STERILE NEUTRINOS*

Carlo Giunti

INFN, Sezione di Torino, Via P. Giuria 1, 10125 Torino, Italy

(Received September 21, 2016)

We present a review of the theory of active–sterile neutrino mixing and of the phenomenology of short-baseline neutrino oscillations induced by light sterile neutrinos. We discuss the results of the global fits of short-baseline neutrino oscillation data in 3+1 and 3+2 neutrino mixing schemes.

DOI:10.5506/APhysPolBSupp.9.711

1. Introduction

The current experimental and theoretical research of the physics of massive neutrinos is based on the standard paradigm of three-neutrino mixing which describes the oscillations of neutrino flavors measured in solar, atmospheric and long-baseline experiments (see Refs. [1–3]). In the three-neutrino mixing framework (3ν), the three active neutrinos ν_e, ν_μ, ν_τ are superpositions of three massive neutrinos ν_1, ν_2, ν_3 with respective masses m_1, m_2, m_3, which determine two independent squared-mass differences: the small solar squared-mass difference

$$\Delta m^2_{\rm SOL} = \Delta m^2_{21} \simeq 7.5 \times 10^{-5}\,{\rm eV}^2\,, \tag{1.1}$$

and the larger atmospheric squared-mass difference

$$\Delta m^2_{\rm ATM} = |\Delta m^2_{31}| \simeq \left|\Delta m^2_{32}\right| \simeq 2.3 \times 10^{-3}\,{\rm eV}^2\,, \tag{1.2}$$

with $\Delta m^2_{kj} = m^2_k - m^2_j$ [4–6].

The completeness of the 3ν mixing paradigm has been challenged by the following indications in favor of short-baseline neutrino oscillations:

1. The reactor antineutrino anomaly [7], which is an about 2.8σ deficit of the rate of $\bar{\nu}_e$ observed in several short-baseline reactor neutrino experiments in comparison with that expected from the calculation of the reactor neutrino fluxes [8, 9].

* Presented at the 52nd Winter School of Theoretical Physics, “Theoretical Aspects of Neutrino Physics”, Lądek Zdrój, Poland, February 14–21, 2016.

2. The gallium neutrino anomaly [10–14], consisting in a short-baseline disappearance of ν_e measured in the gallium radioactive source experiments GALLEX [15] and SAGE [16] with a statistical significance of about 2.9σ.

3. The LSND experiment, in which a signal of short-baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations has been observed with a statistical significance of about 3.8σ [17, 18].

A neutrino oscillation explanation of these anomalies requires the existence of at least one additional squared-mass difference

$$\Delta m^2_{\mathrm{SBL}} \gtrsim 1\,\mathrm{eV}^2\,, \tag{1.3}$$

which is much larger than the solar and atmospheric squared-mass differences and requires the existence of at least one massive neutrino ν_4 in addition to the three standard massive neutrinos ν_1, ν_2, ν_3 (see the review in Ref. [19]). Since from the LEP measurement of the invisible width of the Z boson we know that there are only three active neutrinos (see Ref. [20]), in the flavor basis the additional massive neutrinos correspond to sterile neutrinos [21], which do not have standard weak interactions.

Sterile neutrinos are singlets of the Standard Model gauge symmetries which can couple to the active neutrinos through the Lagrangian mass term. In practice, there are bounds on the active–sterile mixing, but there is no bound on the number of sterile neutrinos and on their mass scales. Therefore, the existence of sterile neutrinos is investigated at different mass scales. This review is devoted to the discussion of sterile neutrinos at the eV scale, which can explain the indications in favor of short-baseline neutrino oscillations listed above. However, there are other very interesting possibilities which are under study: very light sterile neutrinos at a mass scale smaller than 0.1 eV, which could affect the oscillations of solar [22–24] and reactor [25–31] neutrinos; sterile neutrinos at the keV scale, which could constitute warm dark matter according to the Neutrino Minimal Standard Model (νMSM) [32–37]; sterile neutrinos at the MeV scale [38–41]; sterile neutrinos at the electroweak scale [42, 43] or above it [43, 44], whose effects may be seen at the LHC and other high-energy colliders. Let us also note that there are several interesting models with sterile neutrinos at different mass scales [45–61].

The possible existence of sterile neutrinos is very interesting, because they are new particles which could give us precious information on the physics beyond the Standard Model (see Refs. [62, 63]).

2. Beyond three-neutrino mixing

The Standard Model of electroweak interactions [64–66] based on the $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$ gauge symmetry is a superb theory which can explain the majority of terrestrial experimental observations. However, it does not account for neutrino masses, whose existence have been proved without doubt by the measurement of neutrino oscillations in solar, atmospheric and long-baseline neutrino oscillation experiments (see Refs. [1, 20, 67–70]). The simplest way to extend the Standard Model in order to take into account neutrino masses is through the introduction of $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$ singlet fields which are traditionally called "right-handed neutrino" fields or "sterile neutrino" fields. The adjective "right-handed" indicates that they do not belong to $\mathrm{SU}(2)_{\mathrm{L}}$ left-handed multiplets. Therefore, they are "sterile", because they do not have Standard Model weak interactions. Moreover, assuming that they have zero hypercharge[1], they are neutral and can be called "neutrino" fields. Many models which extend the Standard Model include these right-handed sterile neutrino fields (see Refs. [62, 63, 73–76]). In the following, we consider the general theory of neutrino mixing in which we have the three standard active left-handed flavor neutrino fields $\nu_{e\mathrm{L}}$, $\nu_{\mu\mathrm{L}}$, $\nu_{\tau\mathrm{L}}$ and N_{s} sterile right-handed flavor neutrino fields $\nu_{s_1\mathrm{R}}, \ldots, \nu_{N_{\mathrm{s}}\mathrm{R}}$. The most general Lagrangian mass term which can be written with these fields is (the superscript (F) indicates the flavor basis)

$$\mathcal{L}_{\mathrm{mass}} = \frac{1}{2}\, \nu_{\mathrm{L}}^{(\mathrm{F})\,T}\, \mathcal{C}^{\dagger}\, M\, \nu_{\mathrm{L}}^{(\mathrm{F})} + \mathrm{H.c.}\,, \tag{2.1}$$

where (the superscripts (a) and (s) indicate, respectively, the column matrices of active and sterile neutrino fields)

$$\nu_{\mathrm{L}}^{(\mathrm{F})} = \begin{pmatrix} \nu_{\mathrm{L}}^{(\mathrm{a})} \\ \nu_{\mathrm{R}}^{(\mathrm{s})\,c} \end{pmatrix}, \qquad \nu_{\mathrm{L}}^{(\mathrm{a})} = \begin{pmatrix} \nu_{e\mathrm{L}} \\ \nu_{\mu\mathrm{L}} \\ \nu_{\tau\mathrm{L}} \end{pmatrix}, \qquad \nu_{\mathrm{R}}^{(\mathrm{s})\,c} = \begin{pmatrix} \nu_{s_1\mathrm{R}}^{c} \\ \vdots \\ \nu_{s_{N_{\mathrm{s}}}\mathrm{R}}^{c} \end{pmatrix}, \tag{2.2}$$

and $\mathcal{C}$ is the unitary charge-conjugation matrix[2], such that $\mathcal{C}\,\gamma_{\mu}^{T}\,\mathcal{C}^{-1} = -\gamma_{\mu}$ and $\mathcal{C}^{T} = -\mathcal{C}$. For any field ψ, the charge-conjugated field ψ^{c} is given by $\psi^{c} = \mathcal{C}\overline{\psi}^{T}$ and charge conjugation transforms the chirality of a field (*e.g.* ψ_{R}^{c} is left-handed). In general, M is a complex symmetric mass matrix, which can be diagonalized with the unitary transformation (the superscript (M) indicates the mass basis)

$$\nu_{\mathrm{L}}^{(\mathrm{F})} = \mathcal{U}\,\nu_{\mathrm{L}}^{(\mathrm{M})} \qquad \text{with} \qquad \nu_{\mathrm{L}}^{(\mathrm{M})} = \begin{pmatrix} \nu_{1\mathrm{L}} \\ \vdots \\ \nu_{N\mathrm{L}} \end{pmatrix}, \tag{2.3}$$

---

[1] We do not consider here the exotic possibility of a small nonzero hypercharge of the right-handed neutrino fields, which would imply that neutrinos are Dirac and millicharged particles (see Refs. [71, 72]).

[2] We use the notations and conventions from Ref. [20].

where $N = 3 + N_{\rm s}$ is the total number of neutrino fields. The matrix $\mathcal{U}$ is a $N \times N$ unitary matrix such that

$$\mathcal{U}^T M \mathcal{U} = \mathrm{diag}\,(m_1, \ldots, m_N) \ , \tag{2.4}$$

with real and positive masses $m_1, \ldots, m_N$ (see Refs. [20, 77]). The Lagrangian mass term (2.1) becomes

$$\begin{aligned} \mathcal{L}_{\rm mass} &= \frac{1}{2} \sum_{k=1}^{N} m_k \nu_{k\rm L}^T \mathcal{C}^\dagger \nu_{k\rm L} + \text{H.c.} = -\frac{1}{2} \sum_{k=1}^{N} m_k \overline{\nu_{k\rm L}^c} \nu_{k\rm L} + \text{H.c.} \\ &= -\frac{1}{2} \sum_{k=1}^{N} m_k \overline{\nu_k} \nu_k \ , \end{aligned} \tag{2.5}$$

with the massive Majorana neutrino fields $\nu_k = \nu_{k\rm L} + \nu_{k\rm L}^c$ which satisfy the Majorana constraint $\nu_k = \nu_k^c$. Hence, in the general case of active–sterile neutrino mixing, the massive neutrinos are Majorana particles[3].

The physical effects of the unitary transformation (2.3) are due to the noninvariance of the weak interaction Lagrangian. Let us first consider the leptonic charged-current weak interaction Lagrangian. In the flavor basis where the mass matrix of the charged leptons $\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$ is diagonal, we have

$$\begin{aligned} \mathcal{L}_{\rm CC} &= -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \overline{\ell_{\alpha\rm L}} \gamma^\rho \nu_{\alpha\rm L} W_\rho^\dagger + \text{H.c.} \\ &= -\frac{g}{\sqrt{2}} \sum_{k=1}^{N} \sum_{\alpha=e,\mu,\tau} \overline{\ell_{\alpha\rm L}} \gamma^\rho \mathcal{U}_{\alpha k} \nu_{k\rm L} W_\rho^\dagger + \text{H.c.} \end{aligned} \tag{2.6}$$

It is convenient to write $\mathcal{L}_{\rm CC}$ in the following matrix form

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} \overline{\ell_{\rm L}} \gamma^\rho \nu_{\rm L}^{(\rm a)} W_\rho^\dagger + \text{H.c.} = -\frac{g}{\sqrt{2}} \overline{\ell_{\rm L}} \gamma^\rho U \nu_{\rm L}^{(\rm M)} W_\rho^\dagger + \text{H.c.} \tag{2.7}$$

with

$$\ell_{\rm L} = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} , \qquad \nu_{\rm L}^{(\rm a)} = U \nu_{\rm L}^{(\rm M)} \qquad \text{and} \qquad U = \mathcal{U}|_{3\times N} \ . \tag{2.8}$$

The mixing matrix U is a $3 \times N$ rectangular matrix formed by the first three rows of $\mathcal{U}$. Therefore, the number of physical mixing parameters is smaller

[3] However, it is not excluded that the mixing is such that there are pairs of Majorana neutrino fields with exactly the same mass which form Dirac neutrino fields.

than the number necessary to parameterize the unitary matrix $\mathcal{U}$. This is due to the arbitrariness of the mixing in the sterile sector, which does not affect weak interactions. A careful analysis (see Ref. [20]) shows that the $3 \times N$ mixing matrix U can be parameterized in terms of $3 + 3N_{\mathrm{s}}$ mixing angles and $3 + 3N_{\mathrm{s}}$ physical phases, of which $1 + 2N_{\mathrm{s}}$ are Dirac phases and $N - 1$ are Majorana phases. For such parameterization, it is convenient to use the scheme

$$U = \left[\left(\prod_{a=1}^{3} \prod_{b=4}^{N} W^{ab} \right) R^{23} W^{13} R^{12} \right]_{3\times N} \operatorname{diag}\left(1, e^{i\lambda_{21}}, \ldots, e^{i\lambda_{N1}}\right) . \qquad (2.9)$$

The unitary $N \times N$ matrix $W^{ab} = W^{ab}(\theta_{ab}, \eta_{ab})$ represents a complex rotation in the a–b plane by a mixing angle θ_{ab} and a Dirac phase η_{ab}. Its components are

$$\begin{aligned} \left[W^{ab}(\vartheta_{ab}, \eta_{ab})\right]_{rs} &= \delta_{rs} + (c_{ab} - 1)\left(\delta_{ra}\delta_{sa} + \delta_{rb}\delta_{sb}\right) \\ &\quad + s_{ab}\left(e^{i\eta_{ab}}\delta_{ra}\delta_{sb} - e^{-i\eta_{ab}}\delta_{rb}\delta_{sa}\right) , \end{aligned} \qquad (2.10)$$

where $c_{ab} \equiv \cos\vartheta_{ab}$ and $s_{ab} \equiv \sin\vartheta_{ab}$. The order of the product of W^{ab} matrices in Eq. (2.9) is arbitrary. The orthogonal matrix $R^{ab} = W^{ab}(\theta_{ab}, 0)$ represents a real rotation in the a–b plane. The square brackets with subscript $3 \times N$ indicate that the enclosed $N \times N$ matrix is truncated to the first three rows. The Majorana phases $\lambda_{21}, \ldots \lambda_{N1}$, which are physical only if massive neutrinos are Majorana particles, are collected in a diagonal matrix on the right[4]. Moreover, not all the phases η_{ab} in the product of W^{ab} matrices in Eq. (2.9) are physical, but one can eliminate an unphysical phase for each value of the index $b = 4, \ldots, N$ (see Ref. [20]).

Scheme (2.9) has the advantage that in the limit of vanishing active–sterile mixing, the mixing matrix reduces to the three-neutrino (3ν) mixing matrix in the standard parameterization

$$\begin{aligned} U^{(3\nu)} &= \left[R^{23} W^{13} R^{12}\right]_{3\times 3} \operatorname{diag}\left(1, e^{i\lambda_{21}}, e^{i\lambda_{31}}\right) \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\eta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\eta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\eta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\eta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\eta_{13}} & c_{23}c_{13} \end{pmatrix} \\ &\quad \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix} . \end{aligned} \qquad (2.11)$$

[4] It is possible to choose any other diagonal matrix with $N - 1$ phases, as for example $\operatorname{diag}\left(e^{i\lambda_{12}}, 1, e^{i\lambda_{32}}, \ldots, e^{i\lambda_{N2}}\right)$, *etc.*

It is convenient to choose in Eq. (2.9) the order of the real or complex rotations for each index $b \geq 4$ such that the rotations in the 3-b, 2-b and 1-b planes are ordered from left to right. In this way, the first two lines, which are relevant for the study of the oscillations of the experimentally more accessible flavor neutrinos ν_e and ν_μ, are independent of the mixing angles and Dirac phases corresponding to the rotations in all the 3-b planes for $b \geq 4$. Moreover, the first line, which is relevant for the study of ν_e disappearance, is independent also of the mixing angles and Dirac phases corresponding to the rotations in the 2-b planes for $b \geq 3$. For example, one can choose

$$\begin{aligned} U &= \left[W^{3N} R^{2N} W^{1N} \cdots W^{34} R^{24} W^{14} R^{23} W^{13} R^{12}\right]_{3\times N} \\ &\quad \times \mathrm{diag}\left(1, e^{i\lambda_{21}}, \ldots, e^{i\lambda_{N1}}\right) , \end{aligned} \tag{2.12}$$

or

$$\begin{aligned} U &= \left[W^{3N} \cdots W^{34} W^{2N} \cdots W^{24} R^{1N} \cdots R^{14} R^{23} W^{13} R^{12}\right]_{3\times N} \\ &\quad \times \mathrm{diag}\left(1, e^{i\lambda_{21}}, \ldots, e^{i\lambda_{N1}}\right) . \end{aligned} \tag{2.13}$$

Let us now consider the neutrino neutral-current Lagrangian

$$\mathcal{L}_{\mathrm{NC}} = -\frac{g}{2\cos\vartheta_{\mathrm{W}}} \overline{\nu_{\mathrm{L}}^{(\mathrm{a})}} \gamma^\rho \nu_{\mathrm{L}}^{(\mathrm{a})} Z_\rho = -\frac{g}{2\cos\vartheta_{\mathrm{W}}} \overline{\nu_{\mathrm{L}}^{(\mathrm{M})}} \gamma^\rho U^\dagger U \nu_{\mathrm{L}}^{(\mathrm{M})} Z_\rho . \tag{2.14}$$

Since the rectangular $3 \times N$ mixing matrix U is formed by the first three rows of the unitary matrix $\mathcal{U}$, we have

$$UU^\dagger = \mathbf{1}_{3\times 3} \qquad \text{but} \qquad U^\dagger U \neq \mathbf{1}_{N\times N} . \tag{2.15}$$

Therefore, the GIM mechanism [78] does not work in neutral-current weak interactions [79] and it is possible to have neutral-current transitions among different massive neutrinos[5].

The introduction of sterile neutrinos is allowed by the fact that it has no effect or small effects[6] on the effective number of active neutrinos which contributes to the decay of the Z-boson. This number has been determined with high precision to be close to three by the LEP experiments [84]

$$N_\nu^{(Z)} = 2.9840 \pm 0.0082 . \tag{2.16}$$

[5] This is a special case of the general theorem that the weak leptonic neutral current is nondiagonal in the mass basis if the leptons of a given charge and chirality have different weak isospins [80].

[6] Sterile neutrinos at mass scales larger than the muon mass affect the determination of the Fermi constant G_{F} through muon decay [81]. Those at mass scales larger than $m_Z/2$ can induce a kinematical suppression of $N_\nu^{(Z)}$ [82, 83].

In this review, we will consider sterile neutrinos at the eV scale, for which $N_{\nu}^{(Z)}$ is given by [82, 83]

$$N_{\nu}^{(Z)} = \sum_{j,k=1}^{N} \left| \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^{*} \, U_{\alpha k} \right|^{2} = 3 \,. \tag{2.17}$$

Hence, there is no constraint on the number and mixing of these light sterile neutrinos from the high-precision LEP measurement of $N_{\nu}^{(Z)}$.

3. Short-baseline neutrino oscillations

The standard framework of 3ν mixing can be extended with the introduction of non-standard massive neutrinos only if their mixing with the active neutrinos is sufficiently small in order not to spoil the successful 3ν mixing explanation of solar, atmospheric and long-baseline neutrino oscillation measurements [4–6]. In other words, the non-standard massive neutrinos must be mostly sterile, *i.e.*

$$|U_{\alpha k}|^{2} \ll 1 \qquad (\alpha = e, \mu, \tau;\; k = 4, \ldots, N) \,. \tag{3.1}$$

In the following, we will always assume this constraint.

In this review, we consider mainly the so-called $3+1$ scheme in which there is a non-standard massive neutrino (mostly sterile) at the eV scale which generates a new squared-mass difference

$$\Delta m_{\mathrm{SBL}}^{2} \sim 1 \, \mathrm{eV}^{2} \,, \tag{3.2}$$

in order to explain the anomalies found in some short-baseline (SBL) neutrino oscillation experiments (see Section 3). We assume that the three standard massive neutrinos are much lighter than the eV scale. We will consider also the so-called $3+2$ scheme in which there are two non-standard massive neutrinos (mostly sterile) at the eV scale[7]. We do not consider schemes in which $\Delta m_{\mathrm{SBL}}^{2}$ is obtained with one or more very light (or massless) non-standard massive neutrinos and the three standard massive neutrinos have almost degenerate masses at the eV scale (*e.g.*, the $1+3$, $1+3+1$ and $2+3$ schemes), because this possibility is strongly disfavored by cosmological measurements [94] and by the experimental bound on neutrinoless double-β decay (assuming that massive neutrinos are Majorana particles; see Refs. [95, 96]). Let us emphasize that these mixing schemes must be considered as effective, in the sense that the existence of more non-standard

[7] In the literature, one can also find studies of the $3+3$ [85, 86], $3+1+1$ [87–91], and $1+3+1$ [92, 93] schemes.

massive neutrinos is allowed, as long as their mixing with the three active neutrinos is sufficiently small to be negligible in the analysis of the data of current experiments.

For the study of neutrino oscillations in vacuum, it is convenient to use the following general expression of the probability of $\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta$ oscillations [97, 98]

$$P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} = \delta_{\alpha\beta} - 4 \sum_{k \neq p} |U_{\alpha k}|^2 \left(\delta_{\alpha\beta} - |U_{\beta k}|^2 \right) \sin^2 \varDelta_{kp} + 8 \sum_{\substack{j>k \\ j,k \neq p}} |U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k}| \sin \varDelta_{kp} \sin \varDelta_{jp} \cos \left(\varDelta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk} \right) , \tag{3.3}$$

where

$$\varDelta_{kp} = \frac{\Delta m^2_{kp} L}{4E} , \qquad \eta_{\alpha\beta jk} = \arg \left[U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \right] , \tag{3.4}$$

and p is an arbitrary fixed index, which can be chosen in the most convenient way depending on the case under consideration. In the case of three-neutrino mixing, there is only one interference term in Eq. (3.3), because for any choice of p, there is only one possibility for j and k such that $j > k$.

We are interested in the effective oscillation probabilities in short-baseline experiments, for which $\varDelta_{21} \ll \varDelta_{31} \ll 1$. Let us consider the general $3 + N_{\rm s}$ case in which $\Delta m^2_{k1} \approx \Delta m^2_{\rm SBL}$ and $\varDelta_{k1} \approx 1$ for $k \geq 4$. Choosing $p = 1$ in Eq. (3.4), we obtain

$$P^{(\rm SBL)}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \delta_{\alpha\beta} - 4 \sum_{k=4}^{N} |U_{\alpha k}|^2 \left(\delta_{\alpha\beta} - |U_{\beta k}|^2 \right) \sin^2 \varDelta_{k1} + 8 \sum_{k=4}^{N} \sum_{j=k+1}^{N} |U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k}| \sin \varDelta_{k1} \sin \varDelta_{j1} \cos \left(\varDelta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk} \right) . \tag{3.5}$$

Considering the survival probabilities of active neutrinos, let us define the effective amplitudes

$$\sin^2 2\vartheta^{(k)}_{\alpha\alpha} = 4|U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2 \right) \simeq 4|U_{\alpha k}|^2 \qquad (\alpha = e, \mu, \tau;\ k \geq 4) , \tag{3.6}$$

where we have taken into account the constraint in Eq. (3.1). Dropping the quadratically suppressed terms also in the survival probabilities, we obtain

$$P^{(\rm SBL)}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sum_{k=4}^{N} \sin^2 2\vartheta^{(k)}_{\alpha\alpha} \sin^2 \varDelta_{k1} \qquad (\alpha = e, \mu, \tau) . \tag{3.7}$$

Hence, each effective mixing angle $\vartheta_{\alpha\alpha}^{(k)}$ parameterizes the disappearance of $\overset{(-)}{\nu}_{\alpha}$ due to its mixing with $\overset{(-)}{\nu}_{k}$.

Let us now consider the probabilities of short-baseline $\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}$ transitions between two different active neutrinos or an active and a sterile neutrino. We define the transition amplitudes

$$\sin^2 2\vartheta_{\alpha\beta}^{(k)} = 4|U_{\alpha k}|^2|U_{\beta k}|^2 \qquad (\alpha \neq \beta;\, k \geq 4)\,, \tag{3.8}$$

which allow us to write the transition probabilities as

$$\begin{aligned} P^{(\mathrm{SBL})}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}} \simeq\ & \sum_{k=4}^{N} \sin^2 2\vartheta_{\alpha\beta}^{(k)} \sin^2 \varDelta_{k1} \\ & + 2\sum_{k=4}^{N}\sum_{j=k+1}^{N} \sin 2\vartheta_{\alpha\beta}^{(k)} \sin 2\vartheta_{\alpha\beta}^{(j)} \sin\varDelta_{k1} \sin\varDelta_{j1} \cos\left(\varDelta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk}\right). \end{aligned} \tag{3.9}$$

From the first line, one can see that each effective mixing angle $\vartheta_{\alpha\beta}^{(k)}$ parameterizes the amount of $\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}$ transitions due to the mixing of $\overset{(-)}{\nu}_{\alpha}$ and $\overset{(-)}{\nu}_{\beta}$ with $\overset{(-)}{\nu}_{k}$. The second line in Eq. (3.9) is the interference between the $\overset{(-)}{\nu}_{k}$ and $\overset{(-)}{\nu}_{j}$ contributions, which depends on the same effective mixing angles.

Considering now the transitions between two different active neutrinos, from Eqs. (3.6) and (3.8), one can see that for each value of $k \geq 4$, the transition amplitude $\sin 2\vartheta_{\alpha\beta}^{(k)}$ and the disappearance amplitudes $\sin 2\vartheta_{\alpha\alpha}^{(k)}$ and $\sin 2\vartheta_{\beta\beta}^{(k)}$ depend only on the elements in k^{th} column of the mixing matrix and are related by[8] [101]

$$\sin^2 2\vartheta_{\alpha\beta}^{(k)} \simeq \tfrac{1}{4}\, \sin^2 2\vartheta_{\alpha\alpha}^{(k)}\, \sin^2 2\vartheta_{\beta\beta}^{(k)} \qquad (\alpha = e, \mu, \tau)\,. \tag{3.10}$$

This relation is very important, because it constrains the oscillation signals that can be observed in short-baseline appearance and disappearance experiments in any $3 + N_{\mathrm{s}}$ mixing scheme with sterile neutrinos. Its experimental test is crucial for the acceptance or rejection of these schemes. In particular, since both $\sin^2 2\vartheta_{\alpha\alpha}^{(k)}$ and $\sin^2 2\vartheta_{\beta\beta}^{(k)}$ are small for $\alpha, \beta = e, \mu, \tau$, the amplitudes of the short-baseline transition probabilities between active neutrinos are quadratically suppressed. We will see in Section 4 that the current short-baseline data have an appearance–disappearance tension due to the constraint in Eq. (3.10).

[8] This relation was derived in the case of $3+1$ mixing (see Eq. (3.13)) in Refs. [99,100].

In the following part of this section, we discuss briefly the main peculiar characteristics of short-baseline oscillations in the cases of $3+1$ and $3+2$ neutrino mixing.

3.1. 3+1 mixing

In the case of $3+1$ neutrino mixing [99, 100, 102, 103], we have $\Delta m^2_{41} = \Delta m^2_{\mathrm{SBL}}$ and $\varDelta_{41} \sim 1$ in short-baseline experiments. The transition and survival probabilities can be written as

$$\begin{aligned}
P^{(\mathrm{SBL})}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} &\simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2 \varDelta_{41} \qquad (\alpha \neq \beta)\,, \\
P^{(\mathrm{SBL})}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} &\simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2 \varDelta_{41}\,,
\end{aligned} \tag{3.11}$$

with the transition and survival amplitudes

$$\begin{aligned}
\sin^2 2\vartheta_{\alpha\beta} &= 4|U_{\alpha4}|^2|U_{\beta4}|^2 \qquad (\alpha \neq \beta)\,, \\
\sin^2 2\vartheta_{\alpha\alpha} &= 4|U_{\alpha4}|^2 \left(1 - |U_{\alpha4}|^2\right)\,,
\end{aligned} \tag{3.12}$$

and with the appearance–disappearance constraint [99, 100]

$$\sin^2 2\vartheta_{\alpha\beta} \simeq \tfrac{1}{4} \sin^2 2\vartheta_{\alpha\alpha} \sin^2 2\vartheta_{\beta\beta} \qquad (\alpha = e, \mu, \tau)\,. \tag{3.13}$$

The transition and survival probabilities in Eq. (3.11) depend only on the largest squared-mass difference $\Delta m^2_{41} = \Delta m^2_{\mathrm{SBL}}$ and on the absolute values of the elements in the fourth column of the mixing matrix. The transition probabilities of neutrinos and antineutrinos are equal, because the absolute values of the elements in the fourth column of the mixing matrix do not depend on the CP-violating phases in the mixing matrix. Hence, even if there are CP-violating phases in the mixing matrix, CP violation cannot be measured in short-baseline experiments. In order to measure the effects of these phases, it is necessary to perform experiments sensitive to the oscillations generated by the smaller squared-mass differences $\Delta m^2_{\mathrm{ATM}}$ [104–106] or $\Delta m^2_{\mathrm{SOL}}$ [107].

3.2. 3+2 mixing

In the case of $3+2$ neutrino mixing [85, 108–111], we have $\Delta m^2_{51} \approx \Delta m^2_{41} = \Delta m^2_{\mathrm{SBL}}$ and $\varDelta_{51} \approx \varDelta_{41} \sim 1$ in short-baseline experiments. From Eq. (3.7), we obtain the short-baseline survival probabilities of active neutrinos

$$P^{(\mathrm{SBL})}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sin^2 2\vartheta^{(4)}_{\alpha\alpha} \sin^2 \varDelta_{41} - \sin^2 2\vartheta^{(5)}_{\alpha\alpha} \sin^2 \Delta_{51} \qquad (\alpha = e, \mu, \tau)\,, \tag{3.14}$$

and from Eq. (3.9), we obtain the short-baseline transition probabilities

$$P^{(\mathrm{SBL})}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sin^2 2\vartheta^{(4)}_{\alpha\beta} \sin^2 \Delta_{41} + \sin^2 2\vartheta^{(5)}_{\alpha\beta} \sin^2 \Delta_{51}$$
$$+2 \sin 2\vartheta^{(4)}_{\alpha\beta} \sin 2\vartheta^{(5)}_{\alpha\beta} \sin \Delta_{41} \sin \Delta_{51} \cos\left(\Delta_{54} \overset{(+)}{-} \eta_{\alpha\beta 54}\right) \qquad (\alpha \neq \beta)\,. \tag{3.15}$$

The appearance and disappearance amplitudes are related by the general constraint in Eq. (3.10). The $3+2$ scheme has the important characteristic that CP violation is observable in short-baseline experiments through the asymmetries

$$\begin{aligned} A^{(\mathrm{SBL})}_{\alpha\beta} &= P^{(\mathrm{SBL})}_{\nu_\alpha \to \nu_\beta} - P^{(\mathrm{SBL})}_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} \\ &\simeq 4 \sin 2\vartheta^{(4)}_{\alpha\beta} \sin 2\vartheta^{(5)}_{\alpha\beta} \sin \Delta_{41} \sin \Delta_{51} \sin \Delta_{54} \sin \eta_{\alpha\beta 54} \end{aligned} \tag{3.16}$$

for $\alpha \neq \beta$.

4. Global fits of short-baseline data

The results of several analyses of short-baseline neutrino oscillation data have been published after the discovery of the LSND anomaly in the middle 90s [85, 87, 99, 100, 102, 108, 110, 112–125]. The interest in short-baseline neutrino oscillations was renewed after the discovery in 2006 of the gallium neutrino anomaly [10–14, 126–132] and especially after the discovery in 2011 of the reactor antineutrino anomaly [7, 14, 19, 89, 91–93, 101, 133–144].

Here, we review the results of the global fit of short-baseline neutrino oscillation data presented in Ref. [19], in which the data of the following three groups of experiments have been considered:

(A) The $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ appearance data of the LSND [18], MiniBooNE [146], BNL-E776 [147], KARMEN [148], NOMAD [149], ICARUS [150] and OPERA [151] experiments[9].

(B) The following $\overset{(-)}{\nu}_e$ disappearance data: (1) the data of the Bugey-4 [154], ROVNO91 [155], Bugey-3 [156], Gosgen [157], ILL [158], Krasnoyarsk [159], Rovno88 [160], SRP [161], Chooz [162], Palo Verde [163], Double Chooz [164], and Daya Bay [165] reactor antineutrino experiments with the new theoretical fluxes [7–9, 75]; (2) the data of the

[9] The correct but more complicated analysis of the ICARUS and OPERA data presented in Ref. [152] (see also Ref. [153]) have not been considered because it would not change significantly the results of the global fits.

GALLEX [15] and SAGE [16] gallium radioactive source experiments with the statistical method discussed in Ref. [13], considering the recent ${}^{71}\mathrm{Ga}({}^{3}\mathrm{He}, {}^{3}\mathrm{H}){}^{71}\mathrm{Ge}$ cross-section measurement in Ref. [166]; (3) the solar neutrino constraint on $\sin^2 2\vartheta_{ee}$ [14, 167–170]; (4) the KARMEN [171] and LSND [172] $\nu_e + {}^{12}\mathrm{C} \to {}^{12}\mathrm{N}_{\mathrm{gs}} + e^-$ scattering data [134], with the method discussed in Ref. [137].

(C) The constraints on $\overset{(-)}{\nu}_{\mu}$ disappearance obtained from the data of the CDHSW experiment [173], from the analysis [85] of the data of atmospheric neutrino oscillation experiments, from the analysis [136, 174] of the MINOS neutral-current data [175] and from the analysis of the SciBooNE–MiniBooNE data neutrino [176] and antineutrino [177] data.

The MiniBooNE data require a special treatment, because they show an anomalous excess in the low-energy bins [146, 178] which, as explained later, induces a tension in the global analysis of the data of short-baseline neutrino oscillation experiments [136, 137]. Hence, we will discuss two types of global fits: "total" (TotGLO) and "pragmatic" (PrGLO). In the total fits, all the data listed above of short-baseline neutrino oscillation experiments are taken into account. In the pragmatic fits [91], the anomalous low-energy bins of the MiniBooNE experiment [146, 178] are omitted.

Table I summarizes the statistical results obtained from global fits of the data above in the $3+1$ and $3+2$ schemes. Besides the total and pragmatic fits, there is also a $3+1$-noMB fit without MiniBooNE data and a $3+1$-noLSND fit without LSND data which are explained below.

From Table I, one can see that in all fits which include the LSND data, the absence of short-baseline oscillations is nominally disfavored by about 6σ, because the improvement of the χ^2 with short-baseline oscillations is much larger than the number of oscillation parameters.

In both the $3+1$ and $3+2$ schemes, the goodness-of-fit in the total analysis is significantly worse than that in the pragmatic analysis and the appearance–disappearance parameter goodness-of-fit is much worse. This result confirms the fact that the MiniBooNE low-energy anomaly is incompatible with neutrino oscillations, because it would require a small value of Δm^2_{41} and a large value of $\sin^2 2\vartheta_{e\mu}$ [136, 137], which are excluded by the data of other experiments (see Ref. [91] for further details)[10]. Note that the appearance–disappearance tension in the $3+2$-TotGLO fit is even worse

[10] One could fit the three anomalous MiniBooNE low-energy bins in a $3+2$ scheme [142] by considering the appearance data without the ICARUS [150] and OPERA [151] constraints, but the required large transition probability is excluded by the disappearance data.

TABLE I

Results of the global fit of short-baseline data taking into account all MiniBooNE data (TotGLO), only the MiniBooNE data above 475 MeV (PrGLO), without MiniBooNE data (noMB) and without LSND data (noLSND) in the $3+1$ and $3+2$ schemes. The first three lines give the minimum χ^2 ($(\chi^2_{\rm min})_{\rm GLO}$), the number of degrees of freedom ($\mathrm{NDF}_{\rm GLO}$) and the goodness-of-fit ($\mathrm{GoF}_{\rm GLO}$) of the global fit (GLO). The following five lines give the quantities relevant for the appearance–disappearance (APP–DIS) parameter goodness-of-fit (PG) [145]. The last three lines give the difference $\Delta\chi^2_{\rm NO}$ between the χ^2 without short-baseline oscillations and $(\chi^2_{\rm min})_{\rm GLO}$, the corresponding difference of number of degrees of freedom ($\mathrm{NDF}_{\rm NO}$) and the resulting number of σ's ($n\sigma_{\rm NO}$) for which the absence of oscillations is disfavored.

	3+1 TotGLO	3+1 PrGLO	3+1 noMB	3+1 noLSND	3+2 TotGLO	3+2 PrGLO
$(\chi^2_{\rm min})_{\rm GLO}$	306.0	276.3	251.2	291.3	299.6	271.1
$\mathrm{NDF}_{\rm GLO}$	268	262	230	264	264	258
$\mathrm{GoF}_{\rm GLO}$	5%	26%	16%	12%	7%	28%
$(\chi^2_{\rm min})_{\rm APP}$	98.9	77.0	50.9	91.8	86.0	69.6
$(\chi^2_{\rm min})_{\rm DIS}$	194.4	194.4	194.4	194.4	192.9	192.9
$\Delta\chi^2_{\rm PG}$	13.0	5.3	6.2	5.3	20.7	8.6
$\mathrm{NDF}_{\rm PG}$	2	2	2	2	4	4
$\mathrm{GoF}_{\rm PG}$	0.1%	7%	5%	7%	0.04%	7%
$\Delta\chi^2_{\rm NO}$	49.2	47.7	48.1	11.4	55.7	52.9
$\mathrm{NDF}_{\rm NO}$	3	3	3	3	7	7
$n\sigma_{\rm NO}$	6.4σ	6.3σ	6.4σ	2.6σ	6.1σ	5.9σ

than that in the $3+1$-TotGLO fit, since the $\Delta\chi^2_{\rm PG}$ is so much larger that it cannot be compensated by the additional degrees of freedom[11]. Therefore, we think that it is very likely that the MiniBooNE low-energy anomaly has an explanation which is different from neutrino oscillations[12]. The cause of the MiniBooNE low-energy excess of ν_e-like events is going to be investigated in the MicroBooNE experiment at Fermilab [182], which is a large Liquid Argon Time Projection Chamber (LArTPC) in which electrons and photons can be distinguished[13].

[11] This behavior has been explained in Ref. [179]. It was found also in the analysis presented in Ref. [93].

[12] Only some part of the MiniBooNE low-energy anomaly can be explained by taking into account nuclear effects in the energy reconstruction [144, 180, 181].

[13] In the MiniBooNE, mineral-oil Cherenkov detector, ν_e-induced events cannot be distinguished from ν_μ-induced events which produce only a visible photon (for example, neutral-current π^0 production in which only one of the two decay photons is visible).

In the following, we adopt the "pragmatic approach" advocated in Ref. [91] which considers the PrGLO fits without the anomalous MiniBooNE low-energy bins as more reliable than the TotGLO fits, which include the anomalous MiniBooNE low-energy bins.

The $3+2$ mixing scheme was considered to be interesting in 2010 when the MiniBooNE neutrino [178] and antineutrino [183] data showed a CP-violating tension, but this tension almost disappeared in the final MiniBooNE data [146]. In fact, from Table I, one can see that there is little improvement of the $3+2$-PrGLO fit with respect to the $3+1$-PrGLO fit, in spite of the four additional parameters and the additional possibility of CP violation. Moreover, the p-value obtained by restricting the $3+2$ scheme to $3+1$ disfavors the $3+1$ scheme only at 1.1σ. Therefore, we think that considering the larger complexity of the $3+2$ scheme is not justified by the data and in the following, we consider only the $3+1$ mixing scheme.

Figure 1 shows the allowed regions in the $\sin^2 2\vartheta_{e\mu}$–Δm^2_{41}, $\sin^2 2\vartheta_{ee}$–Δm^2_{41} and $\sin^2 2\vartheta_{\mu\mu}$–$\Delta m^2_{41}$ planes obtained in the $3+1$-PrGLO fit. These regions are relevant, respectively, for $\overset{(-)}{\nu}_{\mu} \to \overset{(-)}{\nu}_{e}$ appearance, $\overset{(-)}{\nu}_{e}$ disappearance and $\overset{(-)}{\nu}_{\mu}$ disappearance searches. Figure 1 shows also the region allowed by $\overset{(-)}{\nu}_{\mu} \to \overset{(-)}{\nu}_{e}$ appearance data and the constraints from $\overset{(-)}{\nu}_{e}$ disappearance and $\overset{(-)}{\nu}_{\mu}$ disappearance data. One can see that the combined disappearance constraint in the $\sin^2 2\vartheta_{e\mu}$–Δm^2_{41} plane excludes a large part of the region allowed by $\overset{(-)}{\nu}_{\mu} \to \overset{(-)}{\nu}_{e}$ appearance data, leading to the well-known appearance–disappearance tension [19,91–93,101,135–137,140,142,179] quantified by the parameter goodness-of-fit in Table I. The best-fit values of the oscillation parameters are $(\Delta m^2_{41})_{\rm bf} = 1.6\,{\rm eV}^2$, $(|U_{e4}|^2)_{\rm bf} = 0.028$, $(|U_{\mu 4}|^2)_{\rm bf} = 0.013$, which imply $(\sin^2 2\vartheta_{e\mu})_{\rm bf} = 0.0014$, $(\sin^2 2\vartheta_{ee})_{\rm bf} = 0.11$ and $(\sin^2 2\vartheta_{\mu\mu})_{\rm bf} = 0.050$.

It is interesting to investigate what are the impacts of the MiniBooNE and LSND experiments on the global analysis of short-baseline neutrino oscillation data. With this aim, we consider two additional $3+1$ fits: a $3+1$-noMB fit without MiniBooNE data and a $3+1$-noLSND fit without LSND data. From Table I, one can see that the results of the $3+1$-noMB fit are similar to those of the $3+1$-PrGLO fit and the nominal exclusion of the case of no-oscillations remains at the level of 6σ. On the other hand, in the $3+1$-noLSND fit, without LSND data, the nominal exclusion of the case of no-oscillations drops dramatically to 2.6σ. In fact, in this case, the main indication in favor of short-baseline oscillations is given by the reactor and gallium anomalies which have a similar statistical significance [14]. Therefore, it is clear that the LSND experiment is still crucial for the indication in favor of short-baseline $\bar{\nu}_{\mu} \to \bar{\nu}_{e}$ transitions.

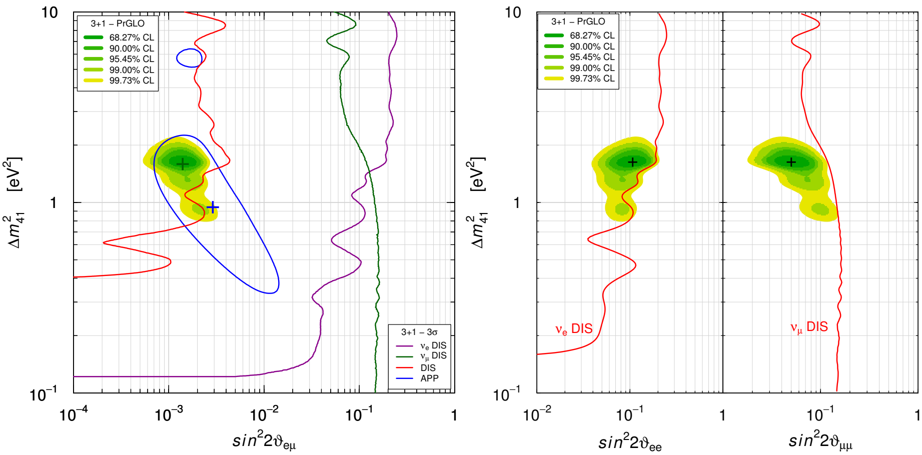

Fig. 1. Allowed regions in the $\sin^2 2\vartheta_{e\mu}$–Δm^2_{41}, $\sin^2 2\vartheta_{ee}$–Δm^2_{41} and $\sin^2 2\vartheta_{\mu\mu}$–$\Delta m^2_{41}$ planes obtained in the pragmatic $3+1$ global fit PrGLO of short-baseline neutrino oscillation data compared with the 3σ allowed regions obtained from $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ short-baseline appearance data (APP) and the 3σ constraints obtained from $\overset{(-)}{\nu}_e$ short-baseline disappearance data (ν_e DIS), $\overset{(-)}{\nu}_\mu$ short-baseline disappearance data (ν_μ DIS) and the combined short-baseline disappearance data (DIS). The best-fit points of the global (PrGLO) and APP fits are indicated by crosses.

5. Conclusions

The reactor, gallium and LSND anomalies can be explained by neutrino oscillations if the standard three-neutrino mixing paradigm is extended with the addition of light sterile neutrinos which can give us important information on the new physics beyond the Standard Model.

The global fits of short-baseline neutrino oscillation data in the framework of mixing schemes with one or more sterile neutrinos suffer from a tension between the results of appearance and disappearance short-baseline neutrino oscillation experiments. This tension can be alleviated adopting the "pragmatic approach" advocated in Ref. [91], in which the anomalous MiniBooNE low-energy excess of ν_e-like events is neglected from the global analysis of short-baseline neutrino oscillation data. The cause of the MiniBooNE low-energy excess is going to be investigated in the MicroBooNE experiment at Fermilab [182].

Moreover, the cosmological data indicate a tension between the necessity to have a sterile neutrino mass at the eV scale and the expected full thermalization of the sterile neutrinos through active–sterile oscillations in the early Universe [140, 179, 184–186]. This problem led several authors to propose new mechanisms that can relieve the tension: a large lepton asymmetry [187–198], new neutrino interactions [199–211], entropy production after neutrino decoupling [212], neutrino decay [213], very low reheating temperature [214, 215], time varying dark energy components [216], a larger cosmic expansion rate at the time of sterile neutrino production [217], inflationary freedom [218].

Because of the scarcity of data and the tensions between different data sets, the possible existence of light sterile neutrinos at the eV scale is controversial and needs new reliable experimental checks. Fortunately, there is an impressive program of new experiments which are planned to check the existence of eV sterile neutrinos (see the reviews in Refs. [19, 219–225]) with high-precision investigations of neutrino oscillations over short baselines by using very accurate detectors for investigating the disappearance of reactor electron antineutrinos (DANSS [226], NEOS [227], Neutrino-4 [228], PROSPECT [229], SoLid [230], STEREO [231]) and electron neutrinos produced by very intense radioactive sources (BEST [232], CeSOX [233]). New accelerator experiments will perform robust investigations of short-baseline $\overset{(-)}{\nu}_{\mu} \to \overset{(-)}{\nu}_{e}$ transitions (JSNS2 [234], SBN [235]) and $\overset{(-)}{\nu}_{\mu}$ disappearance (KPipe [236], SBN [235]). Hence, we are confident that the question of the existence of the light sterile neutrinos indicated by the reactor, gallium and LSND anomalies will be answered in a definitive way in the next years.

Moreover, light sterile neutrinos have important effects that could be observed in β decay experiments [237–242], in neutrinoless double-β decay experiments [14, 243–251], in solar neutrino experiments [14, 93, 107, 168–170, 252], in long-baseline neutrino oscillation experiments [104–106, 253–258], in atmospheric neutrino experiments [259–268], in supernova neutrino experiments [116, 269–276], in indirect dark matter detection [277], in high-energy cosmic neutrinos experiments [278–280], and in cosmology (see Refs. [19, 281–285]).

Let us finally emphasize that the discovery of the existence of sterile neutrinos would be a major discovery which would have a profound impact not only on neutrino physics, but on our whole view of fundamental physics, because sterile neutrinos are elementary particles beyond the Standard Model. The existence of light sterile neutrinos would prove that there is new physics beyond the Standard Model at low energies and their properties can give important information on this new physics.

This work was partially supported by the research grant of the Theoretical Astroparticle Physics number 2012CPPYP7 under the program PRIN 2012 funded by the Ministero dell'Istruzione, Università e della Ricerca (MIUR).

REFERENCES

[1] G. Bellini *et al.*, *Adv. High Energy Phys.* **2014**, 191960 (2014) [arXiv:1310.7858 [hep-ph]].

[2] Y. Wang, Z.-z. Xing, arXiv:1504.06155 [hep-ph].

[3] M. Gonzalez-Garcia, M. Maltoni, T. Schwetz, *Nucl. Phys. B* **908**, 199 (2016) [arXiv:1512.06856 [hep-ph]].

[4] D.V. Forero, M. Tortola, J.W.F. Valle, *Phys. Rev. D* **90**, 093006 (2014) [arXiv:1405.7540 [hep-ph]].

[5] M. Gonzalez-Garcia, M. Maltoni, T. Schwetz, *J. High Energy Phys.* **1411**, 052 (2014) [arXiv:1409.5439 [hep-ph]].

[6] F. Capozzi *et al.*, *Nucl. Phys. B* **908**, 218 (2016) [arXiv:1601.07777 [hep-ph]].

[7] G. Mention *et al.*, *Phys. Rev. D* **83**, 073006 (2011) [arXiv:1101.2755 [hep-ex]].

[8] T.A. Mueller *et al.*, *Phys. Rev. C* **83**, 054615 (2011) [arXiv:1101.2663 [hep-ex]].

[9] P. Huber, *Phys. Rev. C* **84**, 024617 (2011) [arXiv:1106.0687 [hep-ph]].

[10] J.N. Abdurashitov *et al.* [SAGE Collaboration], *Phys. Rev. C* **73**, 045805 (2006) [arXiv:nucl-ex/0512041].

[11] M. Laveder, *Nucl. Phys. Proc. Suppl.* **168**, 344 (2007).

[12] C. Giunti, M. Laveder, *Mod. Phys. Lett. A* **22**, 2499 (2007) [arXiv:hep-ph/0610352].

[13] C. Giunti, M. Laveder, *Phys. Rev. C* **83**, 065504 (2011) [arXiv:1006.3244 [hep-ph]].

[14] C. Giunti *et al.*, *Phys. Rev. D* **86**, 113014 (2012) [arXiv:1210.5715 [hep-ph]].

[15] F. Kaether *et al.*, *Phys. Lett. B* **685**, 47 (2010) [arXiv:1001.2731 [hep-ex]].

[16] J.N. Abdurashitov *et al.* [SAGE Collaboration], *Phys. Rev. C* **80**, 015807 (2009) [arXiv:0901.2200 [nucl-ex]].

[17] C. Athanassopoulos *et al.*, *Phys. Rev. Lett.* **75**, 2650 (1995) [arXiv:nucl-ex/9504002].

[18] A. Aguilar *et al.* [LSND Collaboration], *Phys. Rev. D* **64**, 112007 (2001) [arXiv:hep-ex/0104049].

[19] S. Gariazzo *et al.*, *J. Phys. G* **43**, 033001 (2016) [arXiv:1507.08204 [hep-ph]].

[20] C. Giunti, C.W. Kim, *Fundamentals of Neutrino Physics and Astrophysics*, Oxford University Press, Oxford, UK, 2007.

[21] B. Pontecorvo, *Sov. Phys. JETP* **26**, 984 (1968).

[22] P.C. de Holanda, A.Y. Smirnov, *Phys. Rev. D* **69**, 113002 (2004) [arXiv:hep-ph/0307266].

[23] P.C. de Holanda, A.Y. Smirnov, *Phys. Rev. D* **83**, 113011 (2011) arXiv:1012.5627 [hep-ph]].
[24] C. Das, J. Pulido, M. Picariello, *Phys. Rev. D* **79**, 073010 (2009) [arXiv:0902.1310 [hep-ph]].
[25] S.K. Kang *et al.*, *Adv. High Energy Phys.* **2013**, 138109 (2013) [arXiv:1408.3211 [hep-ph]].
[26] P. Bakhti, Y. Farzan, *J. High Energy Phys.* **1310**, 200 (2013) [arXiv:1308.2823 [hep-ph]].
[27] A. Palazzo, *J. High Energy Phys.* **1310**, 172 (2013) [arXiv:1308.5880 [hep-ph]].
[28] I. Girardi, D. Meloni, *Phys. Rev. D* **90**, 073011 (2014) [arXiv:1403.5507 [hep-ph]].
[29] I. Girardi *et al.*, *J. High Energy Phys.* **1408**, 057 (2014) [arXiv:1405.6540 [hep-ph]].
[30] A.D. Iura, I. Girardi, D. Meloni, *J. Phys. G* **42**, 065003 (2015) [arXiv:1411.5330 [hep-ph]].
[31] F.P. An *et al.* [Daya Bay Collaboration], *Phys. Rev. Lett.* **113**, 141802 (2014) [arXiv:1407.7259 [hep-ex]].
[32] T. Asaka, S. Blanchet, M. Shaposhnikov, *Phys. Lett. B* **631**, 151 (2005) [arXiv:hep-ph/0503065].
[33] T. Asaka, M. Shaposhnikov, *Phys. Lett. B* **620**, 17 (2005) [arXiv:hep-ph/0505013].
[34] T. Asaka, A. Kusenko, M. Shaposhnikov, *Phys. Lett. B* **638**, 401 (2006) [arXiv:hep-ph/0602150].
[35] T. Asaka, M. Laine, M. Shaposhnikov, *J. High Energy Phys.* **0606**, 053 (2006) [arXiv:hep-ph/0605209].
[36] T. Asaka, M. Laine, M. Shaposhnikov, *J. High Energy Phys.* **0701**, 091 (2007) [arXiv:hep-ph/0612182].
[37] R. Adhikari *et al.*, arXiv:1602.04816 [hep-ph].
[38] A. Kusenko, S. Pascoli, D. Semikoz, *J. High Energy Phys.* **0511**, 028 (2005) [arXiv:hep-ph/0405198].
[39] G. Gelmini *et al.*, *J. Cosmol. Astropart. Phys.* **0810**, 029 (2008) [arXiv:0803.2735 [astro-ph]].
[40] M. Masip, P. Masjuan, D. Meloni, *J. High Energy Phys.* **1301**, 106 (2013) [arXiv:1210.1519 [hep-ph]].
[41] H. Ishida, M. Kusakabe, H. Okada, *Phys. Rev. D* **90**, 083519 (2014) [arXiv:1403.5995 [astro-ph.CO]].
[42] S. Antusch, O. Fischer, *J. High Energy Phys.* **1505**, 053 (2015) [arXiv:1502.05915 [hep-ph]].
[43] F.F. Deppisch, P.S.B. Dev, A. Pilaftsis, *New J. Phys.* **17**, 075019 (2015) [arXiv:1502.06541 [hep-ph]].
[44] S. Antusch, O. Fischer, *J. High Energy Phys.* **1410**, 094 (2014) [arXiv:1407.6607 [hep-ph]].

[45] D.K. Ghosh, G. Senjanovic, Y. Zhang, *Phys. Lett. B* **698**, 420 (2011) [arXiv:1010.3968 [hep-ph]].

[46] V. Barger, P.F. Perez, S. Spinner, *Phys. Lett. B* **696**, 509 (2011) [arXiv:1010.4023 [hep-ph]].

[47] Y. Grossman, D.J. Robinson, *J. High Energy Phys.* **1101**, 132 (2011) [arXiv:1009.2781 [hep-ph]].

[48] A. de Gouvea, W.C. Huang, *Phys. Rev. D* **85**, 053006 (2012) [arXiv:1110.6122 [hep-ph]].

[49] C.S. Chen, R. Takahashi, *Eur. Phys. J. C* **72**, 2089 (2012) [arXiv:1112.2102 [hep-ph]].

[50] J. Barry, W. Rodejohann, H. Zhang, *J. Cosmol. Astropart. Phys.* **1201**, 052 (2012) [arXiv:1110.6382 [hep-ph]].

[51] C.S. Fong, R.N. Mohapatra, I. Sung, *Phys. Lett. B* **704**, 171 (2011) [arXiv:1107.4086 [hep-ph]].

[52] H. Zhang, *Phys. Lett. B* **714**, 262 (2012) [arXiv:1110.6838 [hep-ph]].

[53] P.H. Gu, *Phys. Rev. D* **81**, 095002 (2013) [arXiv:1001.1341 [hep-ph]].

[54] M. Nemevsek, G. Senjanovic, V. Tello, *Phys. Rev. Lett.* **110**, 151802 (2013) [arXiv:1211.2837 [hep-ph]].

[55] P.S.B. Dev, A. Pilaftsis, *Phys. Rev. D* **87**, 053007 (2013) [arXiv:1212.3808 [hep-ph]].

[56] M. Duerr, P.F. Perez, M. Lindner, *Phys. Rev. D* **88**, 051701 (2013) [arXiv:1306.0568 [hep-ph]].

[57] W. Rodejohann, H. Zhang, *Phys. Lett. B* **737**, 81 (2014) [arXiv:1407.2739 [hep-ph]].

[58] J.L. Rosner, *Phys. Rev. D* **90**, 035005 (2014) [arXiv:1404.5198 [hep-ph]].

[59] R. Adhikari, D. Borah, E. Ma, arXiv:1411.4602 [hep-ph].

[60] A. Merle, S. Morisi, W. Winter, *J. High Energy Phys.* **1407**, 039 (2014) [arXiv:1402.6332 [hep-ph]].

[61] A. de Gouvea, D. Hernandez, *J. High Energy Phys.* **1510**, 046 (2015) [arXiv:1507.00916 [hep-ph]].

[62] R.R. Volkas, *Prog. Part. Nucl. Phys.* **48**, 161 (2002) [arXiv:hep-ph/0111326].

[63] R.N. Mohapatra, A.Y. Smirnov, *Annu. Rev. Nucl. Part. Sci.* **56**, 569 (2006) [arXiv:hep-ph/0603118].

[64] S.L. Glashow, *Nucl. Phys.* **22**, 579 (1961).

[65] S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967).

[66] A. Salam, Proc. of the 8^{th} Nobel Symposium on "Elementary Particle Theory, Relativistic Groups and Analyticity', Stockholm, Sweden, 1968, edited by N. Svartholm, pp. 367–377.

[67] S. Bilenky, *Introduction to the Physics of Massive and Mixed Neutrinos*, volume 817, Springer-Verlag, Berlin–Heidelberg 2010.

[68] Z.z. Xing, S. Zhou, *Neutrinos in Particle Physics, Astronomy and Cosmology*, Zhejiang University Press, Hangzhou 2011.

[69] M. Gonzalez-Garcia *et al.*, *J. High Energy Phys.* **1212**, 123 (2012) [arXiv:1209.3023 [hep-ph]].

[70] J. Beringer *et al.* [Particle Data Group], *Phys. Rev. D* **86**, 010001 (2012).

[71] R. Foot, H. Lew, R.R. Volkas, *J. Phys. G* **19**, 361 (1993) [arXiv:hep-ph/9209259].

[72] C. Giunti, A. Studenikin, *Rev. Mod. Phys.* **87**, 531 (2015) [arXiv:1403.6344 [hep-ph]].

[73] R.N. Mohapatra, P.B. Pal, *Massive Neutrinos in Physics and Astrophysics*, World Scientific, 2004.

[74] A. Boyarsky, O. Ruchayskiy, M. Shaposhnikov, *Annu. Rev. Nucl. Part. Sci.* **59**, 191 (2009) [arXiv:0901.0011 [hep-ph]].

[75] K.N. Abazajian *et al.*, arXiv:1204.5379 [hep-ph].

[76] M. Drewes, *Int. J. Mod. Phys. E* **22**, 1330019 (2013) [arXiv:1303.6912 [hep-ph]].

[77] S.M. Bilenky, S.T. Petcov, *Rev. Mod. Phys.* **59**, 671 (1987).

[78] S.L. Glashow, J. Iliopoulos, L. Maiani, *Phys. Rev. D* **2**, 1285 (1970).

[79] J. Schechter, J.W.F. Valle, *Phys. Rev. D* **22**, 2227 (1980).

[80] B.W. Lee, R.E. Shrock, *Phys. Rev. D* **16**, 1444 (1977).

[81] S. Antusch *et al.*, *J. High Energy Phys.* **0610**, 084 (2006) [arXiv:hep-ph/0607020].

[82] C. Jarlskog, *Phys. Lett. B* **241**, 579 (1990).

[83] S.M. Bilenky, W. Grimus, H. Neufeld, *Phys. Lett. B* **252**, 119 (1990).

[84] S. Schael *et al.* [ALEPH, DELPHI, L3, OPAL, SLD, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavour Group], *Phys. Rep.* **427**, 257 (2006) [arXiv:hep-ex/0509008].

[85] M. Maltoni, T. Schwetz, *Phys. Rev. D* **76**, 093005 (2007) [arXiv:0705.0107 [hep-ph]].

[86] M. Rahman, arXiv:1510.01452 [hep-ph].

[87] A.E. Nelson, *Phys. Rev. D* **84**, 053001 (2011) [arXiv:1010.3970 [hep-ph]].

[88] J. Fan, P. Langacker, *J. High Energy Phys.* **1204**, 083 (2012) [arXiv:1201.6662 [hep-ph]].

[89] E. Kuflik, S.D. McDermott, K.M. Zurek, *Phys. Rev. D* **86**, 033015 (2012) [arXiv:1205.1791 [hep-ph]].

[90] J. Huang, A.E. Nelson, *Phys. Rev. D* **88**, 033016 (2013) [arXiv:1306.6079 [hep-ph]].

[91] C. Giunti *et al.*, *Phys. Rev. D* **88**, 073008 (2013) [arXiv:1308.5288 [hep-ph]].

[92] J. Kopp, M. Maltoni, T. Schwetz, *Phys. Rev. Lett.* **107**, 091801 (2011) [arXiv:1103.4570 [hep-ph]].

[93] J. Kopp *et al.*, *J. High Energy Phys.* **1305**, 050 (2013) [arXiv:1303.3011 [hep-ph]].

[94] P.A.R. Ade *et al.* [Planck Collaboration], *Astron. Astrophys.* **594**, A13 (2016) arXiv:1502.01589 [astro-ph.CO]].

[95] S. Bilenky, C. Giunti, *Int. J. Mod. Phys. A* **30**, 0001 (2015) [arXiv:1411.4791 [hep-ph]].

[96] S. Dell'Oro *et al.*, *Adv. High Energy Phys.* **2016**, 2162659 (2016) [arXiv:1601.07512 [hep-ph]].

[97] S. Bilenky, arXiv:1208.2497 [hep-ph].

[98] S. Bilenky, *Phys. Part. Nucl. Lett.* **12**, 453 (2015) [arXiv:1502.06158 [hep-ph]].

[99] N. Okada, O. Yasuda, *Int. J. Mod. Phys. A* **12**, 3669 (1997) [arXiv:hep-ph/9606411].

[100] S.M. Bilenky, C. Giunti, W. Grimus, *Eur. Phys. J. C* **1**, 247 (1998) [arXiv:hep-ph/9607372].

[101] C. Giunti, E.M. Zavanin, *Mod. Phys. Lett. A* **31**, 1650003 (2016) arXiv:1508.03172 [hep-ph]].

[102] S.M. Bilenky *et al.*, *Phys. Rev. D* **60**, 073007 (1999) [arXiv:hep-ph/9903454].

[103] M. Maltoni *et al.*, *New J. Phys.* **6**, 122 (2004) [arXiv:hep-ph/0405172].

[104] A. de Gouvea, K.J. Kelly, A. Kobach, *Phys. Rev. D* **91**, 053005 (2015) [arXiv:1412.1479 [hep-ph]].

[105] N. Klop, A. Palazzo, *Phys. Rev. D* **91**, 073017 (2015) [arXiv:1412.7524 [hep-ph]].

[106] J.M. Berryman *et al.*, *Phys. Rev. D* **92**, 073012 (2015) [arXiv:1507.03986 [hep-ph]].

[107] H. Long, Y. Li, C. Giunti, *Phys. Rev. D* **87**, 113004 (2013) [arXiv:1304.2207 [hep-ph]].

[108] M. Sorel, J. Conrad, M. Shaevitz, *Phys. Rev. D* **70**, 073004 (2004) [arXiv:hep-ph/0305255].

[109] G. Karagiorgi *et al.*, *Phys. Rev. D* **75**, 013011 (2007) [arXiv:hep-ph/0609177].

[110] G. Karagiorgi *et al.*, *Phys. Rev. D* **80**, 073001 (2009) [*Erratum ibid.* **81**, 039902 (2010)] [arXiv:0906.1997 [hep-ph]].

[111] M. Blennow, E. Fernandez-Martinez, *Phys. Lett. B* **704**, 223 (2011) [arXiv:1107.3992 [hep-ph]].

[112] J. Gomez-Cadenas, M. Gonzalez-Garcia, *Z. Phys. C* **71**, 443 (1996) [arXiv:hep-ph/9504246].

[113] S. Goswami, *Phys. Rev. D* **55**, 2931 (1997) [arXiv:hep-ph/9507212].

[114] C. Giunti, M.C. Gonzalez-Garcia, C. Pena-Garay, *Phys. Rev. D* **62**, 013005 (2000) [arXiv:hep-ph/0001101].

[115] V.D. Barger *et al.*, *Phys. Lett. B* **489**, 345 (2000) [arXiv:hep-ph/0008019].

[116] O. Peres, A. Smirnov, *Nucl. Phys. B* **599**, 3 (2001) [arXiv:hep-ph/0011054].

[117] W. Grimus, T. Schwetz, *Eur. Phys. J. C* **20**, 1 (2001) [arXiv:hep-ph/0102252].

[118] A. Strumia, *Phys. Lett. B* **539**, 91 (2002) [arXiv:hep-ph/0201134].

[119] M. Maltoni *et al.*, *Nucl. Phys. B* **643**, 321 (2002) [arXiv:hep-ph/0207157].

[120] R. Foot, *Mod. Phys. Lett. A* **18**, 2079 (2003) [arXiv:hep-ph/0210393].

[121] C. Giunti, *Mod. Phys. Lett. A* **18**, 1179 (2003) [arXiv:hep-ph/0302173].

[122] V. Barger, D. Marfatia, K. Whisnant, *Phys. Lett. B* **576**, 303 (2003) [arXiv:hep-ph/0308299].

[123] S. Goswami, W. Rodejohann, *J. High Energy Phys.* **0710**, 073 (2007) [arXiv:0706.1462 [hep-ph]].

[124] T. Schwetz, *J. High Energy Phys.* **0802**, 011 (2008) [arXiv:0710.2985 [hep-ph]].

[125] E. Akhmedov, T. Schwetz, *J. High Energy Phys.* **1010**, 115 (2010) [arXiv:1007.4171 [hep-ph]].

[126] M.A. Acero, C. Giunti, M. Laveder, *Phys. Rev. D* **78**, 073009 (2008) [arXiv:0711.4222 [hep-ph]].

[127] C. Giunti, M. Laveder, *Phys. Rev. D* **77**, 093002 (2008) [arXiv:0707.4593 [hep-ph]].

[128] C. Giunti, M. Laveder, *Phys. Rev. D* **80**, 013005 (2009) [arXiv:0902.1992 [hep-ph]].

[129] C. Giunti, M. Laveder, *Phys. Rev. D* **82**, 053005 (2010) [arXiv:1005.4599 [hep-ph]].

[130] C. Giunti, M. Laveder, *Phys. Rev. D* **82**, 113009 (2010) [arXiv:1008.4750 [hep-ph]].

[131] C. Giunti, M. Laveder, *Phys. Rev. D* **82**, 093016 (2010) [arXiv:1010.1395 [hep-ph]].

[132] C. Giunti, M. Laveder, *Phys. Rev. D* **83**, 053006 (2011) [arXiv:1012.0267 [hep-ph]].

[133] A. Donini *et al.*, *J. High Energy Phys.* **1107**, 105 (2011) [arXiv:1106.0064 [hep-ph]].

[134] J. Conrad, M. Shaevitz, *Phys. Rev. D* **85**, 013017 (2012) [arXiv:1106.5552 [hep-ex]].

[135] C. Giunti, M. Laveder, *Phys. Rev. D* **84**, 073008 (2011) [arXiv:1107.1452 [hep-ph]].

[136] C. Giunti, M. Laveder, *Phys. Rev. D* **84**, 093006 (2011) [arXiv:1109.4033 [hep-ph]].

[137] C. Giunti, M. Laveder, *Phys. Lett. B* **706**, 200 (2011) [arXiv:1111.1069 [hep-ph]].

[138] G. Karagiorgi, M.H. Shaevitz, J.M. Conrad, arXiv:1202.1024 [hep-ph].

[139] A. Donini *et al.*, *J. High Energy Phys.* **1207**, 161 (2012) [arXiv:1205.5230 [hep-ph]].

[140] M. Archidiacono *et al.*, *Phys. Rev. D* **86**, 065028 (2012) [arXiv:1207.6515 [astro-ph.CO]].

[141] C. Giunti *et al.*, *Phys. Rev. D* **87**, 013004 (2013) [arXiv:1212.3805 [hep-ph]].

[142] J. Conrad *et al.*, *Adv. High Energy Phys.* **2013**, 163897 (2013) [arXiv:1207.4765 [hep-ex]].

[143] G.H. Collin *et al.*, *Nucl. Phys. B* **908**, 354 (2016) [arXiv:1602.00671 [hep-ph]].

[144] M. Ericson *et al.*, *Phys. Rev. D* **93**, 073008 (2016) [arXiv:1602.01390 [hep-ph]].

[145] M. Maltoni, T. Schwetz, *Phys. Rev. D* **68**, 033020 (2003) [arXiv:hep-ph/0304176].

[146] A. Aguilar-Arevalo *et al.* [MiniBooNE Collaboration], *Phys. Rev. Lett.* **110**, 161801 (2013) [arXiv:1303.2588 [hep-ex]].

[147] L. Borodovsky *et al.* [BNL-E776 Collaboration], *Phys. Rev. Lett.* **68**, 274 (1992).

[148] B. Armbruster *et al.* [KARMEN Collaboration], *Phys. Rev. D* **65**, 112001 (2002) [arXiv:hep-ex/0203021].

[149] P. Astier *et al.* [NOMAD Collaboration], *Phys. Lett. B* **570**, 19 (2003) [arXiv:hep-ex/0306037].

[150] M. Antonello *et al.* [ICARUS Collaboration], *Eur. Phys. J. C* **73**, 2599 (2013) [arXiv:1307.4699 [hep-ex]].

[151] N. Agafonova *et al.* [OPERA Collaboration], *J. High Energy Phys.* **1307**, 004 (2013) [arXiv:1303.3953 [hep-ex]].

[152] A. Palazzo, *Phys. Rev. D* **91**, 091301 (2015) [arXiv:1503.03966 [hep-ph]].

[153] A. Donini *et al.*, *J. High Energy Phys.* **0712**, 013 (2007) [arXiv:0704.0388 [hep-ph]].

[154] Y. Declais *et al.*, *Phys. Lett. B* **338**, 383 (1994).

[155] A. Kuvshinnikov *et al.*, *JETP Lett.* **54**, 253 (1991).

[156] B. Achkar *et al.*, *Nucl. Phys. B* **434**, 503 (1995).

[157] G. Zacek *et al.* [Caltech-SIN-TUM Collaboration], *Phys. Rev. D* **34**, 2621 (1986).

[158] A. Hoummada *et al.*, *Appl. Radiat. Isotopes* **46**, 449 (1995).

[159] G.S. Vidyakin *et al.*, *Sov. Phys. JETP* **71**, 424 (1990).

[160] A.I. Afonin *et al.*, *Sov. Phys. JETP* **67**, 213 (1988).

[161] Z.D. Greenwood *et al.*, *Phys. Rev. D* **53**, 6054 (1996).

[162] M. Apollonio *et al.* [CHOOZ Collaboration], *Eur. Phys. J. C* **27**, 331 (2003) [arXiv:hep-ex/0301017].

[163] F. Boehm *et al.*, *Phys. Rev. D* **64**, 112001 (2001) [arXiv:hep-ex/0107009].

[164] Y. Abe *et al.* [Double Chooz Collaboration], *J. High Energy Phys.* **1410**, 86 (2014) [*Erratum ibid.* **1502**, 74 (2015)] [arXiv:1406.7763 [hep-ex]].

[165] F.P. An *et al.* [Daya Bay Collaboration], *Phys. Rev. Lett.* **116**, 061801 (2016) [arXiv:1508.04233 [hep-ex]].

[166] D. Frekers *et al.*, *Phys. Lett. B* **706**, 134 (2011).

[167] C. Giunti, Y. Li, *Phys. Rev. D* **80**, 113007 (2009) [arXiv:0910.5856 [hep-ph]].

[168] A. Palazzo, *Phys. Rev. D* **83**, 113013 (2011) [arXiv:1105.1705 [hep-ph]].

[169] A. Palazzo, *Phys. Rev. D* **85**, 077301 (2012) [arXiv:1201.4280 [hep-ph]].

[170] A. Palazzo, *Mod. Phys. Lett. A* **28**, 1330004 (2013) [arXiv:1302.1102 [hep-ph]].

[171] B. Armbruster *et al.* [KARMEN Collaboration], *Phys. Rev. C* **57**, 3414 (1998) [arXiv:hep-ex/9801007].

[172] L.B. Auerbach *et al.* [LSND Collaboration], *Phys. Rev. C* **64**, 065501 (2001) [arXiv:hep-ex/0105068].

[173] F. Dydak *et al.* [CDHSW Collaboration], *Phys. Lett. B* **134**, 281 (1984).

[174] D. Hernandez, A.Y. Smirnov, *Phys. Lett. B* **706**, 360 (2012) [arXiv:1105.5946 [hep-ph]].

[175] P. Adamson *et al.* [MINOS Collaboration], *Phys. Rev. Lett.* **107**, 011802 (2011) [arXiv:1104.3922 [hep-ex]].

[176] K.B.M. Mahn *et al.* [SciBooNE–MiniBooNE Collaboration], *Phys. Rev. D* **85**, 032007 (2012) [arXiv:1106.5685 [hep-ex]].

[177] G. Cheng *et al.* [SciBooNE–MiniBooNE Collaboration], *Phys. Rev. D* **86**, 052009 (2012) [arXiv:1208.0322 [hep-ex]].

[178] A.A. Aguilar-Arevalo *et al.* [MiniBooNE Collaboration], *Phys. Rev. Lett.* **102**, 101802 (2009) [arXiv:0812.2243 [hep-ex]].

[179] M. Archidiacono *et al.*, *Phys. Rev. D* **87**, 125034 (2013) [arXiv:1302.6720 [astro-ph.CO]].

[180] M. Martini, M. Ericson, G. Chanfray, *Phys. Rev. D* **85**, 093012 (2012) [arXiv:1202.4745 [hep-ph]].

[181] M. Martini, M. Ericson, G. Chanfray, *Phys. Rev. D* **87**, 013009 (2013) [arXiv:1211.1523 [hep-ph]].

[182] A.M. Szelc [ArgoNeuT and MicroBooNE collaborations], *AIP Conf. Proc.* **1666**, 180001 (2015).

[183] A.A. Aguilar-Arevalo *et al.* [MiniBooNE Collaboration], *Phys. Rev. Lett.* **105**, 181801 (2010) [arXiv:1007.1150 [hep-ex]].

[184] S. Gariazzo, C. Giunti, M. Laveder, *J. High Energy Phys.* **1311**, 211 (2013) [arXiv:1309.3192 [hep-ph]].

[185] M. Archidiacono *et al.*, *J. Cosmol. Astropart. Phys.* **1406**, 031 (2014) [arXiv:1404.1794 [astro-ph.CO]].

[186] J. Bergstrom *et al.*, *J. High Energy Phys.* **1410**, 104 (2014) [arXiv:1407.3806 [hep-ph]].

[187] R. Foot, R.R. Volkas, *Phys. Rev. Lett.* **75**, 4350 (1995) [arXiv:hep-ph/9508275].

[188] R. Foot, R.R. Volkas, *Phys. Rev. D* **55**, 5147 (1997) [arXiv:hep-ph/9610229].

[189] N.F. Bell, R. Foot, R.R. Volkas, *Phys. Rev. D* **58**, 105010 (1998) [arXiv:hep-ph/9805259].

[190] N.F. Bell, R.R. Volkas, Y.Y. Wong, *Phys. Rev. D* **59**, 113001 (1999) [arXiv:hep-ph/9809363].

[191] X. Shi, G.M. Fuller, K. Abazajian, *Phys. Rev. D* **60**, 063002 (1999) [arXiv:astro-ph/9905259].

[192] P. Di Bari, R. Foot, *Phys. Rev. D* **61**, 105012 (2000) [arXiv:hep-ph/9912215].

[193] P. Di Bari, R. Foot, *Phys. Rev. D* **65**, 045003 (2002) [arXiv:hep-ph/0103192].

[194] Y.Z. Chu, M. Cirelli, *Phys. Rev. D* **74**, 085015 (2006) [arXiv:astro-ph/0608206].

[195] S. Hannestad, I. Tamborra, T. Tram, *J. Cosmol. Astropart. Phys.* **1207**, 025 (2012) [arXiv:1204.5861 [astro-ph.CO]].

[196] A. Mirizzi *et al.*, *Phys. Rev. D* **86**, 053009 (2012) [arXiv:1206.1046 [hep-ph]].

[197] N. Saviano *et al.*, *Phys. Rev. D* **87**, 073006 (2013) [arXiv:1302.1200 [astro-ph.CO]].

[198] S. Hannestad, R.S. Hansen, T. Tram, *J. Cosmol. Astropart. Phys.* **1304**, 032 (2013) [arXiv:1302.7279 [astro-ph.CO]].

[199] L. Bento, Z. Berezhiani, *Phys. Rev. D* **64**, 115015 (2001) [arXiv:hep-ph/0108064].

[200] B. Dasgupta, J. Kopp, *Phys. Rev. Lett.* **112**, 031803 (2014) [arXiv:1310.6337 [hep-ph]].

[201] S. Hannestad, R.S. Hansen, T. Tram, *Phys. Rev. Lett.* **112**, 031802 (2014) [arXiv:1310.5926 [astro-ph.CO]].

[202] T. Bringmann, J. Hasenkamp, J. Kersten, *J. Cosmol. Astropart. Phys.* **1407**, 042 (2014) [arXiv:1312.4947 [hep-ph]].

[203] J.F. Cherry, A. Friedland, I.M. Shoemaker, arXiv:1411.1071 [hep-ph].

[204] P. Ko, Y. Tang, *Phys. Lett. B* **739**, 62 (2014) [arXiv:1404.0236 [hep-ph]].

[205] M. Archidiacono *et al.*, *Phys. Rev. D* **91**, 065021 (2015) [arXiv:1404.5915 [astro-ph.CO]].

[206] N. Saviano *et al.*, *Phys. Rev. D* **90**, 113009 (2014) [arXiv:1409.1680 [astro-ph.CO]].

[207] A. Mirizzi *et al.*, *Phys. Rev. D* **91**, 025019 (2015) [arXiv:1410.1385 [hep-ph]].

[208] Y. Tang, *Phys. Lett. B* **750**, 201 (2015) [arXiv:1501.00059 [hep-ph]].

[209] F. Forastieri, M. Lattanzi, P. Natoli, *J. Cosmol. Astropart. Phys.* **1507**, 014 (2015) [arXiv:1504.04999 [astro-ph.CO]].

[210] X. Chu, B. Dasgupta, J. Kopp, *J. Cosmol. Astropart. Phys.* **1510**, 011 (2015) [arXiv:1505.02795 [hep-ph]].

[211] M. Archidiacono *et al.*, *Phys. Rev. D* **93**, 045004 (2016) [arXiv:1508.02504 [astro-ph.CO]].

[212] C.M. Ho, R.J. Scherrer, *Phys. Rev. D* **87**, 065016 (2013) [arXiv:1212.1689 [hep-ph]].

[213] S. Gariazzo, C. Giunti, M. Laveder, arXiv:1404.6160 [astro-ph.CO].

[214] G. Gelmini, S. Palomares-Ruiz, S. Pascoli, *Phys. Rev. Lett.* **93**, 081302 (2004) [arXiv:astro-ph/0403323].

[215] A.Y. Smirnov, R.Z. Funchal, *Phys. Rev. D* **74**, 013001 (2006) [arXiv:hep-ph/0603009].

[216] E. Giusarma *et al.*, *Phys. Rev. D* **85**, 083522 (2012) [arXiv:1112.4661 [astro-ph.CO]].

[217] T. Rehagen, G.B. Gelmini, *J. Cosmol. Astropart. Phys.* **1406**, 044 (2014) [arXiv:1402.0607 [hep-ph]].

[218] S. Gariazzo, C. Giunti, M. Laveder, *J. Cosmol. Astropart. Phys.* **1504**, 023 (2015) [arXiv:1412.7405 [astro-ph.CO]].

[219] T. Lasserre, *Nucl. Part. Phys. Proc.* **265–266**, 281 (2015).

[220] D. Lhuillier, *AIP Conf. Proc.* **1666**, 180003 (2015).

[221] B. Caccianiga, *AIP Conf. Proc.* **1666**, 180002 (2015).

[222] J. Spitz, *AIP Conf. Proc.* **1666**, 180004 (2015).

[223] C. Giunti, *Nucl. Phys. B* **908**, 336 (2016) [arXiv:1512.04758 [hep-ph]].

[224] L. Stanco, arXiv:1604.06769 [hep-ph].

[225] A. Fava, *Rev. Phys.* **1**, 52 (2016).

[226] M. Danilov [DANSS Collaboration], arXiv:1412.0817 [physics.ins-det].

[227] B.R. Kim *et al.*, arXiv:1511.05551 [physics.ins-det].

[228] A.P. Serebrov *et al.*, arXiv:1605.05909 [physics.ins-det].

[229] J. Ashenfelter *et al.* [PROSPECT Collaboration], arXiv:1512.02202 [physics.ins-det].

[230] N. Ryder [SoLid Collaboration], *PoS* **EPS-HEP2015**, 071 (2015) [arXiv:1510.07835 [hep-ex]].

[231] V. Helaine, arXiv:1604.08877 [physics.ins-det].

[232] V. Barinov *et al.*, *Phys. Rev. D* **93**, 073002 (2016) [arXiv:1602.03826 [hep-ph]].

[233] G. Bellini *et al.* [Borexino Collaboration], *J. High Energy Phys.* **1308**, 038 (2013) [arXiv:1304.7721 [physics.ins-det]].

[234] M. Harada *et al.*, arXiv:1601.01046 [physics.ins-det].

[235] R. Acciarri *et al.* [MicroBooNE, LAr1-ND, ICARUS-WA104 collaborations], `arXiv:1503.01520 [physics.ins-det]`.

[236] S.N. Axani *et al.*, `arXiv:1510.06994 [physics.ins-det]`.

[237] A.S. Riis, S. Hannestad, *J. Cosmol. Astropart. Phys.* **1102**, 011 (2011) [`arXiv:1008.1495 [astro-ph.CO]`].

[238] J.A. Formaggio, J. Barrett, *Phys. Lett. B* **706**, 68 (2011) [`arXiv:1105.1326 [nucl-ex]`].

[239] A.S. Riis, S. Hannestad, C. Weinheimer, *Phys. Rev. C* **84**, 045503 (2011) [`arXiv:1105.6005 [nucl-ex]`].

[240] A. Esmaili, O.L.G. Peres, *Phys. Rev. D* **85**, 117301 (2012) [`arXiv:1203.2632 [hep-ph]`].

[241] S. Mertens, talk presented at TAUP 2015, 7–11 September 2015, Torino, Italy.

[242] L. Gastaldo, C. Giunti, E.M. Zavanin, `arXiv:1605.05497 [hep-ph]`.

[243] J. Barry, W. Rodejohann, H. Zhang, *J. High Energy Phys.* **1107**, 091 (2011) [`arXiv:1105.3911 [hep-ph]`].

[244] Y. Li, S. Liu, *Phys. Lett. B* **706**, 406 (2012) [`arXiv:1110.5795 [hep-ph]`].

[245] W. Rodejohann, *J. Phys. G* **39**, 124008 (2012) [`arXiv:1206.2560 [hep-ph]`].

[246] I. Girardi, A. Meroni, S.T. Petcov, *J. High Energy Phys.* **1311**, 146 (2013) [`arXiv:1308.5802 [hep-ph]`].

[247] S. Pascoli, M. Mitra, S. Wong, *Phys. Rev. D* **90**, 093005 (2014) [`arXiv:1310.6218 [hep-ph]`].

[248] A. Meroni, E. Peinado, *Phys. Rev. D* **90**, 053002 (2014) [`arXiv:1406.3990 [hep-ph]`].

[249] A. Abada, V.D. Romeri, A. Teixeira, *J. High Energy Phys.* **1409**, 074 (2014) [`arXiv:1406.6978 [hep-ph]`].

[250] C. Giunti, E.M. Zavanin, *J. High Energy Phys.* **1507**, 171 (2015) [`arXiv:1505.00978 [hep-ph]`].

[251] H. Pas, W. Rodejohann, *New J. Phys.* **17**, 115010 (2015) [`arXiv:1507.00170 [hep-ph]`].

[252] H. Long, Y. Li, C. Giunti, *J. High Energy Phys.* **1308**, 056 (2013) [`arXiv:1306.4051 [hep-ph]`].

[253] R. Gandhi *et al.*, *J. High Energy Phys.* **1511**, 039 (2015) [`arXiv:1508.06275 [hep-ph]`].

[254] A. Palazzo, *Phys. Lett. B* **757**, 142 (2016) [`arXiv:1509.03148 [hep-ph]`].

[255] S.K. Agarwalla *et al.*, *J. High Energy Phys.* **1602**, 111 (2016) [`arXiv:1601.05995 [hep-ph]`].

[256] S.K. Agarwalla, S.S. Chatterjee, A. Palazzo, *J. High Energy Phys.* **1609**, 016 (2016) [`arXiv:1603.03759 [hep-ph]`].

[257] S. Choubey, D. Pramanik, `arXiv:1604.04731 [hep-ph]`.

[258] S.K. Agarwalla, S.S. Chatterjee, A. Palazzo, `arXiv:1605.04299 [hep-ph]`.

[259] S. Razzaque, A.Y. Smirnov, *J. High Energy Phys.* **1107**, 084 (2011) [arXiv:1104.1390 [hep-ph]].

[260] S. Razzaque, A.Y. Smirnov, *Phys. Rev. D* **85**, 093010 (2012) [arXiv:1203.5406 [hep-ph]].

[261] R. Gandhi, P. Ghoshal, *Phys. Rev. D* **86**, 037301 (2012) [arXiv:1108.4360 [hep-ph]].

[262] A. Esmaili, F. Halzen, O.L.G. Peres, *J. Cosmol. Astropart. Phys.* **1211**, 041 (2012) [arXiv:1206.6903 [hep-ph]].

[263] A. Esmaili, F. Halzen, O.L.G. Peres, *J. Cosmol. Astropart. Phys.* **1307**, 048 (2013) [arXiv:1303.3294 [hep-ph]].

[264] A. Esmaili, A.Y. Smirnov, *J. High Energy Phys.* **1312**, 014 (2013) [arXiv:1307.6824 [hep-ph]].

[265] S. Rajpoot, S. Sahu, H.C. Wang, *Eur. Phys. J. C* **74**, 2936 (2014) [arXiv:1310.7075 [hep-ph]].

[266] M. Lindner, W. Rodejohann, X.J. Xu, *J. High Energy Phys.* **1601**, 124 (2016) [arXiv:1510.00666 [hep-ph]].

[267] J. Liao, D. Marfatia, *Phys. Rev. Lett* **117**, 071802 (2016) [arXiv:1602.08766 [hep-ph]].

[268] M.G. Aartsen *et al.* [IceCube Collaboration], *Phys. Rev. Lett.* **117**, 071801 (2016) [arXiv:1605.01990 [hep-ex]].

[269] D.O. Caldwell, G.M. Fuller, Y.Z. Qian, *Phys. Rev. D* **61**, 123005 (2000) [arXiv:astro-ph/9910175].

[270] M. Sorel, J. Conrad, *Phys. Rev. D* **66**, 033009 (2002) [arXiv:hep-ph/0112214].

[271] S. Choubey, N.P. Harries, G. Ross, *Phys. Rev. D* **74**, 053010 (2006) [arXiv:hep-ph/0605255].

[272] S. Choubey, N.P. Harries, G.G. Ross, *Phys. Rev. D* **76**, 073013 (2007) [arXiv:hep-ph/0703092].

[273] I. Tamborra *et al.*, *J. Cosmol. Astropart. Phys.* **1201**, 013 (2012) [arXiv:1110.2104 [astro-ph.SR]].

[274] M.R. Wu *et al.*, *Phys. Rev. D* **89**, 061303 (2014) [arXiv:1305.2382 [astro-ph.HE]].

[275] A. Esmaili, O.L.G. Peres, P.D. Serpico, *Phys. Rev. D* **90**, 033013 (2014) [arXiv:1402.1453 [hep-ph]].

[276] M.L. Warren *et al.*, *Phys. Rev. D* **90**, 103007 (2014) [arXiv:1405.6101 [astro-ph.HE]].

[277] A. Esmaili, O.L. Peres, *J. Cosmol. Astropart. Phys.* **1205**, 002 (2012) [arXiv:1202.2869 [hep-ph]].

[278] M. Cirelli *et al.*, *Nucl. Phys. B* **708**, 215 (2005) [arXiv:hep-ph/0403158].

[279] A. Donini, O. Yasuda, arXiv:0806.3029 [hep-ph]].

[280] J. Barry, R.N. Mohapatra, W. Rodejohann, *Phys. Rev. D* **83**, 113012 (2011) [arXiv:1012.1761 [hep-ph]].

[281] J. Lesgourgues *et al.*, *Neutrino Cosmology*, Cambridge University Press, 2013.

[282] S. Riemer-Sorensen, D. Parkinson, T.M. Davis, *Publ. Astron. Soc. Austral.* **30**, e029 (2013) [arXiv:1301.7102 [astro-ph.CO]].

[283] M. Archidiacono *et al.*, *Adv. High Energy Phys.* **2013**, 191047 (2013) [arXiv:1307.0637 [astro-ph.CO]].

[284] J. Lesgourgues, S. Pastor, *New J. Phys.* **16**, 065002 (2014) [arXiv:1404.1740 [hep-ph]].

[285] S. Hannestad, arXiv:1605.03829 [astro-ph.CO].