Polyakov loop potential and a simple approximation for the grand potential, we determine and compare with lattice results the pressure and thermodynamical observables derived from it. We also determine the location of the critical end point of the μ_B–T phase diagram.

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1. Introduction

In view of the planned experimental facilities (FAIR and NICA), where the strongly interacting matter can be investigated at high densities and nonzero temperatures, effective model studies of certain aspects of the chiral phase transition could be of some relevance. In the present work, we use the $\mathrm{SU}(3)_{\rm L} \times \mathrm{SU}(3)_{\rm R}$ (axial-)vector meson extended LσM, for which a parametrization method was developed in [1], and, as in [2], we incorporate further degrees of freedom in the form of constituent quarks and Polyakov loop variables. The study of the model done in [2] under the assumption that the mesons are $q\bar{q}$ states showed that in order to have a thermodynamics

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consistent with continuum extrapolated lattice results light scalar particles should be assigned to the scalar nonet. In this work, we use the assignment $a_0^{\bar{q}q} \to a_0(980)$, $K_0^{\star,\bar{q}q} \to K_0^\star(800)$, $f_0^{\mathrm{L},\bar{q}q} \to f_0(500)$, $f_0^{\mathrm{H},\bar{q}q} \to f_0(980)$ (L/H denotes the lower/higher mass state in the mixing sector) and study the thermodynamics of the model with a different form of the Polyakov loop potential than the one used in [2]. We have chosen here the above assignment because the minimization procedure of [2] showed that the parameters which lead to acceptable values of the pseudocritical temperature at $\mu_B = 0$ give, in this case, the smallest χ^2 value among all the 40 possible ways of assigning five f_0, two a_0 and two $K_0^\star$ particles to the states of the scalar nonets.

In Section 2, we discuss the parametrization of the model and the approximations we use to solve it; in Section 3, we present our results and we conclude in Section 4.

2. The model and the approximation used to solve it

The Lagrangian of the model is

$$\mathcal{L} = \mathcal{L}_{\mathrm{M}}(S_a, P_a, V_a, A_a, A_e^\mu) + \bar{\Psi} i\gamma_\mu D^\mu \Psi - g_{\mathrm{F}} \bar{\Psi} \sum_{a=0}^{8} (S_a + i\gamma_5 P_a)\, T_a \Psi\,, \quad (1)$$

where $\mathcal{L}_{\mathrm{M}}$, depending on the pseudoscalar (P), scalar (S), vector (V), axialvector (A) nonet and the electromagnetic field (A_e^μ), can be found in [1]. The constituent quark fields Ψ are only coupled to the (pseudo)scalars. The covariant derivative $D^\mu = \partial^\mu - iG^\mu$ contains the gluonic field G^μ. In the mean field approximation, the gauge field is taken to have only a temporal component G^4, which is constant and diagonal in color space. This gives rise to the Polyakov loop Φ and its conjugate $\bar{\Phi}$ when calculating the partition function $\mathcal{Z}$ (for details, see [2]). For the Polyakov loop potential, we use

$$\frac{U\left(\Phi, \bar{\Phi}\right)}{T^4} = -\frac{a(T)}{2} \Phi\bar{\Phi} + b(T) \ln M_{\mathrm{H}}\left(\Phi, \bar{\Phi}\right) + \frac{c(T)}{2}\left(\Phi^3 + \bar{\Phi}^3\right) + d(T)\left(\Phi\bar{\Phi}\right)^2\,, \quad (2)$$

where $M_{\mathrm{H}}(\Phi, \bar{\Phi}) = 1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2$ is the SU(3) Haar measure. The parameters are determined in [3] from lattice data on the equation of state, the Polyakov loop expectation value and its susceptibilities. To the above Polyakov loop potential, we apply the improvement of Ref. [4], which takes into account also the effect of the quarks on the pure gluonic sector. This potential, which in addition to the transition temperature of the YM theory involves a new temperature scale T_c^{glue}, was previously applied at finite densities in Ref. [5].

The grand potential $\Omega(T,\mu_q) = -(T\ln\mathcal{Z})/V$ of the symmetric quark matter ($\mu_q = \mu_B/3$, for $q = u, d, s$) is constructed in the simplest approximation: the mesonic part of the potential $U(\phi_{\rm N}, \phi_{\rm S})$ is classical and written in terms of the strange and nonstrange scalar condensates $\phi_{\rm S}$ and $\phi_{\rm N}$, while the fermionic part (determinant) is computed also neglecting the mesonic fluctuations. Adding the Polyakov loop potential, one has

$$\Omega\left(T,\mu_q\right) = U\left(\phi_{\rm N},\phi_{\rm S}\right) + U\left(\Phi,\bar{\Phi}\right) + \Omega_{\bar{q}q}^{(0)\rm v} + \Omega_{\bar{q}q}^{(0)\rm T}\left(T,\mu_q\right)\,, \tag{3}$$

where the renormalization of the vacuum part of the fermionic determinant $\Omega_{\bar{q}q}^{(0)\rm v}$ introduces the renormalization scale M_0 and the thermal part can be written in terms of modified Fermi–Dirac distribution functions involving the Polyakov loop and its conjugate. The approximation we use involves also the replacing the Polyakov loops by their expectation values (for which we use the same notation for simplicity) which at $\mu_q \neq 0$ are treated as independent and real quantities. The values of Φ, $\bar{\Phi}$, $\phi_{\rm N}$, and $\phi_{\rm S}$ are obtained from the 4 coupled field equations

$$\frac{\partial\Omega}{\partial\phi_{\rm N}} = \frac{\partial\Omega}{\partial\phi_{\rm S}} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\bar{\Phi}} = 0\,. \tag{4}$$

The explicit expression of these equations can be found in [2].

2.1. Parametrization

The parameters of the model are determined using the multiparametric χ^2 minimization method of [6] in which, given a set of parameters p and a set of observables $\mathcal{O}$, one computes

$$\chi^2(p) = \sum_{i=1}^{|\mathcal{O}|}\left[\frac{Q_i(p) - Q_i^{\rm exp}}{\delta Q_i}\right]^2\,, \tag{5}$$

where for the $i^{\rm th}$ observable, $Q_i(p)$ is the value calculated within the model, $Q_i^{\rm exp}$ is the experimental value taken from the Particle Data Group and δQ_i is the assigned error, which typically is 20% for scalar masses and their decay widths, 10% for the constituent quark masses and 5% for other vacuum quantities. For observables, we use in addition to 29 vacuum quantities also the pseudocritical temperature $T_{\rm c}$ at $\mu_B = 0$, with 10% error. The inclusion of $T_{\rm c}$ drastically improved the efficiency of the scanning through the parameter space. The vacuum quantities we used are: 8 (pseudo)scalar curvatures masses, 5 tree-level (axial-)vector masses, 2 tree-level constituent quark masses, 12 tree-level decays width and 2 PCAC relations. For further details, we refer to [2].

In the present work, using 5×10^4 points of the parameter space, we did a χ^2 minimization at fixed renormalization scale $M_0 = 0.3$ GeV, then we minimized also for M_0. We followed this procedure for three values of $T_{\rm c}^{\rm glue}$, namely 182, 210, and 240 MeV ($T_{\rm c}^{\rm glue}$ influences the value of $T_{\rm c}$) and, in each case, we used the parameters giving the minimal χ^2 value to obtain the figures of the next section[1].

3. Results

Now, we briefly summarize our results which are very similar to those obtained in [2], where a more detailed investigation was done using a different form of the Polyakov loop potential. In Fig. 1, we compare with the lattice result of [7] the subtracted condensate $\Delta_{l,s} = \frac{(\phi_{\rm N} - h_{\rm N}\phi_{\rm S}/h_{\rm S})|_T}{(\phi_{\rm N} - h_{\rm N}\phi_{\rm S}/h_{\rm S})|_{T=0}}$ obtained at three different values of $T_{\rm c}^{\rm glue}$. For $T_{\rm c}^{\rm glue} = 182$ MeV, we also show the effect of the χ^2 minimization by comparing at fixed $M_0 = 0.3$ GeV the value of $\Delta_{l,s}$ obtained with the minimal χ^2 (χ^2/d.o.f. $= 1.14$) with the one given by the 100$^{\rm th}$ best solution (χ^2/d.o.f. $= 1.42$) of the minimization procedure.

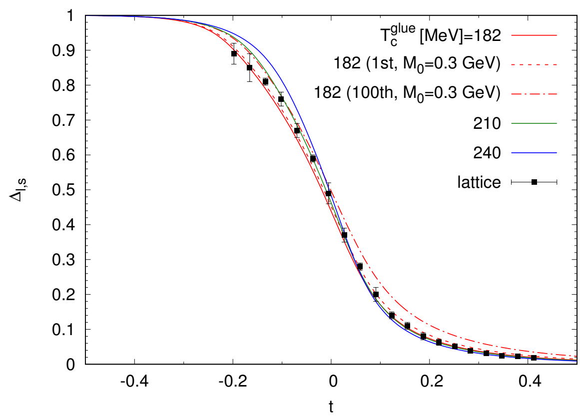

Fig. 1. The subtracted chiral condensate $\Delta_{l,s}$ at $\mu_B = 0$ as a function of the reduced temperature $t = T/T_{\rm c} - 1$.

In Fig. 2, we compare with the lattice result of [8] the pressure obtained at $\mu_B = 0$ from (3) to which we also added the thermal contribution of the π, K and $f_0^{\rm L}$ mesons. We see that the pressure overshoots the lattice data, but it is interesting to note that if one scales the pressure by a given factor

[1] For the sake of completeness, we give the parameters obtained for $T_{\rm c}^{\rm glue} = 210$ MeV: $\lambda_1 = -1.692$, $\lambda_2 = 24.279$, $h_1 = 29.274$, $h_2 = 4.625$, $h_3 = 5.227$, $g_1 = 5.635$, $g_2 = 2.551$, $g_{\rm F} = 4.671$, $\phi_{\rm N} = 0.1373$ GeV, $\phi_{\rm S} = 0.1393$ GeV, $c_1 = 1.404$ GeV, $M_0 = 0.368$ GeV, $\delta_s = 0.1135$ GeV, $m_0^2 = 2.908{\rm E}{-3}$ GeV^2, $m_1^2 = 1.575{\rm E}{-6}$ GeV^2.

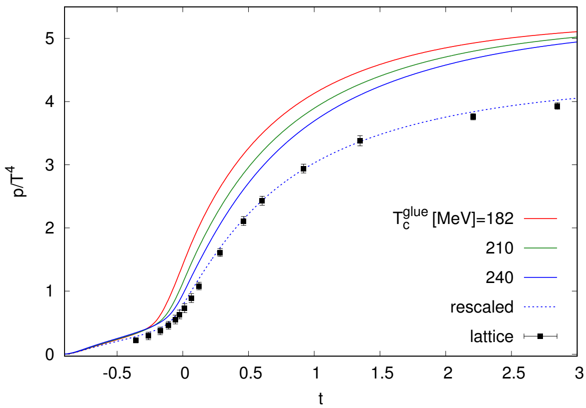

Fig. 2. (Color online) The normalized pressure as a function of the reduced temperature (see the text for the dashed/blue curve).

(0.82 for $T_c^{\rm glue} = 240$ MeV), then one almost recovers the lattice data (see the dashed/blue curve). However, as seen in Fig. 3, the interaction measure cannot be completely reproduced with this rescaling. A rescaling was also used in [9], where the reduction of the hadronic part in the pressure was motivated with a decrease in the degrees of freedom, an idea supported by a calculation in a field theoretical model constructed based on the spectral function. We use this observation just to explain why, although the pressure overshoots the lattice data, the lattice data for $c_{\rm s}^2(t) = {\rm d}p/{\rm d}\epsilon$ and $p/\epsilon(t)$ is relatively well-reproduced, as shown in Fig. 3: this is because the scaling factor drops out from these quantities.

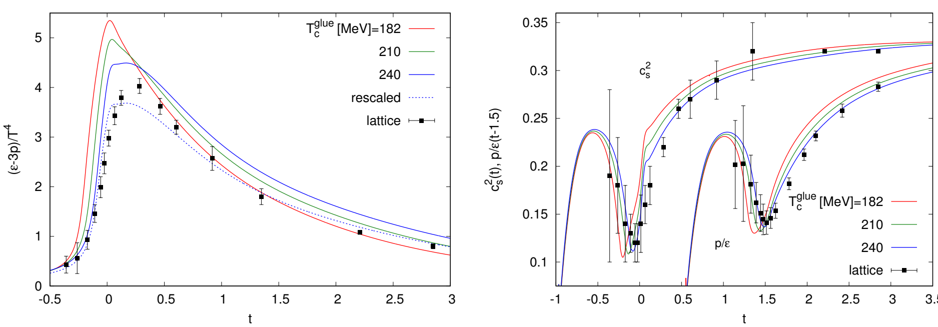

Fig. 3. The interaction measure (left panel) and the square of the speed of sound $c_{\rm s}^2$ and the appropriately shifted ratio of the pressure to the energy density p/ϵ (right panel) as a function of $t = T/T_{\rm c} - 1$ for three values of $T_{\rm c}^{\rm glue}$.

In Fig. 4, we show the phase diagram in the μ_B–T plane in comparison with the chemical freeze-out curve of [10]. Using $T_c^{\rm glue} = 210$ MeV, the CEP is located at $(\mu_B, T)_{\rm CEP} = (864, 67)$ MeV. For the curvature κ of the chiral crossover transition curve at $\mu_B = 0$, we find $\kappa = 0.0197$, which is compatible with the lattice result $\kappa = 0.020(4)$ reported in [11].

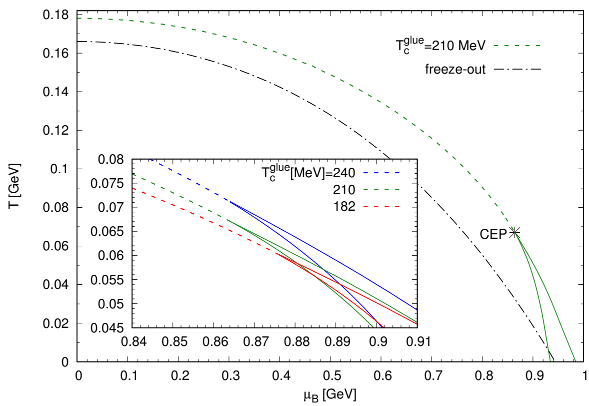

Fig. 4. The phase diagram obtained with $T_c^{\rm glue} = 210$ MeV. The inset shows the influence of the $T_c^{\rm glue}$ on the location of the CEP.

4. Conclusions

The $(2+1)$ flavor LσM extended with (axial-)vector meson, constituent quarks and Polyakov loop degrees of freedom is able to reproduce, to an acceptable degree, the QCD pressure obtained on the lattice at $\mu_B = 0$ and the thermodynamical observables derived from it, even with a rather crude approximation for the computation of the pressure, provided that the quark improved Polyakov loop potential of Ref. [4] is used. The best parametrization of the model based on curvature (pseudo)scalar meson masses and tree-level decay widths allows for the existence of a CEP in the μ_B–T plane with low T and high μ_B coordinates. Since the mass of the lightest scalar–isoscalar state $m_\sigma \equiv m_{f_0^{\rm L}}$ corresponding to the best set of parameter is below 300 MeV, an extension of the model with tetraquark states seems necessary. It would be also interesting to couple the constituent quarks to the (axial-)vector mesons in order to see the influence of the fermions on the curvature mass of these mesons.

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