NEUTRINOS AND THEIR INTERACTIONS IN THE STANDARD MODEL*

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A basic introduction to the theory of neutrino interactions with matter is presented. I review the relevant ingredients of the Standard Model of electroweak and strong interactions, highlighting the role of the flavor structure of electroweak currents and chiral symmetry. The general expression of the inclusive neutrino-interaction cross section is also derived and discussed. Charged-current quasielastic scattering on nucleons and the derivation of the nucleon weak current in terms of form factors are considered in detail.

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1. Introduction

Neutrino interaction are at the heart of many relevant processes in astrophysics, nuclear and particle physics. First and foremost, neutrino interactions are our doorway to neutrino properties. They have allowed to detect neutrinos from many different sources, identify their flavor and discover neutrino oscillations. The presence of oscillations implies that neutrinos have non-zero albeit small masses. Oscillation experiments, currently evolving from the discovery to the precision stage, require a good understanding of neutrino interactions at the detectors to distinguish signal from background and minimize systematic uncertainties. The aim is to establish the neutrino mass ordering and discover CP violation in the lepton sector. These experiments might also reveal the presence of physics beyond the Standard Model in the form of non-standard neutrino interactions or the existence of sterile neutrinos. Hadron physics might also benefit from more precise neutrino cross-section measurements as a source of information about the axial structure of the nucleon and baryon resonances.

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After this introduction, I summarize the properties of electroweak interactions for leptons and quarks. Next, I discuss strong interactions paying special attention to some of the approximate symmetries of QCD such as flavor and chiral symmetries, which are important for the modeling of lepton–hadron scattering. This is followed by a general description of the inclusive cross sections for electroweak processes. The focus is then set on charged current quasielastic scattering on the nucleon and the derivation of the nucleon electroweak current. In particular, it is shown how weak vector form factors can be related to electromagnetic ones.

In the preparation of this article, I have made extensive use of the monograph about the structure of the nucleon written by Thomas and Weise [1]. Useful information and inspiration has also been found in several monographs, text books and review articles [2–6].

2. Electroweak interactions in the Standard Model

After the spontaneous breaking of the SU(2) × U(1) gauge symmetry through the Higgs mechanism, the Lagrangian density of the electroweak sector of the Standard Model (SM) can be cast as

$$\mathcal{L}_{\mathrm{EW}} = -eJ^{\mu}_{\mathrm{EM}}A_{\mu} - \frac{g}{2\cos\theta_{\mathrm{W}}}J^{\mu}_{\mathrm{NC}}Z_{\mu} - \frac{g}{2\sqrt{2}}J^{\mu}_{\mathrm{CC}}W^{\dagger}_{\mu} + \mathrm{h.c.} \tag{1}$$

Massless photons act as mediators of the electromagnetic (EM) interactions among charged particles (quarks and leptons except neutrinos). Massive vector $W^{\pm}$ and Z bosons are responsible for charged current (CC) and neutral current (NC) electroweak interactions. The strength of the EM interaction e is related to the weak coupling g by the weak angle θ_{W}, which also defines the ratio of vector boson masses

$$\sin\theta_{\mathrm{W}} = \frac{e}{g}\,, \qquad \cos\theta_{\mathrm{W}} = \frac{M_W}{M_Z}\,. \tag{2}$$

These interactions can be written in terms of currents coupled to the corresponding gauge bosons. In the lepton sector

$$J^{\mu}_{\mathrm{EM}} = \bar{l}_i\gamma^{\mu}l_i\,, \qquad i = e,\mu,\tau\,, \tag{3}$$

$$J^{\mu}_{\mathrm{CC}} = \bar{\nu}_i\gamma^{\mu}\left(1-\gamma_5\right)l_i\,, \tag{4}$$

$$J^{\mu}_{\mathrm{NC}} = \tfrac{1}{2}\bar{l}_i\gamma^{\mu}\left(g_{\mathrm{V}} - g_{\mathrm{A}}\gamma_5\right)l_i + \tfrac{1}{2}\bar{\nu}_i\gamma^{\mu}\left(1-\gamma_5\right)\nu_i\,. \tag{5}$$

The EM current is a Lorentz vector, while the weak ones have the well-known V$-$A structure responsible for parity violation. Notice that flavor changing NC are absent in the SM. The NC couplings of charged leptons

$$g_{\mathrm{V}} = -1 + 4\sin^2\theta_{\mathrm{W}}\,, \qquad g_{\mathrm{A}} = -1 \tag{6}$$

are such that $|g_{\mathrm{V}}| \approx 0.04 \ll |g_{\mathrm{A}}|$ so that the axial part is dominant.

With these ingredients, it is straightforward to calculate (anti)neutrino–electron elastic scattering amplitudes at tree level, which is precise enough for most applications in neutrino physics due to the weakness of the interaction. Unlike the case for $\overset{(-)}{\nu}_{\mu,\tau}$, $\overset{(-)}{\nu}_e\, e^- \to \overset{(-)}{\nu}_e\, e^-$ proceeds not only via NC but also through CC interactions. This difference is responsible for matter effects in neutrino oscillations.

In the quark sector, one has

$$\begin{aligned} J^\mu_{\rm EM} &= Q_i \bar{q}_i \gamma^\mu q_i = \tfrac{2}{3}\bar{q}_u\gamma^\mu q_u - \tfrac{1}{3}\left(\bar{q}_d\gamma^\mu q_d + \bar{q}_s\gamma^\mu q_s\right) + \dots\,, \qquad (7)\\ J^\mu_{\rm NC} &= \bar{q}_u\gamma^\mu\left[\tfrac{1}{2} - \left(\tfrac{2}{3}\right)2\sin^2\theta_{\rm W} - \tfrac{1}{2}\gamma_5\right]q_u + (u\to c) + (u\to t)\\ &\quad + \bar{q}_d\gamma^\mu\left[-\tfrac{1}{2} - \left(-\tfrac{1}{3}\right)2\sin^2\theta_{\rm W} + \tfrac{1}{2}\gamma_5\right]q_d + (d\to s) + (d\to b) \qquad (8) \end{aligned}$$

which are diagonal in flavor, in contrast to CC where flavor is mixed

$$J^\mu_{\rm CC} = \left(\bar{q}_u\bar{q}_c\bar{q}_t\right)\gamma^\mu\left(1-\gamma_5\right)U\begin{pmatrix} q_d\\ q_s\\ q_b\end{pmatrix}. \qquad (9)$$

For two families, the Cabibbo–Kobayashi–Maskawa matrix U reduces to a 2×2 rotation matrix defined by a single Cabibbo angle $\theta_{\rm C}\approx 13^\circ$. Such a mixing implies that strangeness can be produced in neutrino–proton interactions, for example

$$\begin{aligned} W^-\,p(u\boldsymbol{u}d) &\to n(u\boldsymbol{d}d) & &\sim \cos^2\theta_{\rm C}\,,\\ W^-\,p(u\boldsymbol{u}d) &\to \Lambda(u\boldsymbol{s}d) & &\sim \sin^2\theta_{\rm C}\,,\\ W^-\,p(u\boldsymbol{u}d) &\to p(uud)\,K^-(\bar{u}\boldsymbol{s}) & &\sim \sin^2\theta_{\rm C}\,, \end{aligned}$$

albeit at the price of much smaller cross sections because $\sin^2\theta_{\rm C}\ll\cos^2\theta_{\rm C}$.

3. Strong interactions in the Standard Model

Although neutrinos do not engage in strong interactions, they collide with strong interacting matter, so these interactions play an indirect but nonetheless essential role. Strong interactions in the SM are characterized by a gauge theory with color SU(3) symmetry

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_q\left(i\gamma^\mu D_\mu - m_q\right)\psi_q - \tfrac{1}{4}G^{\mu\nu}_a G_{a\mu\nu}\,, \qquad q = u,d,s,\dots\,, \qquad a = 1-8 \qquad (10)$$

with

$$D_\mu\psi = \left(\partial_\mu - ig\frac{\lambda_a}{2}A^a_\mu\right)\psi\,, \qquad G^{\mu\nu}_a = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a + g f_{abc}A^\mu_b A^\nu_c \qquad (11)$$

written in terms of 3 colored quark fields (for each flavor) and 8 gluons. In contrast to electroweak interactions, QCD has a running coupling which is large at low energies but becomes small at high ones (asymptotic freedom). Perturbation theory cannot be then used to describe hadrons. It is also confining: free quarks and gluons are not observed as asymptotic states.

Except for very high-energy neutrinos, like those observed at IceCube, neutrino scattering with hadronic matter has to be modeled using effective descriptions of strong interactions, in terms of the actual degrees of freedom at low energy, which are not quarks and gluons but mesons and baryons. Nevertheless, the QCD Lagrangian exhibits several (approximate) symmetries that should be taken into account in any realistic description of neutrino collisions with strongly interacting particles.

3.1. Approximate symmetries of QCD: $SU(N_f)$

Let us consider the hypothetical case of $N_\mathrm{f} = 3$ with equal masses: $m_u = m_d = m_s$. In this limit, QCD has a global SU(3) flavor symmetry[1] and 8 conserved currents

$$V_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q \Leftrightarrow \partial_\mu V_a^\mu = 0\,, \qquad a = 1-8\,, \qquad \bar{q} = (\bar{q}_u\, \bar{q}_d\, \bar{q}_s)\ , \qquad q = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix}, \tag{12}$$

where λ_{1-8} denote the Gell-Mann matrices. These masses, however, are actually different. At the scale of 1 GeV,

$$\begin{aligned} m_u(1\ \mathrm{GeV}) &= 4 \pm 2\ \mathrm{MeV}\,, \\ m_d(1\ \mathrm{GeV}) &= 8 \pm 4\ \mathrm{MeV}\,, \\ m_s(1\ \mathrm{GeV}) &= 164 \pm 33\ \mathrm{MeV}\,. \end{aligned} \tag{13}$$

In this situation, flavor currents are not conserved but, instead, one has that

$$\partial_\mu V_a^\mu = \bar{q}\left[m, \frac{\lambda_a}{2}\right] q\,, \qquad m = \mathrm{diag}(m_u, m_d, m_s)\,. \tag{14}$$

From the quark mass values listed in Eq. (13), it is clear that flavor symmetry is a better approximation for $N_\mathrm{f} = 2$. This is nothing but isospin symmetry ($m_u = m_d$) for which

$$V_a^\mu = \bar{q}'\gamma^\mu \frac{\tau_a}{2} q' \Leftrightarrow \partial_\mu V_a^\mu = 0\,, \qquad a = 1-3\,, \qquad q' = \begin{pmatrix} q_u \\ q_d \end{pmatrix}\ ; \tag{15}$$

τ_{1-3} are the Pauli matrices.

[1] Not to be confused with the local SU(3) color symmetry discussed above.

These symmetries have implications for the electroweak currents that become apparent when we consider their flavor structure. Restricting ourselves to three flavors, $J_{\rm EM}$ of Eq. (7) can be cast as

$$\begin{aligned} J^\mu_{\rm EM} &= \frac{2}{3}\bar{q}_u\gamma^\mu q_u - \frac{1}{3}\left(\bar{q}_d\gamma^\mu q_d + \bar{q}_s\gamma^\mu q_s\right) = \bar{q}Q\gamma^\mu q \\ &= \frac{1}{2}\bar{q}\frac{\lambda_8}{\sqrt{3}}\gamma^\mu q + \bar{q}\frac{\lambda_3}{2}\gamma^\mu q = \frac{1}{2}V^\mu_Y + V^\mu_3\,, \end{aligned} \tag{16}$$

where $Q = \mathrm{diag}(2/3, -1/3, -1/3)$; V^μ_Y is known as the hypercharge current. For the vector part of $J_{\rm CC}$, Eq. (9),

$$V^\mu_{\rm CC} = \bar{q}_u\gamma^\mu\left(q_d\cos\theta_{\rm C} + q_s\sin\theta_{\rm C}\right)\,, \tag{17}$$

one has that

$$\bar{q}_u\gamma^\mu q_d = \bar{q}\gamma^\mu\frac{\lambda_1 + i\lambda_2}{2}q = V^\mu_1 + iV^\mu_2\,, \tag{18}$$

$$\bar{q}_u\gamma^\mu q_s = \bar{q}\gamma^\mu\frac{\lambda_4 + i\lambda_5}{2}q = V^\mu_4 + iV^\mu_5\,. \tag{19}$$

Analogously, the vector part of the NC, Eq. (8),

$$\begin{aligned} V^\mu_{\rm NC} &= \bar{q}_u\gamma^\mu\left[\tfrac{1}{2} - \left(\tfrac{2}{3}\right)2\sin^2\theta_{\rm W}\right]q_u \\ &\quad + \bar{q}_d\gamma^\mu\left[-\tfrac{1}{2} - \left(-\tfrac{1}{3}\right)2\sin^2\theta_{\rm W}\right]q_d + (d\to s) \\ &= \left(1 - 2\sin^2\theta_{\rm W}\right)V^\mu_3 - 2\sin^2\theta_{\rm W}\tfrac{1}{2}V^\mu_Y - \tfrac{1}{2}\bar{q}_s\gamma^\mu q_s\,. \end{aligned} \tag{20}$$

We can see from these expressions that part of the EM and NC, as well as the CC, are components V_{1-5} of the same flavor current, which is conserved in the limit of equal quark masses. In the same way, for only two flavors, CC and the isovector part of the EM current can be obtained from one another by isospin rotations. This is a remarkable property that allows to relate the vector hadronic form factors (FF) that appear in neutrino-induced reactions to EM FF present in electron scattering.

A similar exercise can be performed for the axial current. Indeed

$$A^\mu_{\rm CC} = \bar{q}_u\gamma^\mu\gamma_5\left(q_d\cos\theta_{\rm C} + q_s\sin\theta_{\rm C}\right) \tag{21}$$

so that

$$\bar{q}_u\gamma^\mu\gamma_5 q_d = \bar{q}\gamma^\mu\gamma_5\frac{\lambda_1 + i\lambda_2}{2}q = A^\mu_1 + iA^\mu_2\,, \tag{22}$$

$$\bar{q}_u\gamma^\mu\gamma_5 q_s = \bar{q}\gamma^\mu\gamma_5\frac{\lambda_4 + i\lambda_5}{2}q = A^\mu_4 + iA^\mu_5\,, \tag{23}$$

and for the NC,

$$\begin{aligned} A^\mu_{\rm NC} &= \bar{q}_u \tfrac{1}{2}\gamma^\mu\gamma_5 q_u - \bar{q}_d \tfrac{1}{2}\gamma^\mu\gamma_5 q_d - (d \to s) \\ &= A^\mu_3 - \tfrac{1}{2}\bar{q}_s\gamma^\mu\gamma_5 q_s \,. \end{aligned} \tag{24}$$

Now, A_{1-5} form a multiplet of axial currents which are related by flavor rotations. But, in contrast to the vector case, these currents are not conserved, only partially conserved. The meaning of this statement is explained in the forthcoming section.

3.2. Approximate symmetries of QCD: chiral symmetry

Another no less important symmetry of QCD arises in the limit of massless quarks: $m_u = m_d = m_s = 0$. In this case, the QCD Lagrangian, which can be written using left and right chiral quark fields

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_{q\rm L} i\gamma^\mu D_\mu \psi_{q\rm L} + \bar{\psi}_{q\rm R} i\gamma^\mu D_\mu \psi_{q\rm R} - m_q\left(\bar{\psi}_{q\rm L}\psi_{q\rm R} + \bar{\psi}_{q\rm L}\psi_{q\rm R}\right) + \ldots \tag{25}$$

is symmetric under $\mathrm{SU}(3)_{\rm L} \times \mathrm{SU}(3)_{\rm R}$ transformations. This is known as chiral symmetry, which is more general than the symmetry discussed in the previous section; it has 8 left-handed and the same number of right-handed conserved currents

$$R^\mu_a = \bar{q}_{\rm R}\gamma^\mu \frac{\lambda_a}{2} q_{\rm R}\,, \tag{26}$$

$$L^\mu_a = \bar{q}_{\rm L}\gamma^\mu \frac{\lambda_a}{2} q_{\rm L}\,, \tag{27}$$

which can be rearranged to give conserved vector and axial currents

$$V^\mu_a = R^\mu_a + L^\mu_a = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q\,, \tag{28}$$

$$A^\mu_a = R^\mu_a - L^\mu_a = \bar{q}\gamma^\mu\gamma_5 \frac{\lambda_a}{2} q\,. \tag{29}$$

Quark masses break chiral symmetry explicitly and the divergence of the vector current is given by Eq. (14), while for the axial one

$$\partial_\mu A^\mu_a = i\bar{q}\left\{m, \frac{\lambda_a}{2}\right\}\gamma_5 q\,. \tag{30}$$

Notice that unlike the vector case, the divergence of the axial current does not vanish for equal quark masses. The fact that the axial current is *almost conserved* for small quark masses is called partial conservation of the axial current (PCAC).

Besides explicit breaking, the chiral symmetry of QCD is also spontaneously broken. The hadron spectrum is not chirally symmetric even in the presence of only u and d quarks, for which the explicit breaking is small. For example, chiral partners such as the vector meson ρ and the axial a_1 have very different masses ($m_\rho = 770$ MeV $\neq m_{a_1} = 1230$ MeV) rather than being degenerated as chiral symmetry would dictate. Indeed $\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}$ is spontaneously broken down to $\mathrm{SU}(3)_\mathrm{V}$. This is reflected by a gap in the hadron spectrum between the vector mesons (ρ, ω, ϕ) and the light pseudoscalar mesons (π, K, η), which are the Goldstone bosons of the symmetry breaking and, as such, would be massless if it was not for the explicit breaking driven by the quark masses. The pattern of spontaneous chiral symmetry breaking is a crucial ingredient in the formulation of effective models of neutrino interactions with hadrons (and, in general, of interactions of hadrons among themselves and with external probes). The fact that Goldstone bosons interact weakly (for strong interactions) at low energies allows to use perturbative methods. Such a framework is known as chiral perturbation theory (see Ref. [7] for a detailed introduction for both meson and baryon sectors).

4. Inclusive electroweak cross section

Let us consider lepton scattering with an extended object (nucleon or nucleus) assuming that only the final lepton is detected *i.e.* a sum over all final hadronic states is performed. The reaction is

$$l(k) + N(p) \to l'\left(k'\right) + X\left(p'\right) \tag{31}$$

in which the initial lepton with four-momentum $k = (k_0, \vec{k}\,)$ hits the target with $p = (E, \vec{p}\,)$ producing a final lepton with $k' = (k'_0, \vec{k}\,)$ and a hadronic state generically denoted as X with momentum $p' = (E', \vec{p}\,')$. The four-momentum transfer $q = k - k' = p' - p = (\omega, \vec{q}\,)$ is such that $q^2 < 0$.

For CC interactions in the Laboratory frame, $p = (M, \vec{0}\,)$,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}k'_0 \mathrm{d}\Omega\left(\vec{k}'\right)} = \frac{G_\mathrm{F}^2}{(2\pi)^2} \frac{\left|\vec{k}'\right|}{k_0} L_{\mu\nu} W^{\mu\nu} \,. \tag{32}$$

This expression is valid when $|q^2| \ll M_W^2$, so that the static limit for the boson propagator

$$D_{\mu\nu} = \frac{1}{q^2 - M_W^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2} \right) \approx -\frac{g_{\mu\nu}}{M_W^2} \tag{33}$$

can be taken. Hence, the Fermi constant enters as

$$\left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2} = \frac{G_{\mathrm{F}}}{\sqrt{2}}\,. \tag{34}$$

The cross section contains the contraction of the leptonic L and the hadronic W tensors. The leptonic tensor, obtained directly using J_{CC} of Eq. (4),

$$L_{\mu\nu} = k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu} k\cdot k' + i\epsilon_{\mu\nu\alpha\beta} k'^\alpha k^\beta \tag{35}$$

has a symmetric part and an antisymmetric term characteristic of weak interactions.

The hadronic tensor with the general structure

$$\begin{aligned} W^{\mu\nu} &= \frac{1}{2M}\overline{\sum_{\mathrm{polar}}}\prod_i\left(\int\frac{\mathrm{d}^3p_i}{2E'_i(2\pi)^3}\right)(2\pi)^3\delta^4\left(k'+p'-k-p\right) \\ &\quad\times \langle X|\, J^\mu\,|N\rangle\,\langle X|\, J^\nu\,|N\rangle^* \end{aligned} \tag{36}$$

can be rather involved. Nevertheless, for inclusive processes, it can be built from tensors $g^{\mu\nu}$, $\epsilon^{\mu\nu\alpha\beta}$ and two independent momenta such as q^μ and p^μ, taking the following simple form [3,8]

$$\begin{aligned} W^{\mu\nu} &= -W_1 g^{\mu\nu} + W_2\frac{p^\mu p^\nu}{M^2} + W_4\frac{q^\mu q^\nu}{M^2} + W_5\frac{p^\mu q^\nu + q^\mu p^\nu}{M^2} \\ &\quad + W_3 i\epsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2M^2} + W_6\frac{p^\mu q^\nu - q^\mu p^\nu}{M^2}\,, \end{aligned} \tag{37}$$

with four symmetric and two antisymmetric terms. Structure functions W_{1-6} depend on all possible scalar quantities: $W_i = W_i(p^2 = M^2, q\cdot p = \omega_{\mathrm{L}} M, q^2) = W_i(\omega_{\mathrm{L}}, q^2)$, where ω_{L} denotes the energy transferred to the target in the Laboratory frame. All details of the hadronic dynamics are encoded in these functions.

The corresponding cross section for EM scattering is readily obtained from Eq. (32) by the replacement

$$L_{\mu\nu}\to L_{\mu\nu}^{(\mathrm{sym})}\,, \qquad \frac{G_{\mathrm{F}}^2}{(2\pi)^2}\to\frac{\alpha^2}{q^4}\,, \tag{38}$$

where α is the fine structure constant and the factor q^{-4} arises from the photon propagator squared.

For EM interactions, current conservation $q_\mu J^\mu = 0$ implies that

$$q_\mu W^{\mu\nu}_{\mathrm{EM}} = W^{\mu\nu}_{\mathrm{EM}} q_\nu = 0\,.$$

This condition reduces the number of structure functions to two (details of the derivation can be found in Chapter 11 of Ref. [3])

$$W^{\mu\nu}_{\rm EM} = W_1 \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) + \frac{W_2}{M^2} \left(p^\mu - \frac{p\cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p\cdot q}{q^2} q^\nu \right) . \quad (39)$$

The expression for EM scattering of massless leptons in terms of $W_{1,2}$ is quite simple and can be easily found in the literature. Here, instead, the corresponding one for CC scattering is provided

$$\frac{\mathrm{d}\sigma}{\mathrm{d}k'_0 \mathrm{d}\Omega(\vec{k}\,')} = \frac{G_{\rm F}^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} \Big\{ W_1 2k\cdot k' + W_2 \left(2k'_0 k_0 - k\cdot k' \right)$$
$$+2\frac{m_l^2}{M^2} \left[W_4 k\cdot k' - W_5 M k_0 \right] + \frac{W_3}{M} \left[\left(k_0 + k'_0 \right) k\cdot k' - k_0 m_l^2 \right] \Big\} . \quad (40)$$

Notice that W_6 does not contribute to the cross section. In the $m_l \to 0$ limit, only three structure functions remain and one gets the familiar result

$$\frac{\mathrm{d}\sigma}{\mathrm{d}k'_0 \mathrm{d}\Omega(\vec{k}')} = \frac{G_{\rm F}^2}{2\pi^2} \left(k'_0 \right)^2 \left[W_1 2\sin^2\frac{\theta'}{2} + W_2 \cos^2\frac{\theta'}{2} \pm W_3 \frac{(k_0 + k'_0)}{M} \sin^2\frac{\theta'}{2} \right] , \quad (41)$$

where the $+(-)$ sign stands for $\nu(\bar{\nu})$ scattering.

5. Quasielastic scattering on the nucleon (I)

By quasielastic (QE) (or elastic) lepton scattering on nucleons, one understands the following reactions:

$$\begin{aligned}
\mathrm{EM}: l^\pm(k) + N(p) &\to l^\pm\left(k'\right) + N\left(p'\right) ,\\
\mathrm{CC}: \nu(k) + n(p) &\to l^-\left(k'\right) + p\left(p'\right) ,\\
\bar{\nu}(k) + p(p) &\to l^+\left(k'\right) + n\left(p'\right) ,\\
\mathrm{NC}: \nu(k) + N(p) &\to \nu\left(k'\right) + N\left(p'\right) ,\\
\bar{\nu}(k) + N(p) &\to \bar{\nu}\left(k'\right) + N\left(p'\right)
\end{aligned}$$

with a lepton and a single nucleon in the final state. In this situation, taking proton and neutron masses as equal (isospin limit),

$$\begin{aligned}
&(q+p)^2 = (p')^2 ,\\
&q^2 + 2M\omega + M^2 = M^2 ,\\
&\omega = -\frac{q^2}{2M} \Rightarrow x = -\frac{q^2}{2M\omega} = 1 .
\end{aligned}$$

In QE scattering, energy and momentum transfers are not independent.

For the cross section, Eq. (32) holds now with the hadron tensor taking the simple form

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{\mathrm{d}^3 p'}{2E'} \delta^4 \left(k' + p' - k - p\right) H^{\nu\mu} \,, \tag{42}$$

with

$$H^{\alpha\beta} = \mathrm{Tr}\left[\left(\not{p} + M\right) \gamma^0 \left(\Gamma^\alpha\right)^\dagger \gamma^0 \left(\not{p}' + M\right) \Gamma^\beta\right] \,, \tag{43}$$

where Γ^μ is defined as

$$\left\langle N' \right| J^\mu \left| N \right\rangle = \bar{u}(p') \Gamma^\mu u(p) \,. \tag{44}$$

If the nucleon were a point-like fermion, one would simply have $\Gamma^\mu = \gamma^\mu(1-\gamma^5)$ but the actual structure is richer because of strong interactions.

5.1. Electroweak nucleon current

Here, I sketch the derivation of the most general expression for the matrix element of the nucleon electroweak current. To begin with, one realizes that the 4-vector Γ^μ can be built using the following structures:

1. p^μ , p'^μ,
2. $\epsilon_{\alpha\beta\mu\nu}$, $g^{\mu\nu}$,
3. γ_μ , γ_5 , $\gamma_\mu\gamma_5$, $\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right]$.

Any other combination of γ matrices can be reduced to the set above. Using Dirac algebra (for example Gordon identities) and the Dirac equation $(\not{p} - M)u(p) = \bar{u}(p')(\not{p}' - M) = 0$, one finds that

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu} q_\nu F_2 + \frac{q^\mu}{M} F_\mathrm{S} - \gamma^\mu \gamma_5 F_\mathrm{A} - \frac{i}{2M}\sigma^{\mu\nu} q_\nu \gamma_5 F_\mathrm{T} - \frac{q^\mu}{M}\gamma_5 F_\mathrm{P} \,. \tag{45}$$

The FF $F_{1,2,\mathrm{S},\mathrm{A},\mathrm{T},\mathrm{P}}$ are scalar functions of all independent scalars in the problem, which in the case of the QE kinematics reduce to only one: q^2.

The next step is to consider the time reversal transformation (T). Taking into account the antilinear property of T, it is obtained that

$$T\left(\bar{u}\Gamma^\mu u\right) T^\dagger = \sum_i F_i^* \bar{u}(\mathcal{O}_i)_\mu u \,, \tag{46}$$

while for the leptonic currents $l^\mu = J^\mu_{\mathrm{EM,CC,NC}}$ introduced in Eqs. (3), (5), one has

$$T l_\mu T^\dagger = l^\mu \,. \tag{47}$$

The interaction amplitude is proportional to

$$l_\mu \bar{u} \Gamma^\mu u = \sum_i F_i l_\mu \bar{u} \mathcal{O}_i^\mu u \,, \tag{48}$$

and then

$$T \left(l_\mu \bar{u} \Gamma^\mu u \right) T^\dagger = \sum_i F_i^* l_\mu \bar{u} \mathcal{O}_i^\mu u \,. \tag{49}$$

From these two equations, it is clear that T invariance of the interaction implies $F_i = F_i^*$. Therefore, all the FFs are real. As nucleons are made of light quarks, T invariance can be safely assumed but this is not, in general, true for systems with heavier quarks.

We can further explore the properties of the nucleon current by applying the parity P transformation

$$P \bar{u} \left(p_0', \vec{p}\,' \right) \Gamma^\mu (q_0, \vec{q}) u(p_0, \vec{p}) P^\dagger = \bar{u} \left(p_0', -\vec{p}\,' \right) \gamma_0 \Gamma^\mu (q_0, -\vec{q}) \gamma_0 u \left(p_0, -\vec{p} \right) \,. \tag{50}$$

This exercise reveals that γ^μ, $\sigma^{\mu\nu} q_\nu$ and q^μ transform as vectors, while $\gamma^\mu \gamma_5$, $\sigma^{\mu\nu} \gamma_5 q_\nu$, $q^\mu \gamma_5$ do as axial vectors. In the case of EM interactions, only the first set of operators is present, while parity-violating weak interactions demand a combination of both. Therefore,

$$\left\langle N' \right| J^\mu \left| N \right\rangle = \bar{u} \left(p' \right) \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu \tag{51}$$

with

$$\mathcal{V}^\mu = \bar{u} \left(p' \right) \left[\gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 + \frac{q^\mu}{M} F_{\mathrm{S}} \right] u(p) \,, \tag{52}$$

$$\mathcal{A}^\mu = \bar{u} \left(p' \right) \left[\gamma^\mu \gamma_5 F_{\mathrm{A}} + \frac{i}{2M} \sigma^{\mu\nu} q_\nu \gamma_5 F_{\mathrm{T}} + \frac{q^\mu}{M} \gamma_5 F_{\mathrm{P}} \right] u(p) \,. \tag{53}$$

Finally, let us consider the G-parity transformation which combines charge conjugation with isospin rotation: $G = C e^{i\pi \frac{\tau_2}{2}}$. It can be shown that $G \mathcal{V}^\mu G^\dagger = \mathcal{V}^\mu$ except for the term proportional to F_{S} which transforms differently. Analogously, $G \mathcal{A}^\mu G^\dagger = -\mathcal{A}^\mu$ except for the term proportional to F_{T}. One should bear in mind that in the absence of strong interactions, only the term proportional to γ^μ ($\gamma^\mu \gamma^5$) is present in the vector (axial) current, while others are induced by G-parity conserving strong interactions. We then demand that the whole vector (axial) current transforms under G as the term proportional to F_1 (F_{A}) does, obtaining that $F_{\mathrm{S}} = 0$ ($F_{\mathrm{T}} = 0$).

We have hence arrived at the well-known result for the nucleon current

$$\mathcal{V}^\mu = \bar{u} \left(p' \right) \left[\gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 \right] u(p) \,, \tag{54}$$

$$\mathcal{A}^\mu = \bar{u} \left(p' \right) \left[\gamma^\mu \gamma_5 F_{\mathrm{A}} + \frac{q^\mu}{M} \gamma_5 F_{\mathrm{P}} \right] u(p) \tag{55}$$

written in terms of the Dirac (F_1), Pauli (F_2), axial (F_A) and pseudoscalar (F_P) FF. In the case of the EM current, the Sachs electric and magnetic FF

$$
\begin{aligned}
G_\mathrm{E} &= F_1 + \frac{q^2}{4M^2} F_2 \,, && (56)\\
G_\mathrm{M} &= F_1 + F_2 && (57)
\end{aligned}
$$

are often introduced.

In the following, the consequences of isospin symmetry for the nucleon matrix elements of the EM and weak currents are explored. These considerations follow Sec. 3.1, where the flavor structure of electroweak currents was discussed. I start by introducing the isovector

$$V_a^\alpha = \mathcal{V}^\alpha \frac{\tau_a}{2} \tag{58}$$

and isoscalar (hypercharge) currents

$$V_Y^\alpha = \mathcal{V}_Y^\alpha I \tag{59}$$

as 2×2 matrices in the isospin space of protons and neutrons. In terms of these,

$$
\begin{aligned}
\langle p| V_\mathrm{EM}^\alpha |p\rangle &= \langle p| V_3^\alpha + \frac{1}{2} V_Y^\alpha |p\rangle = \frac{\mathcal{V}^\alpha + \mathcal{V}_Y^\alpha}{2} \equiv \mathcal{V}_p^\alpha \,, && (60)\\
\langle n| V_\mathrm{EM}^\alpha |n\rangle &= \langle n| V_3^\alpha + \frac{1}{2} V_Y^\alpha |n\rangle = \frac{-\mathcal{V}^\alpha + \mathcal{V}_Y^\alpha}{2} \equiv \mathcal{V}_n^\alpha \,. && (61)
\end{aligned}
$$

On the other hand,

$$\langle p| V_\mathrm{CC}^\alpha |n\rangle = \langle p| V_1^\alpha + i V_2^\alpha |n\rangle = \mathcal{V}^\alpha = \mathcal{V}_p^\alpha - \mathcal{V}_n^\alpha \tag{62}$$

and

$$
\begin{aligned}
\langle p| V_\mathrm{NC}^\alpha |p\rangle &= \langle p| \left(1 - 2\sin^2\theta_\mathrm{W}\right) V_3^\alpha - \sin^2\theta_\mathrm{W} V_Y^\alpha |p\rangle \\
&= \left(\frac{1}{2} - \sin^2\theta_\mathrm{W}\right) \mathcal{V}^\alpha - \sin^2\theta_\mathrm{W} \mathcal{V}_Y^\alpha \\
&= \left(\frac{1}{2} - 2\sin^2\theta_\mathrm{W}\right) \mathcal{V}_p^\alpha - \frac{1}{2} \mathcal{V}_n^\alpha \,. && (63)
\end{aligned}
$$

It has been shown that, by virtue of the isospin symmetry, the matrix elements of the vector part of charged and neutral currents can be expressed in terms of the EM ones of protons and neutrons. Therefore, the weak vector FF can be related to the EM ones. This result allows to use input from electron scattering experiments in neutrino cross sections. Nucleon EM FF have been extracted with high precision up to high $q^2 \sim 10$ GeV^2 thanks to polarization transfer techniques (see Ref. [6] for more details).

For the axial part of the current, PCAC together with the assumption of pion-pole dominance of the pseudoscalar FF allows to express the latter in terms of the axial FF

$$F_{\mathrm{P}}\left(q^2\right) = F_{\mathrm{A}}\left(q^2\right)\frac{2M}{-q^2+m_\pi^2}\,. \tag{64}$$

F_{P} has a small impact on CCQE cross sections except for ν_τ because it appears in terms proportional to $(m_l/M)^4$ which is very small for $l=e,\mu$. For the same reason, it does not contribute to NC elastic cross sections. F_{P} can, however, be studied and has been studied in muon capture $\mu^- p \to \nu_\mu n$ and found to be consistent with pion-pole dominance [9].

6. Quasielastic scattering on the nucleon (II)

With the ingredients discussed in the previous section, it is possible to write the QE cross section as a function of the four FFs in a compact way (see, for instance, Eq. (57) of Ref. [10]). Alternatively, it is instructive to write this cross section as an expansion in small variables[2] $q^2, m_l^2 \ll M^2, E_\nu^2$. For CCQE, one finds that [11]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^2} = \frac{1}{2\pi}G^2\cos^2\theta_{\mathrm{C}}\left[R-\frac{m_l^2}{4E_\nu^2}S+\frac{q^2}{4E_\nu^2}T\right]+\mathcal{O}\left(q^4,m_l^4,m_l^2q^2\right) \tag{65}$$

with

$$R = 1+g_{\mathrm{A}}^2\,, \tag{66}$$

$$S = \frac{2E_\nu+M}{M}+g_{\mathrm{A}}^2\frac{2E_\nu-M}{M}\,, \tag{67}$$

$$\begin{aligned} T &= 1-g_{\mathrm{A}}^2+2\frac{E_\nu}{M}\left(1\mp g_{\mathrm{A}}\right)^2\mp 4\frac{E_\nu}{M}g_{\mathrm{A}}\kappa^{\mathrm{V}}-\left(\frac{E_\nu}{M}\kappa^{\mathrm{V}}\right)^2 \\ &\quad +4E_\nu^2\left[\frac{1}{3}\left(\langle r_p^2\rangle-\langle r_n^2\rangle+g_{\mathrm{A}}^2\langle r_{\mathrm{A}}^2\rangle\right)-\frac{1}{2M^2}\kappa^{\mathrm{V}}\right]. \end{aligned} \tag{68}$$

Remarkably, a sizable fraction of the CCQE cross section depends on a small number of nucleon properties: charges, axial coupling (g_{A}, known from β decay), magnetic moments ($\kappa^{\mathrm{V}}=\mu_p-\mu_n-1$), EM and axial mean squared radii

$$\langle r_p^2\rangle = \frac{6}{G_{\mathrm{E}}^{(p)}(0)}\left.\frac{\mathrm{d}G_{\mathrm{E}}^{(p)}\left(q^2\right)}{\mathrm{d}q^2}\right|_{q^2=0}\,,\qquad \langle r_n^2\rangle = 6\left.\frac{\mathrm{d}G_{\mathrm{E}}^{(n)}\left(q^2\right)}{\mathrm{d}q^2}\right|_{q^2=0} \tag{69}$$

[2] Close to threshold ($E_\nu \sim m_l$) and for CCQE with τ-neutrinos (due to the large m_τ value), the counting is different.

and

$$\langle r_{\mathrm{A}}^{2} \rangle = \frac{6}{F_{\mathrm{A}}(0)} \frac{\mathrm{d}F_{\mathrm{A}}\left(q^{2}\right)}{\mathrm{d}q^{2}} \bigg|_{q^{2}=0} . \tag{70}$$

Among these quantities, a relevant one but still not well-constrained by experiments is the axial radius. It has been extracted from early CCQE measurements on deuterium and, to lesser extent, hydrogen targets, and from single pion electroproduction with compatible results. In both cases, the axial FF has been parametrized with the common dipole Ansatz

$$F_{\mathrm{A}}\left(q^{2}\right) = g_{\mathrm{A}} \left(1 - \frac{q^{2}}{M_{\mathrm{A}}^{2}}\right)^{-2} . \tag{71}$$

For such a one-parameter function, the so-called axial mass M_{A} is directly related to the axial radius: $\langle r_{\mathrm{A}}^{2} \rangle = 12/M_{\mathrm{A}}^{2}$. In spite of the fact that deviations from the dipole have not been observed so far, it is worth stressing that the dipole form is not well-justified from a theoretical point of view. Furthermore, it can be argued that because of the model-dependent relation between $\langle r_{\mathrm{A}}^{2} \rangle$ and M_{A}, a low q^2 property like $\langle r_{\mathrm{A}}^{2} \rangle$ is extracted from the whole experimentally available kinematic range, leading to an artificially small error. A new extraction of F_{A} from neutrino CCQE data has been recently undertaken using a model-independent representation of the FF based on conformal mapping (z-expansion). The resulting $\langle r_{\mathrm{A}}^{2} \rangle = 0.46(22)$ fm^2 [12] agrees with previous values but with a much larger error. More precise determinations of $F_{\mathrm{A}}(q^2)$ might become available from lattice QCD simulations in the future.

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REFERENCES

[1] A.W. Thomas, W. Weise, *The Structure of the Nucleon*, Wiley-VCH, Berlin 2001.

[2] F. Halzen, A.D. Martin, *Quarks and Leptons*, John Wiley and Sons, 1984.

[3] J.D. Walecka, *Electron Scattering for Nuclear and Nucleon Structure*, Cambridge University Press, 2001.

[4] E.D. Commins, P.H. Bucksbaum, *Weak Interactions of Leptons and Quarks*, Cambridge University Press, 1983.

[5] M. Nowakowski, E.A. Paschos, J.M. Rodriguez, *Eur. J. Phys.* **26**, 545 (2005).

[6] C.F. Perdrisat, V. Punjabi, M. Vanderhaeghen, *Prog. Part. Nucl. Phys.* **59**, 694 (2007).

[7] S. Scherer, M.R. Schindler, *Lect. Notes Phys.* **830**, 1 (2012).

[8] J. Nieves, J.E. Amaro, M. Valverde, *Phys. Rev. C* **70**, 055503 (2004) [*Erratum ibid.* **72**, 019902 (2005)].

[9] V. Bernard, L. Elouadrhiri, U.-G. Meißner, *J. Phys. G* **28**, R1 (2002).

[10] J.A. Formaggio, G.P. Zeller, *Rev. Mod. Phys.* **84**, 1307 (2012).

[11] L. Alvarez-Ruso, Y. Hayato, J. Nieves, *New J. Phys.* **16**, 075015 (2014).

[12] A.S. Meyer, M. Betancourt, R. Gran, R.J. Hill, *Phys. Rev. D* **93**, 113015 (2016).