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SOFT-WALL MODELLING OF MESON SPECTRA*

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The holographic methods inspired by the gauge/gravity correspondence from string theory have been actively applied to the hadron spectroscopy in the last eleven years. Within the phenomenological bottom–up approach, the linear Regge-like trajectories for light mesons are naturally reproduced in the so-called "soft-wall" holographic models. I will give a very short review of the underlying ideas and technical aspects related to the meson spectroscopy. A generalization of soft-wall description of Regge trajectories to arbitrary intercept is proposed. The problem of incorporation of the chiral symmetry breaking is discussed.

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1. Introduction

The hypothesis of AdS/CFT correspondence from the string theory [1] (also referred to as gauge/gravity duality or holographic duality) has led to unexpected and challenging ways for description of strongly coupled systems. Such descriptions are given in terms of weakly-coupled higher-dimensional gravitational theories. The holographic ideas have penetrated to many branches of physics. I will consider a traditional (and one of the first) application of AdS/CFT correspondence — the hadron physics. This field is very extensive and in the given short report, I discuss only some basic elements for description of Regge-like meson spectrum within a phenomenological bottom–up holographic approach.

The physics of hadrons composed of light quarks is highly non-perturbative. Such hadrons represent typical strongly coupled systems. The masses of hadron states appear in poles of correlation functions of QCD currents interpolating these states. The AdS/CFT correspondence provides a practical recipe for calculation of correlation functions in strongly coupled gauge

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theories via the semiclassical expansion of the action of higher-dimensional dual theory [2]. The correlation functions encode various dynamical information. If we are interested only in the mass spectrum (representing a part of this information), it is sufficient to solve the equations of motion using the plane-wave Ansatz for physical particles. The obtained infinite tower of Kaluza–Klein states represents a model for an infinite number of meson resonances expected in the large-N limit. In the bottom–up approach, one assumes the existence of a dual theory for QCD, guesses an action for this theory and uses it to do all necessary calculations in the semiclassical limit. Below, some examples are demonstrated.

2. The soft-wall model

2.1. Vector mesons

The first and mostly used in practice bottom–up holographic model that describes the linear Regge-like meson spectrum was constructed in Ref. [3]. The simplest action of this model describing the vector mesons is

$$S = -\frac{1}{4g_5^2}\int \mathrm{d}^4x\,\mathrm{d}z\sqrt{g}\,e^{-az^2}F_{MN}F^{MN}\,, \tag{1}$$

where $F_{MN} = \partial_M V_N - \partial_N V_M$, $M = 0,1,2,3,4$ (the metric is mostly negative), and g_5 represents the 5D vector coupling. Action (1) is defined in the 5D anti-de Sitter (AdS_5) space. A widely used parametrization of its metric is: $\mathrm{d}s^2 = \frac{R^2}{z^2}(\mathrm{d}x_\mu \mathrm{d}x^\mu - \mathrm{d}z^2)$. Here, R is the AdS_5 radius and $z > 0$ denotes the holographic coordinate which has the physical sense of inverse energy scale. On the boundary of the AdS_5 space, $z = \epsilon \to 0$, the vector field V_M corresponds to the source for a QCD operator interpolating the vector mesons, we will put $V_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu q$. The AdS/CFT correspondence provides various prescriptions for connections between a gauge theory and its holographic duality [2]. One of them yields the masses of dual fields in the AdS_5 space,

$$m_5^2 R^2 = (\Delta - J)(\Delta + J - 4)\,, \tag{2}$$

where Δ is the dimension of the corresponding J-form operator. In practice, J means just spin. In the case under consideration, $\Delta = 3$, $J = 1$, and we have $m_5 = 0$ in (1).

The linear Regge-like meson spectrum emerges due to the background e^{-az^2} in action (1). This background resembles the dilaton coupling in some string theories. It provides a "soft" way for introducing the mass scale a. The name "soft-wall" (SW) model is used for such actions in order to distinguish from the "hard-wall" holographic models proposed earlier [4], where the mass scale appears via a hard cutoff at some z_0 and the spectrum depends on the boundary conditions imposed at z_0. The meson spectrum of hard-wall models is not Regge-like. In the SW models, the only boundary condition is that the action must be finite.

The gauge invariance of action (1) allows to choose a convenient gauge $V_z=0$. The physical particles correspond to the plane-wave Ansatz $V_\mu(x,z)=v(q,z)e^{iqx}\varepsilon_\mu$. The equation of motion for the scalar function $v(q,z)$ following from (1) reads

$$\partial_z\left(\frac{e^{-az^2}}{z}\partial_z v\right)+\frac{e^{-az^2}}{z}q^2v=0\,. \tag{3}$$

The eigenfunctions of Eq. (3) satisfy $v_n(q,0)=0$ and the discrete mass spectrum is given by the corresponding eigenvalues $q_n^2=m_n^2$

$$m_n^2=4|a|\,(n+1)\;,\qquad n=0,1,2,\ldots \tag{4}$$

The spectrum does not depend on the sign of a. Note, however, that the choice $a<0$ results in emergence of a non-normalizable massless state [3,5]. This mode gives a finite contribution to the action as usual normalizable states and, for this reason, leads to an unphysical massless pole in the two-point vector correlator [5].

2.2. Scalar mesons

The action of SW model for free 5D scalar fields is

$$S=\tfrac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}z\sqrt{g}\,e^{-az^2}\left(\partial_M\Phi\partial^M\Phi-m_5^2\Phi^2\right)\,. \tag{5}$$

Solving the ensuing equation of motion, one arrives at the spectrum

$$m_n^2=2|a|\left(2n+1+\sqrt{4+m_5^2R^2}+\frac{a}{|a|}\right)\,,\qquad n=0,1,2,\ldots \tag{6}$$

Here, the problem with unphysical zero mode at $a<0$ does not appear since its contribution to the action is infinite. In addition, Eq. (6) suggests that if $m_5^2R^2=-4$ (the minimal allowed value of the mass squared in the AdS_5 space [6]), one has a physical massless state. The given observation provides a possibility to incorporate the pseudogoldstone π meson [7]. According to prescription (2), such a 5D mass would correspond to the canonical dimension $\Delta=2$ of the interpolating operator. The gauge-invariant local operators in QCD cannot have this dimension. One might speculate that $\Delta=2$ corresponds to the current $\partial_\mu\pi$ in the low-energy strong interactions, and that the choice $a<0$ in the dilaton background somehow mimics the dominance of pion background in the deep infrared $z\to\infty$.

2.3. Higher spin mesons

Two different descriptions of the Higher Spin Fields (HSF) were used in the SW models: the gauge-invariant one [3] and a more general description of Ref. [8]. I will discuss the first method.

The free massless HSF are described by symmetric double traceless tensors $\varPhi_{M_1\ldots M_J}$. The corresponding action is invariant under the gauge transformations $\delta\varPhi_{M_1\ldots M_J} = \nabla_{(M_1}\xi_{M_2\ldots M_J)}$, where ∇ denotes the covariant derivative with respect to the general coordinate transformations and the gauge parameter ξ is a traceless symmetric tensor. The action for free HSF in the AdS_5 space reads

$$S^{(J)} = \tfrac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}z\sqrt{g}\,e^{-az^2}\left(\nabla_N\varPhi_{M_1\ldots M_J}\nabla^N\varPhi^{M_1\ldots M_J} + \ldots\right), \tag{7}$$

where further terms are omitted. In the gauge $\varPhi_{z\ldots} = 0$, the action for a rescaled field,

$$\varPhi = \left(\frac{z}{R}\right)^{2(1-J)}\tilde{\varPhi}\,, \tag{8}$$

contains only the first kinetic term displayed in (7). The equation of motion for $\tilde{\varPhi}(x)$ results in the mass spectrum [3]

$$m_{n,J}^2 = 4a(n+J)\,, \qquad n = 0,1,2,\ldots \tag{9}$$

Note that $a > 0$ in spectrum (9). For $a < 0$, the spectrum is given by relation (4) for any J. This was another reason in favour of the choice $a > 0$ in Ref. [3]. In the framework of a more general description of HSF, the restriction $a > 0$ is not necessary [8].

The rescaled field $\tilde{\varPhi}_{M_1\ldots M_J}$ corresponds to a twist-two operator with the canonical dimension $\varDelta = J+2$ [3]. Such operators have the lowest dimension for given spin and play the dominant role in interpolating the hadron states. It is interesting to observe that action (7) for rescaled fields $\tilde{\varPhi}$ can be then written in a compact form using relation (2)

$$S^{(J)} = \tfrac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}z\sqrt{g}\left(\frac{R}{z}\right)^{R^2m_5^2} e^{-az^2}\nabla_N\tilde{\varPhi}_{M_1\ldots M_J}\nabla^N\tilde{\varPhi}^{M_1\ldots M_J}\,, \tag{10}$$

where $R^2m_5^2 = 4(J-1)$.

3. A solvable extension of SW model to arbitrary intercept

The linear Regge spectrum (9) reproduces the prediction of Veneziano amplitude and the spectral law $m_{n,J}^2 \sim n+J$ seems to agree with available experimental data on the light non-strange mesons [9]. However, the intercept of real Regge trajectories differs from that of (9)

$$m_{n,J}^2 = 4a(n+J+b)\,, \qquad n=0,1,2,\dots\,, \tag{11}$$

where b should be a phenomenological parameter regulating the phenomenological intercept. The question appears, how to reproduce spectrum (11) in simple solvable SW models? A seemingly straightforward idea is to change the 5D masses. However, these masses are already fixed by the requirement that we interpolate the meson states by the quark currents of lowest canonical dimension. It looks more reasonable to modify the dilaton background in the SW action. A method for derivation of modified background that leads to spectrum (11) in the vector case $J=1$ was proposed in Ref. [10]. This method can be extended to arbitrary spin. Below, we give the final answer for $a>0$ that generalizes expression (10)

$$S^{(J)} = \frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}z\sqrt{g}\left(\frac{R}{z}\right)^{R^2m_5^2} U^2\left(b,-|J-1|;az^2\right)e^{-az^2}\mathcal{L}^{(J)}\,, \tag{12}$$

where U is the Tricomi hypergeometric function, $J=0,1,2,\dots$, and we must set $R^2m_5^2=2$ in the case of $J=0$. The HSF in the Lagrangian $\mathcal{L}^{(J)}$ are implied in the rescaled form (8) as in (10). It should be noted that the background in (7) is universal for any spin J. However, in terms of "physical" rescaled fields (8), it becomes spin-dependent as can be seen in (10). The generalized background in action (12) is also spin-dependent.

4. On the chiral symmetry breaking

In the presented approach, the hadron masses depend on the dimension Δ and spin J of the corresponding interpolating operator. This leads to degeneracy of states with opposite space parity when they are described by operators with equal Δ and J, *e.g.* the ρ and a_1 mesons. It is commonly accepted that the observable large mass splittings between such states are caused by the Chiral Symmetry Breaking (CSB) in QCD. The hard-wall model incorporates the CSB in much the same way as traditional chiral effective theories [4]. Such a description of the CSB dynamics does not work however, in simple SW models [3]. The description of CSB in Ref. [4] requires introduction of cubic, quartic and higher vertices which give rise to meson decays and interactions. As the CSB effect is (most likely) not suppressed by large number of colours N, it is difficult to adjust such a description with the large-N limit, in which the holographic duality is formulated. In addition, this description does not provide the mass splittings between parity partners among higher spin mesons in which apparently the splittings are of the same order as between ρ and a_1 mesons [9].

In my opinion, a self-consistent holographic description of CSB should be based on quadratic in fields 5D actions. But this is an open problem. A solution might lie in reinterpretation of what we call spin in the holographic models. An interesting possibility was put forward by the light-front holographic QCD [8] where, similarly to the non-relativistic potential models, the mass degeneracy of parity partners is lifted due to different orbital momentum of quark constituents. An alternative option could consist in interpretation of intercept parameter b in spectrum (11): As was advocated in Ref. [10] for vector mesons, the sign of b depends on parity.

5. Conclusion

In summary, the bottom–up holographic approach is an interesting and mathematically nice method for description of excited hadron spectrum even if in reality, it is not rigorously related with some underlying string theory.

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