Vol. 9 (2016) *Acta Physica Polonica B Proceedings Supplement* No 3

ATTEMPT TO DETERMINE MASS DIFFERENCES $m_{K_{\rm L}} - m_{K_{\rm S}}$ AND $m_{K_2} - m_{K_1}$ FROM CPLEAR DATA ON SEMI-LEPTONIC DECAY OF K^0 AND $\bar{K}^0$*

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(Received July 19, 2016)

It is demonstrated that the theoretical formula for asymmetry, which has been used to determine mass difference $m_{K_{\rm L}} - m_{K_{\rm S}}$ by its comparison with the CPLEAR Collaboration data, is incorrect. If one considers the $K^0 \leftrightarrow \bar{K}^0$ oscillations through $K_{\rm L}$ and $K_{\rm S}$ mesons to be defined in order to take into account CP violation in K-meson physics, the correct theoretical formula for asymmetry is calculated and by its comparison with the CPLEAR data, slightly different value for $m_{K_{\rm L}} - m_{K_{\rm S}}$ is expected to be found. If $K^0 \leftrightarrow \bar{K}^0$ oscillations through K_2 and K_1 mesons, which reflect CP conservation in K-meson physics, are calculated, formally the identical theoretical formula for asymmetry is found with that used by the CPLEAR Collaboration, however, it depends on $m_{K_2} - m_{K_1}$ mass difference and also on the Γ_{K_2} and Γ_{K_1} decay widths as unknown parameters. We expect that by a comparison of the latter with the CPLEAR Collaboration data, one can, in principle, find mass difference $m_{K_2} - m_{K_1}$ and also the decay widths of K_2, K_1 mesons for the first time.

DOI:10.5506/APhysPolBSupp.9.627

In paper [1], the following theoretical expression for the asymmetry, assuming a validity of the $\Delta S = \Delta Q$ rule,

$$A_{\rm th}(t) = \frac{2 \times \cos[(m_{K_{\rm L}} - m_{K_{\rm S}})t] e^{-\frac{\left(\Gamma_{K_{\rm S}} + \Gamma_{K_{\rm L}}\right)}{2} t}}{e^{-\Gamma_{K_{\rm S}} t} + e^{-\Gamma_{K_{\rm L}} t}} \tag{1}$$

* Presented at “Excited QCD 2016”, Costa da Caparica, Lisbon, Portugal, March 6–12, 2016.

has been used to determine the mass difference $m_{K_L} - m_{K_S}$ from the CPLEAR Collaboration data (Fig. 1), which have been obtained by measurements of the semi-leptonic decay rates of neutral K mesons

$$\begin{aligned} R_+(\tau) &= R\left(K^0_{t=0} \to \pi^- e^+ \nu_{t=\tau}\right) , && (2)\\ \bar{R}_-(\tau) &= R\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_{t=\tau}\right) , && (3)\\ R_-(\tau) &= R\left(K^0_{t=0} \to \pi^+ e^- \bar{\nu}_{t=\tau}\right) , && (4)\\ \bar{R}_+(\tau) &= R\left(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_{t=\tau}\right) , && (5) \end{aligned}$$

"tagging" the "strangeness" S of neutral K mesons by the charge sign of the "positron" and "electron" in final state, and afterwards an evaluation of the relation

$$A_{\rm exp}(\tau) = \frac{\left[R_+(\tau) + \bar{R}_-(\tau)\right] - \left[\bar{R}_+(\tau) + R_-(\tau)\right]}{\left[R_+(\tau) + \bar{R}_-(\tau)\right] + \left[\bar{R}_+(\tau) + R_-(\tau)\right]} , \qquad (6)$$

where τ is the decay eigen-time of the neutral K^0_S meson. Further, we show that the applied formula (1) is incorrect.

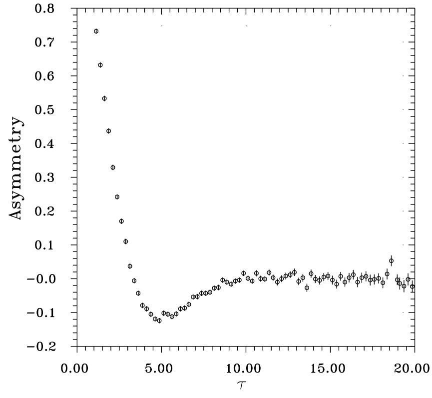

Fig. 1. The data on asymmetry to be obtained by the CPLEAR Collaboration.

The neutral K mesons K^0 and $\bar{K}^0$, with the "strangeness" $S = +1$ and $S = -1$, respectively, are produced by the $\bar{p}$ annihilation at rest in a hydrogen target $\bar{p}p \to K^-\pi^+K^0$ or $K^+\pi^-\bar{K}^0$ (CPLEAR at CERN) each having the branching ratio 2×10^{-3}. The "strangeness" S of these K mesons is "tagged" by measuring the charge sign of the accompanying $K^\pm$ mesons, therefore, it is known "event by event". The quantum number "strangeness" S is conserved in strong and EM interactions. The violation of S in weak interactions is responsible not only for decays of K^0 and $\bar{K}^0$ mesons, but also gives rise to the so-called "oscillations" of neutral K mesons $K^0 \leftrightarrow \bar{K}^0$

in time. Both K^0 and $\bar{K}^0$ can decay into two pions $\pi^0\pi^0$, $\pi^+\pi^-$ and also into three pions $\pi^0\pi^0\pi^0$, $\pi^+\pi^-\pi^0$, whereby these pion systems possess well-defined "CP-parity", +1 and −1, respectively. However, neither K^0 nor $\bar{K}^0$ are eigen-states of CP operator, because $\mathrm{CP}|K^0\rangle = -|\bar{K}^0\rangle$ and $\mathrm{CP}|\bar{K}^0\rangle = -|K^0\rangle$.

On account of this reason, new particles K_1^0 and K_2^0 have been defined to exist as a superposition of K^0 and $\bar{K}^0$

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right), \qquad |K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \tag{7}$$

with definite CP parity values

$$\mathrm{CP}\,|K_1^0\rangle = +\,|K_1^0\rangle, \qquad \mathrm{CP}\,|K_2^0\rangle = -\,|K_2^0\rangle. \tag{8}$$

As a result of the latter, K_1^0 can decay into two pions and K_2^0 can decay into three pions. But Christenson, Cronin, Fitch and Turlay in 1964 [2] have revealed the decay of K_2^0 into two pions $K_2^0 \to \pi^+\pi^-, \pi^0\pi^0$ with some small probability, which violates CP invariance.

As a consequence, another two neutral K mesons, K_S^0 and K_L^0 to be a linear combinations of K_1^0 and K_2^0, have been introduced to exist

$$\begin{aligned} |K_\mathrm{S}^0\rangle &= \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_1^0\rangle + \varepsilon\,|K_2^0\rangle\right), \\ |K_\mathrm{L}^0\rangle &= \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\varepsilon\,|K_1^0\rangle + |K_2^0\rangle\right), \end{aligned} \tag{9}$$

where ε is a complex CP-violation parameter with $|\varepsilon| = 2,3 \times 10^{-3}$ and the CP-violation phase $\Phi = 43,5^\circ$.

Substituting (7) into (9), one obtains relations between K_S^0, K_L^0 and K^0, $\bar{K}^0$

$$\begin{aligned} |K_\mathrm{S}^0\rangle &= \frac{1}{\sqrt{2}\sqrt{1+|\,\varepsilon\,|^2}}\left[(1+\varepsilon)\,|K^0\rangle - (1-\varepsilon)\,|\bar{K}^0\rangle\right], \\ |K_\mathrm{L}^0\rangle &= \frac{1}{\sqrt{2}\sqrt{1+|\,\varepsilon\,|^2}}\left[(1+\varepsilon)\,|K^0\rangle + (1-\varepsilon)\,|\bar{K}^0\rangle\right]. \end{aligned} \tag{10}$$

The inverse to them relations take the forms

$$\begin{aligned} |K^0\rangle &= \frac{\sqrt{1+|\,\varepsilon\,|^2}}{\sqrt{2}(1+\varepsilon)}\left[|K_\mathrm{S}^0\rangle + |K_\mathrm{L}^0\rangle\right], \\ |\bar{K}^0\rangle &= \frac{\sqrt{1+|\varepsilon|^2}}{\sqrt{2}(1-\varepsilon)}\left[-\,|K_\mathrm{S}^0\rangle + |K_\mathrm{L}^0\rangle\right], \end{aligned} \tag{11}$$

which are suitable for a calculation of the theoretical expression for the asymmetry containing the mass difference of $K^0_{\rm L}$ and $K^0_{\rm S}$ mesons as free parameter.

The time evolution of state vectors $|K^0_{\rm S}\rangle$ and $|K^0_{\rm L}\rangle$ is given by the expressions

$$\begin{aligned} \left|K^0_{\rm S}(t)\right\rangle &= e^{-im_{\rm S}t-\mathit{\Gamma}_{\rm S}/2t}\left|K^0_{\rm S}(0)\right\rangle , \\ \left|K^0_{\rm L}(t)\right\rangle &= e^{-im_{\rm L}t-\mathit{\Gamma}_{\rm L}/2t}\left|K^0_{\rm L}(0)\right\rangle , \end{aligned} \quad (12)$$

with $m_{\rm S}, m_{\rm L}$ and $\mathit{\Gamma}_{\rm S}, \mathit{\Gamma}_{\rm L}$ the masses and decay widths of $K^0_{\rm S}, K^0_{\rm L}$, respectively, and finally for $|\ K^0(t)\rangle$ and $|\ \bar{K}^0(t)\rangle$, one can then write

$$\begin{aligned} \left|K^0(t)\right\rangle &= \frac{1}{2}\Bigg[\left(e^{-im_{\rm S}t-\mathit{\Gamma}_{\rm S}/2t}+e^{-im_{\rm L}t-\mathit{\Gamma}_{\rm L}/2t}\right)\left|K^0(0)\right\rangle \\ &\quad +\frac{(1-\varepsilon)}{(1+\varepsilon)}\left(-e^{-im_{\rm S}t-\mathit{\Gamma}_{\rm S}/2t}+e^{-im_{\rm L}t-\mathit{\Gamma}_{\rm L}/2t}\right)\left|\bar{K}^0(0)\right\rangle\Bigg], \quad (13) \\ \left|\bar{K}^0(t)\right\rangle &= \frac{1}{2}\Bigg[\frac{(1+\varepsilon)}{(1-\varepsilon)}\left(-e^{-im_{\rm S}t-\mathit{\Gamma}_{\rm S}/2t}+e^{-im_{\rm L}t-\mathit{\Gamma}_{\rm L}/2t}\right)\left|K^0(0)\right\rangle \\ &\quad +\left(e^{-im_{\rm S}t-\mathit{\Gamma}_{\rm S}/2t}+e^{-im_{\rm L}t-\mathit{\Gamma}_{\rm L}/2t}\right)\left|\bar{K}^0(0)\right\rangle\Bigg]. \quad (14) \end{aligned}$$

The probability that the K^0 meson produced at the moment $t=0$ will be at the moment $t\neq 0$ in the state of $\bar{K}^0$ meson is given by the absolute value squared of the product $\langle K^0(0)\ |\ \bar{K}^0(t)\rangle$, *i.e.*

$$P\Big(K^0(0)\to\bar{K}^0(t)\Big) = \frac{1}{4}\frac{\left(1+|\varepsilon|^2+2{\rm Re}\,\varepsilon\right)}{\left(1+|\varepsilon|^2-2{\rm Re}\,\varepsilon\right)}\left[e^{-\mathit{\Gamma}_{\rm S}t}+e^{-\mathit{\Gamma}_{\rm L}t}-2\cos[(m_{\rm L}-m_{\rm S})t]e^{-\frac{(\mathit{\Gamma}_{\rm S}+\mathit{\Gamma}_{\rm L})}{2}t}\right], \quad (15)$$

whereby the orthogonality of $|K^0(0)\rangle$ and $|\bar{K}^0(0)\rangle$ states is exploited.

Similarly, the probability of inverse transition is

$$P\Big(\bar{K}^0(0)\to K^0(t)\Big) = \frac{1}{4}\frac{\left(1+|\varepsilon|^2-2{\rm Re}\,\varepsilon\right)}{\left(1+|\varepsilon|^2+2{\rm Re}\,\varepsilon\right)}\left[e^{-\mathit{\Gamma}_{\rm S}t}+e^{-\mathit{\Gamma}_{\rm L}t}-2\cos[(m_{\rm L}-m_{\rm S})t]e^{-\frac{(\mathit{\Gamma}_{\rm S}+\mathit{\Gamma}_{\rm L})}{2}t}\right]. \quad (16)$$

One can see immediately that $P(K^0(0)\to\bar{K}^0(t))\neq P(\bar{K}^0(0)\to K^0(t))$ as now we consider CP violation.

In order to calculate the theoretical asymmetry, one has to calculate also $P(K^0(0) \to K^0(t))$ and $P(\bar{K}^0(0) \to \bar{K}^0(t))$ in a similar way.

They are

$$P\Big(K^0(0) \to K^0(t)\Big) \equiv P\Big(\bar{K}^0(0) \to \bar{K}^0(t)\Big) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2\cos\left[(m_L - m_S)\,t\right] e^{-\frac{(\Gamma_S+\Gamma_L)}{2}t}\right] . \quad (17)$$

Such asymmetry is then [3]

$$A_{\rm th}^{\rm CP\ viol}(t) = \frac{2\cos\left[(m_L - m_S)\,t\right] e^{-\frac{(\Gamma_S+\Gamma_L)}{2}t} - \frac{4{\rm Re}^2\varepsilon}{(1+|\varepsilon|^2)^2}\left(e^{-\Gamma_S t}+e^{-\Gamma_L t}\right)}{\left(e^{-\Gamma_S t}+e^{-\Gamma_L t}\right) - \frac{4{\rm Re}^2\varepsilon}{(1+|\varepsilon|^2)^2} 2\cos\left[(m_L - m_S)\,t\right] e^{-\frac{(\Gamma S+\Gamma_L)}{2}}t} , \quad (18)$$

however, completely different from (1), so its comparison with the CPLEAR data have to give different value for the mass difference $m_{K_L} - m_{K_S}$.

If we would like to calculate the oscillations of neutral K mesons through the K_1^0 and K_2^0, one has to start with the relations

$$\left|K^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K_1^0\right\rangle + \left|K_2^0\right\rangle\right) , \qquad \left|\bar{K}^0\right\rangle = \frac{1}{\sqrt{2}}\left(-\left|K_1^0\right\rangle + \left|K_2^0\right\rangle\right) , \quad (19)$$

which are just the inverse transformations to (7) by means of which the existence of $|K_1^0\rangle$ and $|K_2^0\rangle$ has been introduced.

The time dependence of state vectors of K_1^0, K_2^0 is

$$\begin{aligned} \left|K_1^0(t)\right\rangle &= e^{-im_1 t - \Gamma_1/2t}\left|K_1^0(0)\right\rangle , \\ \left|K_2^0(t)\right\rangle &= e^{-im_2 t - \Gamma_2/2t}\left|K_2^0(0)\right\rangle , \end{aligned} \quad (20)$$

with m_1, m_2 and Γ_1, Γ_2 the masses and decay widths of K_1^0, K_2^0, respectively.

Then, for the state vectors $|\,K^0(t)\rangle, |\,\bar{K}^0(t)\rangle$, one can write expressions

$$\begin{aligned} \left|K^0(t)\right\rangle &= \frac{1}{2}\left[e^{-im_1 t-\Gamma_1/2t} + e^{-im_2 t-\Gamma_2/2t}\right]\left|K^0(0)\right\rangle \\ &\quad + \frac{1}{2}\left[e^{-im_2 t-\Gamma_2/2t} - e^{-im_1 t-\Gamma_1/2t}\right]\left|\bar{K}^0(0)\right\rangle , \end{aligned} \quad (21)$$

$$\begin{aligned} \left|\bar{K}^0(t)\right\rangle &= \frac{1}{2}\left[e^{-im_2 t-\Gamma_2/2t} - e^{-im_1 t-\Gamma_1/2t}\right]\left|K^0(0)\right\rangle \\ &\quad + \frac{1}{2}\left[e^{-im_1 t-\Gamma_1/2t} + e^{-im_2 t-\Gamma_2/2t}\right]\left|\bar{K}^0(0)\right\rangle \end{aligned} \quad (22)$$

to be ready for calculations of the $K^0 \leftrightarrow \bar{K}^0$ oscillations.

In order to find an explicit form of the theoretical asymmetry $A_{\rm th}^{\rm CP\,consd}(t)$, one has to calculate probabilities of the following transitions: $P(K^0(0) \to \bar{K}^0(t)), P(\bar{K}^0(0) \to K^0(t)), P(K^0(0) \to K^0(t))$ and $P(\bar{K}^0(0) \to \bar{K}^0(t))$.

The probability that the K^0 meson produced at the moment $t=0$ will be at the moment $t \neq 0$ in the state of $\bar{K}^0$ meson is given by the absolute value squared of the product $\langle K^0(0) \mid\mid \bar{K}^0(t)\rangle$.

Similarly is the reversed probability, whereby the orthogonality of $K^0(0)$ and $\bar{K}^0(0)$ states is exploited.

The result is

$$\begin{aligned} & P\Big(K^0(0) \to \bar{K}^0(t)\Big) \equiv P\Big(\bar{K}^0(0) \to K^0(t)\Big) \\ & = \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2\cos\left[(m_2 - m_1)\,t\right] e^{-\frac{(\Gamma_1+\Gamma_2)}{2}t}\right] \end{aligned} \tag{23}$$

as we consider the CP invariance which creates T invariance because of the CPT conservation.

One has to calculate also $P(K^0(0) \to K^0(t))$ and $P(\bar{K}^0(0) \to \bar{K}^0(t))$ transitions in a similar way, which are taking the following form

$$\begin{aligned} & P\Big(K^0(0) \to K^0(t)\Big) \equiv P\Big(\bar{K}^0(0) \to \bar{K}^0(t)\Big) \\ & = \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2\cos\left[(m_2 - m_1)\,t\right] e^{-\frac{(\Gamma_1+\Gamma_2)}{2}t}\right]. \end{aligned} \tag{24}$$

Substituting all these probabilities of the corresponding transitions into the theoretical asymmetry

$$A = \frac{\left[P_{K^0(0)\to K^0(t)} + P_{\bar{K}^0(0)\to \bar{K}^0(t)}\right] - \left[P_{\bar{K}^0(0)\to K^0(t)} + P_{K^0(0)\to \bar{K}^0(t)}\right]}{\left[P_{K^0(0)\to K^0(t)} + P_{\bar{K}^0(0)\to \bar{K}^0(t)}\right] + \left[P_{\bar{K}^0(0)\to K^0(t)} + P_{K^0(0)\to \bar{K}^0(t)}\right]}, \tag{25}$$

where $A \equiv A_{\rm th}(t)$ and finally, the following expression

$$A_{\rm th}^{\rm CP\,consd}(t) = \frac{2 \times \cos\left[(m_2 - m_1)\,t\right] e^{-\frac{(\Gamma_1+\Gamma_2)}{2}t}}{e^{-\Gamma_1 t} + e^{-\Gamma_2 t}} \tag{26}$$

is found, which formally is identical with formula (1), but depending on the mass difference $m_{K_2} - m_{K_1}$ and moreover, also on the widths Γ_{K_2} and Γ_{K_1} as free parameters.

So, we expect that by a comparison of this expression with the CPLEAR Collaboration data, one can determine $m_{K_2} - m_{K_1}$, Γ_{K_1} and Γ_{K_2}, for the first time.

Unfortunately, the CPLEAR Collaboration data have never been published in a numerical form and they are presented only in the figure in [1].

The support of the Slovak Grant Agency for Sciences VEGA under grant No. 2/0158/13 and of the Slovak Research and Development Agency under the contract No. APVV-0463-12 is acknowledged by S.D. and A.Z.D.

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