QUANTUM STATISTICS IN THE STATISTICAL MODEL OF MULTIPLE PRODUCTION OF PARTICLES

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It is shown that the normalization volume (V_2) which is arbitrary when using the standard versions of the statistical model can be determined from experiment, when quantum statistics are applied to the particles. $V_2 \to \infty$ corresponds to the classical slimit of quantum statistics. Then in the thermodynamic limit Boltzmann's formula should be used. The use of Planck's formula is unjustified. If V_2 is finite and such that Planck's formula applies for average multiplicities, Einstein's condensation should be observed for high multiplicities. The absence of a significant surplus of very low energy pions in high multiplicity events yields a lower bound for V_2. The isospin factor can be separated from the phase spac efactor only in the classical limit

I. The phase space integral

According to the statistical model of multiple production the probability of reaching an n-particle final state is proportional to the so-called phase space integral

$$P(n, E) = \frac{V^n}{(2\pi)^{3n}} \int d^{3n}p \; \delta^4 \left(P - \sum_i p_i\right). \tag{1}$$

Here P is the total four-momentum in the initial state and p_i is the four-momentum of the i-th final state particle. It is understood that the integral should be evaluated in the centre-of-mass system where $P = (E, 0, 0, 0)$.

The proportionality factor, which is not written in (1), is zero if the transition to the final state is forbidden by some conservation laws. For allowed final states it contains a factor which depends on the total isospin and on the number and isospins of the final state particles, but not on energy, and a normalizing factor which does not depend on n. The normalizing factor has no effect on the discussion presented in this paper. The isospin factor is discussed in Section VII.

An important point for our further analysis is the interpretation of the coefficient V^n.

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II. Interpretation of the parameter V

In Fermi's original paper (Fermi 1950) V was introduced as a product of two factors. A normalization volume V_2, supposed large, came from the conversion of the summation over final states into an integral over momentum space

$$\sum_{p_1, \ldots, p_n} \ldots \rightarrow \frac{V_2^n}{(2\pi)^{3n}} \int d^{3n}p \ldots \tag{2}$$

This formula is exact in the limit $V_2 \rightarrow \infty$. For V_2 finite it may be used as an approximation which, however, becomes very bad when V_2 is so small that either the energy differences between neighbouring energy levels become comparable to the average energy of a particle, or the Einstein condensation occurs. Besides, a statistical weight

$$V_1 = V/V_2 \tag{3}$$

was introduced for each one-particle state.

So as to get a crude estimate of V the following argument was used. In order to take part in the production process each particle must be in an interaction volume surrounding the point where the collision takes place. The radius of the interaction volume should be of the order of the range of the meson forces *i.e.* of the Compton wave length of the π meson. Since one-particle states are plane waves normalized in the volume V_2, the probability that a particle takes part in the process is V_1 with V understood as the interaction volume. Accordingly a plausible estimate is

$$V = \frac{4}{3} \pi m_\pi^{-3} \tag{4}$$

where m_π denotes the rest mass of the π meson and, as before, the units $\hbar = 1$, $c = 1$ are used. Note that in this argument the final state particles are described as a gas of non-interacting particles (perfect gas). Fermi suggested also that for very high energies the Lorentz contraction of the meson clouds of the colliding particles should be taken into account. Thus for a collision of two particles of mass M each, (4) should be replaced by

$$V = \frac{4}{3} \pi m_\pi^{-3} \left(\frac{2M}{\mathrm{E}}\right). \tag{4a}$$

Formulae (4) and (4a) give results in reasonably good agreement with experiment (*cf. e.g.* Kretzschmar 1961).

Let us note that the notion of an interaction volume is not essential for this interpretation of formula (1). It is possible to interpret $(V/V_2)^n$ as an average of the squared modulus of the transition matrix element with no reference to the interaction volume. V must have the dimension of volume, in order to make Vd^3p dimensionless (*e. g.* Kretzschmar 1961).

Once the notion of interaction volume was introduced, it became possible to give a different interpretation of formula (1). According to this interpretation the interaction volume should be visualised as a kind of box, in which the particles are produced and stay for some time. In our previous notation this corresponds to $V_2 = V$ and $V_1 = 1$. Such an approach is open to some objections. From the purely mathematical point of view transition (2) is unjustified for V_2 as small as (4) or (4a). This difficulty was discussed by Auluck and Kothari (1953), (1954), Kothari (1954) and Nanda (1954). The corrections which should be added to the phase space integral may be large, especially if assumption (4a) is made, and are very sensitive to the boundary conditions on the boundary of volume V (Nanda 1954). Since it is known from quantum field theory that a sharp boundary of the meson field would cause infinite energy fluctuations, the problem of finding suitable boundary conditions is nontrivial. A more fundamental difficulty was pointed out by Pomeranchuk (1951). In the box version of the model it is not clear why the production process should stop. When two strongly interacting particles get within a distance of order m_π^{-1} from each other, they start creating new particles. These particles should interact strongly both with each other and with the initial particles, presumably creating more particles. Instead, in the model: the final state particles are described as a perfect gas. In order to avoid this difficulty the hydrodynamical model was developed (*e.g.* Belenkij and Landau 1955), but its predictions differ appreciably from those of the statistical model.

In spite of the objections above, formula (1) does not depend on the interpretation of the parameter V, and from the practical point of view the two interpretations are equivalent. We show in the next section that this is no more the case when quantum statistics are applied to the final state particles.

III. Qualitative discussion of the effects of quantum statistics

In the classical phase space integral (1) only the product $V=V_1V_2$ occurs. Therefore doubling V_1 has the same effect as doubling V_2. Let us check that this does not hold true for quantum statistics.

It is seen from formula (2) that the phase space integral is a sum over possible final states. Let us consider a final state with two particles but only one one-particle state. In this state both particles are in state A. We denote this state by AA. For fermions the state AA is forbidden by the Pauli principle, therefore there is no two-particle state. For two particles the statistical weight of a state is V_1^2. Consequently the sums over states for bosons, for classical statistics and for fermions are V_1^2, V_1^2, and 0. Let us double V_2. According to (2) this means doubling the number of one-particle states; thus A splits into two states, say, a and b. Now for bosons there are three possible two-particle states: aa, ab, and bb. For classical statistics there are four states because ba is no longer identical with ab, and for fermions there is only one state, because aa, and bb are forbidden. Thus the sums over states for the three statistics are: $3V_1^2$, $4V_2^2$, and V_1^2. It is seen that only for classical statistics can the result be reproduced by doubling V_1.

Similar results are obtained for systems composed of more particles and having more one-particle states. If, however, the number of one-particle states is very large compared with the number of particles, the probability of finding two or more particles in one one-particle state becomes negligible. Consequently, the Pauli principle becomes irrelevant. For each kind of quantum statistics the error, when using formula (1), consists in counting separately all the permutations of particles among the one-particle states. Thus for n particles each state is counted $n!$ times. The corrected sum over states is

$$P_{cl}(n, E) = \frac{1}{n!} P(n, E). \tag{5}$$

The notation P_{cl} is used to keep in mind that this is the classical limit of quantum statistics.

In particular for V_2 tending to infinity the density of one-particle states goes to infinity according to (2), and the classical limit (5) may be used. This implies that in the thermodynamic limit Boltzmann's formula

$$\overline{n_p} = \text{const } e^{-\beta p} \tag{6}$$

and not Planck's formula

$$\overline{n_p} = \text{const } (e^{\beta p}-1)^{-1}, \tag{7}$$

should be used for pions. In (6) and (7) n_p denotes the number of particles in the one-particle state characterized by four-momentum p, β is a four-vector which in the centre-of-mass system has components $(\beta_0, 0, 0, 0)$ with β_0 inversly proportional to the absolute termeature. The line above n_p denotes averaging. In order to confirm this result the sum of phase space integrals necessary for the theory of large angle proton-proton scattering was evaluated (Zalewski 1965), it is found that the defficiencies of the thermodynamic method found in previous investigations disappear when Planck's distribution is replaced by the Boltzmann's.

For V_2 finite and small, Planck's distribution might apply, but then a qualitatively new effect: the Einstein condensation is likely to occur. This problem is analysed in Section VI.

For distinguishable particles a representation where the momentum of each particle is specified yields for the sum over states the convenient formula (1). For indistinguishable particles the representation of occupation numbers n_p is appropriate. This is well known from statistical physics (*e.g.* Landau and Lifshits 1951). In the next section a formula for the sum over states of indistinguishable particles is derived.

IV. The sum over states of indistinguishable particles

The state of a set of indistinguishable particles is specified when all the occupation numbers n_p are given. Indeed, the occupation numbers define the state up to exchanges of particles, and for indistinguishable particles an exchange of particles does not change the state. We shall denote the full set of occupation numbers by $[n_p]$. Thus the sum over states of n indistinguishable particles is

$$P_Q(n, E) = \sum_{[n_p]} \delta^4 (P-\sum_p pn_p)\delta(n-\sum_p n_p) \prod_p u^{n_p}(p), \tag{8}$$

where $u(p)$ is the statistical weight of a one-particle state. In the ordinary statistical model $u(p) = V_1$, but our results can be easily generalized to the covariant version of the model (Srivastava and Sudarshan 1958), where $u(p) = V_1/\varepsilon$ with ε denoting the energy of a particle with momentum p. The symbol δ is used for both the Dirac and the Kronecker deltas.

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The summation over $[n_p]$ can be carried out. Let us rewrite (8) in the form

$$P_Q = \sum_{[n_p]} \delta^4(P-\sum_p pn_p)\delta(n-\sum_p n_p)\, e^{\beta P+\alpha n}\prod_p (V_1 e^{-\alpha-\beta p})^{n_p}, \tag{9}$$

where a is a constant and β a four-vector. Because of the δ-functions, the exponential factors reduce to unity for any a and β. We assume that β is positive time-like and that for bosons

$$(\beta p)_{\min} + a > \ln V_1, \tag{10}$$

where $(\beta p)_{\min}$ is the smallest possible value of βp. Using the identities

$$\delta(n-\sum_p n_p) = (2\pi)^{-1}\int_{-\pi}^{\pi} d\varphi\, e^{i(n-\sum_p n_p)\varphi}, \tag{11}$$

and

$$\delta(P-\sum_p pn_p) = (2\pi)^{-4}\int_{-\infty}^{\infty} d^4t\, e^{i(P-\sum_p pn_p)t} \tag{12}$$

we can cast $P_Q(n, E)$ in the following form:

$$P_Q(n, E) = (2\pi)^{-5}\int_{-\pi}^{\pi} d\varphi \int_{-\infty}^{\infty} d^4t\, e^{in\varphi+iPt}\prod_p \sum_{[n_p]} g^{n_p}(p;t,\varphi)\, e^{\beta P+\alpha n}, \tag{13}$$

where

$$g(p;t,\varphi) = V_1\, e^{-(\alpha+i\varphi)-(\beta+it)p}. \tag{14}$$

In formula (13) the summations over the n_p are decoupled. For each p there is a simple geometrical progression. For fermions a finite one with $n_p = 0, 1$; for bosons an infinite one which, however, converges because of assumptions (10). Carrying out the summations we obtain

$$P_Q(n, E) = (2\pi)^{-5}\int_{-\pi}^{\pi} d\varphi \int_{-\infty}^{\infty} d^4t\, e^{(\alpha+i\varphi)n+(\beta+it)p+\Phi(t,\varphi)}, \tag{15}$$

where

$$\Phi(t,\varphi) = \mp \sum_p \ln\,[1 \mp g(p;t,\varphi)]. \tag{16}$$

The upper sings apply to a boson gas, the lower signs apply to a fermion gas. The particular case of formula (15) for $a = 0$, $V_2 = V$ and $V_1 = 1$ was derived by Magalinskij and Terletskij (1957).

For further reference we derive the analogue of formula (15) for the ordinary phase space integral (1). Multiplying the integrand of (1) by $1 = \exp\,(P-\sum p_i)\beta$ and using identity (12) we obtain

$$P(n, E) = \frac{V^n}{(2\pi)^{3n}}(2\pi)^{-4}\int_{-\infty}^{\infty} d^4t\left[\int_{-\infty}^{\infty} d^3p\, e^{-(\beta+it)p}\right]^n e^{\beta P}. \tag{17}$$

V. *Discussion of the sum over states $P_Q(n, E)$ for V_2 tending to infinity*

For V_2 tending to infinity formula (2) may be used and (16) yields

$$\Phi(t, \varphi) = \mp \frac{V_2}{(2\pi)^3} \int d^3p \ln [1 \mp V_1 e^{-(\alpha+i\varphi)-(\beta+it)p}]. \tag{18}$$

Since Φ must remain finite, V_1 must tend to zero. Therefore the logarithm may be replaced by the first non-vanishing term of its expansion in powers of V_1. The result is

$$\Phi(t, \varphi) = \frac{V}{(2\pi)^3} e^{-(\alpha+i\varphi)} \int d^3p \, e^{-(\beta+it)p}, \tag{19}$$

where $V = V_1 V_2$ is substituted according to (3). Expanding exp Φ in a power series in Φ, substituting the result into (15), and performing the integration over φ term by term we obtain a non-vanishing contribution only from the n-th term. The result is

$$P_Q(n, E) = (2\pi)^{-4} \int_{-\infty}^{\infty} d^4t \frac{1}{n!} \left[\frac{V}{(2\pi)^3} \int d^3p \, e^{-(\beta+it)p} \right]^n e^{\beta P}. \tag{20}$$

Comparing with (17) we have finally

$$P_Q(n, E) = P_{cl}(n, E), \tag{21}$$

with P_{cl} defined by (5).

We conclude that if the interpretation with a large normalization volume is chosen, the classical limit of the theory may be used. The implications of the assumption that V_2 is finite are discussed in the next section.

VI. *Discussion of the sum over states $P_Q(n, E)$ for V_2 finite*

For V_2 finite intergral (15) can be evaluated by the steepest descents method. A detailed and mathematically rigorous description of this calculation is given in Khinchin's monograph (Khinchin 1951). When applying this to systems containing a finite, not very large, number of particles, one has to include some correction terms. This part of the calculation is quite analogous to the calculations done for the classical case by Fialho (1957) and by Lurçat and Mazur (1964). Here we give only a qualitative discussion of the results.

In order to apply the steepest descents method it is convenient to choose the parameters α and β in such a way that the integrand of (15) has a maximum at $\varphi = 0$, $t = 0$. Then the first derivatives of the exponent should vanish for $\varphi = 0$, $t = 0$. Therefore we determine α and β from the equations

$$P = \sum_p p(V_1 e^{\alpha+\beta p} \mp 1)^{-1}, \tag{22}$$

$$n = \sum_p (V_1 e^{\alpha+\beta p} \mp 1)^{-1}. \tag{23}$$

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If we assume that the centre-of-mass system is the rest system of β, the equations (22) for the spatial components are identically satisfied. Therefore from the four equations (22) only one is left:

$$E = \sum_{p} \varepsilon (V_1 \, e^{\alpha+\beta\varepsilon} \mp 1)^{-1}, \quad (22a)$$

where the symbol β is used to denote the time component of the four-vector β in its rest system. Introducing the notation

$$V_1^{-1} \, e^{\alpha} = e^{-\beta\mu}, \quad (24)$$

where μ is the chemical potential, we obtain the standard formulae of statistical physics. In particular Planck's distribution is obtained when two condition are fulfilled. One of them is that $\mu = 0$. The other one is that V_2 is sufficiently large to justify transition (2). For a detailed discussion of the transition to the thermodynamic limit the reader is referred to Khinchins book (Khinchin 1951).

It is seen from the equation system (22a), (23) that Planck's formula (implying $\mu = 0$) cannot be applicable for all multiplicities at a given energy. In particular for high energies nad low multiplicities $e^{-\beta\mu}$ must be large in order to make n small. In this case the exponential in the denominators is large, unity can be neglected, and the classical limit is recovered.

For bosons at high multiplicities Planck's distribution might be a good approximation, but we would like to point out that if it is a good approximation for average multiplicities, then Einstein's condensation should be expected at high multiplicities.

When V_2 is large it is a good approximation to assume that Einstein's condensation occurs when the actual number of particles exceeds the maximal value of the integral approximation to (23). We keep this estimate, though for our small volume it is less good. Using condition (10) and noticing that n increases when α decreases we obtain for the maximal value of the integral approximation

$$n_{\max} = \frac{V_2}{(2\pi)^3} \int d^3p (e^{\beta\varepsilon - (\beta\varepsilon)_{\min}} - 1)^{-1}, \quad (25)$$

where β should be determined from Eq. (22a) with $V_1 e^{\alpha} = e^{-(\beta\varepsilon)}$ min. The surplus of particles $n - n_{\max}$, concentrates in the lowest energy states. Consequently the energy spectrum of the final state particles acquires an additional maximum in the vicinity of the minimal energy, besides the usual maximum of Planck's distribution.

In order to estimate the orders of magnitude involved, we solve equation system (22a), (25) for the centre-of-mass energy $E = 5$ BeV. Using the integral approximation, and the ultrarelativistic approximation, where $(\beta\varepsilon)_{\min} = 0$, we obtain

$$E = \frac{V_2}{(2\pi)^3} \int d^3p p (e^{\beta p} - 1)^{-1} = \frac{3V_2}{\pi^2} \zeta(4) \beta^{-4}, \quad (26)$$

$$n_{\max} = \frac{V_2}{(2\pi)^3} \int d^3p (e^{\beta p} - 1)^{-1} = \frac{V_2}{\pi^2} \zeta(3) \beta^{-3}, \quad (27)$$

where $\zeta(x)$ is the Riemannian zeta-function. Thus substituting V_2 from (4)

$$(\beta m_\pi)^{-1} = 2.26, \tag{28}$$

$$n > n_{\max} = 6. \tag{29}$$

The ultrarelativistic approximation is justified when $(\beta m)^2 \ll 1$. Thus it should be a reasonable approximation in our case.

If V_2 is identified with the contracted volume (4a), then at high energies the gas is practically two-dimensional, as was pointed out by Nanda (1954). Also then the predictions deviate considerably from those of the standard statistical model.

Estimate (29) is obtained on the assumption that only one kind of particles occurs in the final state. *E. g.* it could be applied to the annihilation of proton-antiproton pairs, if all the final state particles were π° mesons. Strictly speaking for π mesons the isospin conservation should be taken into account, however, its effect is to reduce the number of available final states, thus it would decrease $n_{\max}$, and the condition for n given in (29) and (30) would remain sufficient. Since experimentally processes where all the final state particles are neurtals are difficult to study, an estimate was found for the case when π^+ and π^- mesons are produced. If conservation of charge is neglected, the number of available one-particle states is twice that for indistinguishable particles. Indeed for each momentum the particle may have two different charge states. This correspond formally to a doubling of V_2 and, using formulae (26) and (27), we obtain

$$n > n_{\max} = 7. \tag{30}$$

It is a moderate multiplicity for $E = 5$ BeV. Therefore a study of very low energy mesons produced in high multiplicity annihilation processes should be a critical test for the statistical model with a finite normalization volume.

VII. Effect of isospin conservation

The preceding discussion applies to n identical particles with zero isospin. It is easy, however, to extend it for particles belonging to an arbitrary isomultiplet. This is achieved by introducing the following two changes into formula (8). Firstly, the one-particle states must be labelled by two indices: p and m, where m is the third component of isospin. Secondly, a factor

$$\delta(I - \sum_{p,m} m n_{p,m}) - \delta(I + 1 - \sum_{p,m} m n_{p,m}) \tag{31}$$

must be introduced. The summations over m go from $-j$ to $+j$ where j is the isospin of a single particle. The term with the first δ yields the number of states with $\Sigma m = I$, which is equal to the number of isomultiplets with isospin not smaller than I. The second term yields the number of states with isospin not smaller than $I+1$. Thus the difference is equal to the number of states with isospin I. This is a standard argument (*cf. e.g.* Magalinskij 1959).

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Introducing these modifications we obtain

$$P_Q(n, E) = \sum_{[n_{p,m}]} \delta^4 (P - \sum_{p,m} pn_{p,m}) \delta(n - \sum_{p,m} n_{p,m}) [\delta(I - \sum_{p,m} mn_{p,m}) -$$
$$-\delta(I+1-\sum_{p,m} mn_{p,m})] \prod_{p,m} u^{n_{p,m}}(p, m). \tag{32}$$

This formula is transformed using the identities (11), (12) and

$$\delta(I - \sum_{p,m} mn_{p,m}) - \delta(I+1-\sum_{p,m} mn_{p,m}) =$$
$$= (2\pi)^{-1} \int_{-\pi}^{\pi} d\psi \, [e^{iI\psi} - e^{i(I+1)\psi}] \cdot e^{-i \sum_{p,m} mn_{p,m}\psi}. \tag{33}$$

After introducing convergence factors with a and β, and performing the summation over $n_{p,m}$, exactly as in the previous case, the result is

$$P_Q(n, E, I) = (2\pi)^{-6} \int_{-\pi}^{\pi} d\psi \int_{-\pi}^{\pi} d\varphi \int_{-\infty}^{\infty} d^4t \, e^{(\alpha+i\varphi)n + (\beta+it)P + \Phi(t,\varphi,\psi)} \, [e^{iI\psi} - e^{i(I+1)\psi}], \tag{34}$$

where

$$\Phi(t, \varphi, \psi) = \pm \sum_{p,m} \ln (1 \pm V_1 \, e^{-(\alpha+i\varphi)-(\beta+it)p-i\psi}). \tag{35}$$

Our first conclusion is that in the general case $P_Q(n, E, I)$ does not split into a product of an isospin part and a phase space part. Let us consider the classical limit which, according to the previous discussion, is probably the realistic one. In this limit

$$\Phi = V_1 \sum_{p,m} [e^{-(\beta+it)p}] \, e^{-(\alpha+i\varphi)-im\psi}. \tag{36}$$

Expanding exp Φ and performing the integrations over φ and ψ term by term, we obtain

$$P_Q(n, E, I) = f(I) P_{\text{cl}}(n, E), \tag{37}$$

where

$$f(I) = (2\pi)^{-1} \int_{-\pi}^{\pi} d\psi (e^{iI\psi} - e^{i(I+1)\psi}) \, (\sum_m e^{im\psi})^n \tag{38}$$

is identical with the classical expression for the isospin weight (*cf.* Magalinskij 1959). This formula may also be rewritten in a form familiar from group theory

$$f(I) = (2\pi)^{-1} \int_{-\pi}^{\pi} d\psi (1 - \cos \psi) \, (\chi_j(\psi))^n \chi_I(\psi), \tag{39}$$

where

$$\chi_j(\psi) = \sum_m e^{im\psi} = \frac{\sin (j+1/2)\, \psi}{\sin \psi/2} \tag{40}$$

is the character of the j-th representation of the rotation group.

A generalization to systems containing particles belonging to more than one isomultiplet introduces only minor changes. The qualitative results remain unchanged.

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