M-THEORY AND THE BIRTH OF THE UNIVERSE*

F.R. Klinkhamer

Institute for Theoretical Physics, Karlsruhe Institute of Technology (KIT)
76128 Karlsruhe, Germany
frans.klinkhamer@kit.edu

(Received March 31, 2021; accepted May 24, 2021)

In memory of Martinus J.G. Veltman (1931–2021)

In this review article, we first discuss a possible regularization of the Big Bang curvature singularity of the standard Friedmann cosmology, where the curvature singularity is replaced by a spacetime defect. We then consider the hypothesis that a new physics phase gave rise to this particular spacetime defect. Specifically, we set out on an explorative calculation using the IIB matrix model, which has been proposed as a particular formulation of nonperturbative superstring theory (M-theory).

DOI:10.5506/APhysPolB.52.1007

1. Introduction

As regards the topic of this conference, we submit that the future of particle physics may very well involve the future of gravitation and cosmology. The title of the present contribution is in that spirit, as "M-theory" concerns a possible future theory merging elementary particle physics and gravitational physics, while "the birth of the Universe" impacts on all topics of cosmology.

This review article presents one particular approach to the question of how the Universe got started. First, we discuss a possible regularization of the Big Bang curvature singularity, while staying within the realm of general relativity but allowing for degenerate spacetime metrics (having, for example, a vanishing determinant of the metric at certain spacetime points). This regularization replaces, in fact, the Big Bang curvature singularity by a three-dimensional spacetime defect with a locally vanishing determinant of the metric.

* Presented at XXVII Cracow Epiphany Conference on *Future of Particle Physics*, Cracow, Poland, January 7–10, 2021.

Next, we investigate the hypothesis that a new physics phase produced this spacetime defect. In order to allow for explicit calculations, we turn to the IIB matrix model, which has been suggested as a formulation of non-perturbative type-IIB superstring theory (M-theory). The crucial question, now, is how a classical spacetime might emerge from the IIB matrix model. The answer appears to be that such an emerging classical spacetime may reside in the large-N master field of the IIB matrix model. This master field can, in principle, give the regularized-big-bang metric of general relativity. A word of caution is, however, called for: as the formulation and validity of M-theory have not yet been established, the present review paper provides no definitive answers but only a few suggestive results.

The outline is as follows. In Section 2, we recall the main points of the standard Friedmann cosmology and its Big Bang curvature singularity, primarily to establish our notation. In Section 3, we present a particular regularization of the Friedmann Big Bang singularity and highlight a few subtle points (*e.g.*, differential structure and singularity theorems). In Section 4, we turn to the main topic, namely the hypothetical existence of a new physics phase giving rise to a classical spacetime and possibly to a regularized (tamed) Big Bang. Here, we rely on a particular matrix model realization of M-theory, specifically the IIB matrix model. Our focus is on the basic ideas and technical details are relegated to four appendices. In Section 5, we give a brief summary and point out what the main outstanding task appears to be.

2. Standard Friedmann cosmology

2.1. Robertson–Walker metric and Friedmann equations

The Einstein gravitational field equation of general relativity reads [1]

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu}\, R = -8\pi G\, T^{(\mathrm{M})}_{\mu\nu}\,, \tag{2.1}$$

with $R_{\mu\nu}$ the Ricci curvature tensor, R the Ricci curvature scalar, $T^{(\mathrm{M})}_{\mu\nu}$ the energy-momentum tensor of the matter (described by the Standard Model of elementary particles and possible extensions), and G Newton's gravitational coupling constant. In this section and the next, we use relativistic units with $c = 1$ and the spacetime indices μ, ν run over $\{0, 1, 2, 3\}$.

For the record, we give the energy-momentum tensor of a perfect fluid [1]:

$$T^{(\mathrm{M,\ perfect\ fluid})}_{\mu\nu} = (P_M + \rho_M)\, U_\mu\, U_\nu + P_M\, g_{\mu\nu}\,, \tag{2.2}$$

with a normalized four-velocity U^μ of the comoving fluid element and scalars ρ_M and P_M (corresponding to the energy density and the pressure measured in a localized inertial frame comoving with the fluid).

We will now review a special solution of the Einstein equation with a relativistic perfect fluid, namely the Friedmann–Lemaître–Robertson–Walker expanding universe [2–5]. For a homogeneous and isotropic cosmological model, the spatially flat Robertson–Walker (RW) metric is [4, 5]

$$\mathrm{d}s^2\,\Big|^{(\mathrm{RW})} \equiv g_{\mu\nu}(x)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu\,\Big|^{(\mathrm{RW})} = -\mathrm{d}t^2 + a^2(t)\,\delta_{mn}\,\mathrm{d}x^m\,\mathrm{d}x^n\,, \tag{2.3}$$

with $x^0 = c\,t$ and $c = 1$. The spatial indices m, n run over $\{1, 2, 3\}$.

Consider a homogeneous perfect fluid with energy density $\rho_M(t)$ and pressure $P_M(t)$. Then, the Einstein equation (2.1) with the RW metric *Ansatz* (2.3) and the energy-momentum tensor of a homogeneous perfect fluid (2.2) gives the following spatially flat Friedmann equations:

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\,\rho_M\,, \tag{2.4a}$$

$$\frac{\ddot a}{a} + \frac{1}{2}\left(\frac{\dot a}{a}\right)^2 = -4\pi G\,P_M\,, \tag{2.4b}$$

$$\dot\rho_M + 3\,\frac{\dot a}{a}\,\left(\rho_M + P_M\right) = 0\,, \qquad P_M = P_M\left(\rho_M\right)\,, \tag{2.4c}$$

where the overdot stands for differentiation with respect to t. The first equation in (2.4c) corresponds to energy conservation and the second stands for the equation-of-state (EOS) relation between pressure and energy density of the perfect fluid. The matter is, moreover, assumed to satisfy the standard energy conditions. The null energy condition of the perfect fluid (2.2), for example, corresponds to the inequality $\rho_M + P_M \geq 0$.

2.2. Big Bang singularity

The Friedmann equations (2.4) for relativistic matter with constant EOS parameter $w_M \equiv P_M/\rho_M = 1/3$ give the well-known Friedmann–Lemaître–Robertson–Walker (FLRW) solution [2–5]:

$$a(t)\,\Big|^{(\mathrm{FLRW})}_{(w_M=1/3)} = \sqrt{t/t_0}\,, \qquad \text{for} \quad t > 0\,, \tag{2.5a}$$

$$\rho_M(t)\,\Big|^{(\mathrm{FLRW})}_{(w_M=1/3)} = \rho_{M0}/a^4(t) = \rho_{M0}\;t_0^2/t^2\,, \qquad \text{for} \quad t > 0\,, \tag{2.5b}$$

where the cosmic scale factor has normalization $a(t_0) = 1$ at $t_0 > 0$ and the constant ρ_{M0} is positive [from (2.4a), we have that $G\rho_{M0}$ is proportional to $1/t_0^2$]. This FLRW solution displays the Big Bang singularity for $t \to 0^+$:

$$\lim_{t\to 0^+} a(t) = 0\,, \tag{2.6}$$

with diverging curvature (*e.g.*, a diverging Kretschmann curvature scalar $K \equiv R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} \propto 1/t^4$) and diverging energy density (2.5b).

At $t = 0$, however, the used theory (*i.e.*, general relativity and the standard model of elementary particles) is no longer valid and we can ask what happens *really*? Or, more precisely: *how to describe the birth of the Universe?*

3. Regularized Big Bang

3.1. New metric and modified Friedmann equations

First, we set out to control the divergences of the standard Friedmann cosmology or, in other words, to "regularize" the Big Bang singularity. We do this by using a new *Ansatz* [4–6]

$$ds^2 \,\Big|^{(\mathrm{RWK})} \equiv g_{\mu\nu}(x)\, dx^\mu\, dx^\nu \,\Big|^{(\mathrm{RWK})} = -\frac{t^2}{t^2 + b^2}\, dt^2 + a^2(t)\, \delta_{mn}\, dx^m\, dx^n\,, \tag{3.1a}$$

$$b > 0\,, \qquad a^2(t) > 0\,, \tag{3.1b}$$

$$t \in (-\infty, \infty)\,, \qquad x^m \in (-\infty, \infty)\,, \tag{3.1c}$$

with $x^0 = c\,t$ and $c = 1$. The nonzero length scale b enters the metric component $g_{00}(t)$ and will be seen to act as a regulator. Setting $b = 0$ in the metric (3.1a) formally reproduces the RW metric (2.3) with $g_{00} = -1$, but this does not really hold for the metric (3.1) at $t = 0$, which has $g_{00}(0) = 0$. In short, the limits $t \to 0$ and $b \to 0$ do not commute for $g_{00}(t)$ from (3.1a).

The metric $g_{\mu\nu}(x)$ from (3.1) is *degenerate*, with a vanishing determinant at $t = 0$. Physically, the $t = 0$ slice corresponds to a three-dimensional *spacetime defect* (the terminology has been chosen so as to emphasize the analogy with a defect in an atomic crystal). See Refs. [7–10] for further discussion of the new cosmological metric (3.1) and Refs. [11–14] for some background on this type of spacetime defect.

At this moment, we have two general observations. First, if we replace the symbol t from (2.3) by $\tau \in \mathbb{R}$ and perform surgery on τ (introducing the length scale b by removing the open interval between the points $\tau = -b$ and $\tau = +b$), then there is a coordinate transformation [6] between the resulting τ and $t \in \mathbb{R}$, which transforms the metric from (2.3) into the one from (3.1a). But this coordinate transformation is *not* a diffeomorphism (an invertible C^∞ function), as the two points $\tau = \pm b$ are mapped into the single point $t = 0$. This implies that the differential structure corresponding to (2.3) is different from the one corresponding to (3.1a); see also Ref. [12] and the second remark at the end of this subsection for related comments.

Second, the metric (3.1a) at $t \sim b$ is a *large* perturbation away from the RW metric (2.3) for equal values of $a(t)$, so that $t \gg b$ would be required in Hawking's argument for the occurrence of a singularity (see the sentence "Therefore any perturbation ..." from the top-left column on p. 690 of Ref. [15]). Anyway, more general cosmological singularity theorems [16–18] hold true and we shall comment on their interpretation in the last paragraph of Section 3.3.

The standard Einstein equation (2.1) with the new metric *Ansatz* (3.1) and a homogeneous perfect fluid (2.2) gives *modified* spatially flat Friedmann equations:

$$\left[1+\frac{b^2}{t^2}\right]\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho_M\,, \tag{3.2a}$$

$$\left[1+\frac{b^2}{t^2}\right]\left(\frac{\ddot{a}}{a}+\frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2\right)-\frac{b^2}{t^3}\,\frac{\dot{a}}{a} = -4\pi G\,P_M\,, \tag{3.2b}$$

$$\dot{\rho}_M + 3\,\frac{\dot{a}}{a}\,\left(\rho_M+P_M\right) = 0\,, \qquad P_M = P_M\left(\rho_M\right)\,, \tag{3.2c}$$

where the overdot stands again for differentiation with respect to t. Two remarks are in order. First, the inverse metric from (3.1a) has a component $g^{00} = (t^2+b^2)/t^2$ that diverges at $t=0$ and we must be careful to obtain the reduced field equations at $t=0$ from the limit $t \to 0$ (see Sec. 3.3.1 of Ref. [13] for further details and Ref. [19] for a discussion of the practical advantages of using a first-order formalism). Second, the new b^2 terms in the modified Friedmann equations (3.2a) and (3.2b) are a manifestation of the different differential structure of (3.1a) compared to the differential structure of (2.3) which gives the standard Friedmann equations (2.4a) and (2.4b).

3.2. Regular solution

Having obtained modified Friedmann equations, it is clear that we expect to get modified solutions. In fact, for constant EOS parameter $w_M \equiv P_M/\rho_M = 1/3$, the even solution of (3.2) reads [2–6]

$$a(t)\,\Big|^{(\text{FLRWK})}_{(w_M=1/3)} = \sqrt[4]{\left(t^2+b^2\right)/\left(t_0^2+b^2\right)}\,, \tag{3.3a}$$

$$\rho_M(t)\,\Big|^{(\text{FLRWK})}_{(w_M=1/3)} = \rho_{M0}/a^4(t) = \rho_{M0}\,\left(t_0^2+b^2\right)/\left(t^2+b^2\right)\,, \tag{3.3b}$$

where the cosmic scale factor has normalization $a(t_0)=1$ at $t_0>0$ and the constant ρ_{M0} is positive [from (3.2a), we have $G\rho_{M0} \propto 1/(b^2+t_0^2)$].

The new solution (3.3) is perfectly smooth at $t = 0$ as long as $b \neq 0$, and the same holds for the corresponding Kretschmann curvature scalar $K(t) \propto 1/(b^2 + t^2)^2$. Figure 1 compares this regularized FLRWK solution (full curve) with the singular FLRW solution (dashed curve).

Observe that the function $a(t)$ from (3.3a) is convex over a *finite* interval around $t = 0$, whereas the function $\widetilde{a}(\tau)$ from (2.5a), with t replaced by τ and t_0 by τ_0, is concave for $\tau \geq b > 0$. This different behavior of $a(t)$ just above $t = 0$ (convex) and $\widetilde{a}(\tau)$ just above $\tau = b$ (concave) results from the different differential structures mentioned at the end of Section 3.1.

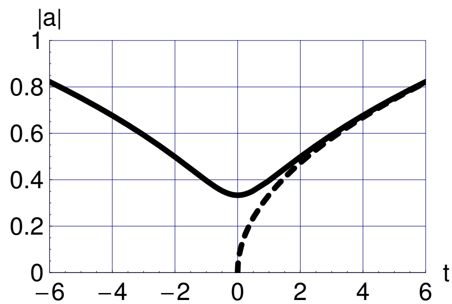

Fig. 1. Cosmic scale factor $a(t)$ of the spatially flat FLRWK universe with $w = 1/3$ matter (full curve), as given by (3.3a) for $b = 1$ and $t_0 = 4\sqrt{5}$. Also shown is the cosmic scale factor of the standard spatially flat FLRW universe with $w = 1/3$ matter (dashed curve), as given by (2.5a) with $t_0 = 4\sqrt{5}$.

3.3. Bounce or new physics phase?

With the regular solution (3.3) in hand, there are now two scenarios:

1. A nonsingular bouncing cosmology, where the cosmic time coordinate t runs from $-\infty$ to $+\infty$. This scenario may hold for $b \gg l_{\text{Planck}}$, so that classical Einstein gravity can be expected to be applicable. A possibly more realistic solution than the one from (3.3) has EOS parameter $w_M = 1$ in the prebounce epoch and $w_M = 1/3$ in the postbounce epoch [8]. Potential experimental signatures may rely on gravitational waves generated in the prebounce epoch, which keep on propagating into the postbounce epoch [9].

2. A new physics phase at $t = 0$, which produces two apparently similar universes. This scenario may hold for $b \sim l_{\text{Planck}}$. A special scenario has the new physics phase at $t = 0$ pair-producing a "universe" for $t > 0$ and an "antiuniverse" for $t < 0$. The relative role of particles and antiparticles in the $t > 0$ branch is then reversed compared to the

one in the separate $t < 0$ branch. In order to obtain this particle–antiparticle behavior, it appears necessary to use the (discontinuous) *odd* solution for $a(t)$, as given by the right-hand side of (3.3a) for $t > 0$ and the same with an overall minus sign for $t < 0$. (The full curve of Fig. 1 also applies to this odd solution, as the figure plots the absolute value of a.) See Ref. [20] for further discussion.

For both scenarios, the $t = 0$ slice corresponds to a three-dimensional *spacetime defect*, which manifests itself as a finite discontinuity at $t = 0$ in the trace $K_{\text{extr}}(t)$ of the extrinsic curvature on constant-t hypersurfaces. Also, there is a finite discontinuity at $t = 0$ in the expansion $\theta(t)$ for a particular congruence of timelike geodesics (for the standard FLRW solution, $\theta(t)$ diverges as $t \to 0^+$; see Sec. 4.2.3 of Ref. [10] for the explicit expressions of θ). The discontinuous behavior of $K_{\text{extr}}(t)$ and $\theta(t)$, in the spacetime with metric (3.1), shows that $K_{\text{extr}}(t)$ and $\theta(t)$ are *ill-defined* at $t = 0$. A related observation is that the $t = 0$ hypersurface of the spacetime with metric (3.1) is not a Cauchy surface (*cf.* Sec. 10.2 of Ref. [18]).

It is unclear for the first scenario, what physical mechanism determines the relatively large value of b and the *raison d'être* of the spacetime defect in the first place. For the second scenario, the hope is that the new physics sets the value of b and also explains the origin of the spacetime defect. The focus of this review article is on the second scenario.

In any case, there is no doubt as to the validity of the Hawking and Hawking–Penrose cosmological singularity theorems [16, 17] and the "singularity" of these theorems may very well correspond to a three-dimensional spacetime defect with a locally degenerate metric, as long as the defect is explained by new physics (if, for example, the defect is produced as a remnant of a new physics phase). We refer the reader to the paragraph starting with "This brings us to the third question: the nature of the singularity" in Sec. 1 of Hawking's paper [16]. Our suggested new physics would, in principle, be physically observable, which would address Hawking's reservations about including the degeneracy points in the definition of spacetime (see, in particular, the sentence with the word "undesirable" in the paragraph mentioned).

4. New phase from M-theory

4.1. Preliminary remarks

We will now explore the idea that a new physics phase gave rise to classical spacetime and matter, as described by general relativity and the Standard Model of elementary particles. The fabric of classical spacetime would emerge from this new physics phase and classical spacetime might resemble an atomic crystal. But, for an atomic crystal, we know that, if the formation of the crystal is rapid enough, there may occur crystal defects. By analogy, it could then be that the hypothetical new physics phase produced the regularized-big-bang spacetime (3.1), with a three-dimensional spacetime defect.

For the moment, we do not know for sure how to describe such a new physics phase. One candidate theory is M-theory. Recall that M-theory is a hypothetical theory that unifies all five consistent versions of superstring theory (*cf.* Refs. [21–23]); see the "nerve-cell" sketch of Fig. 2.

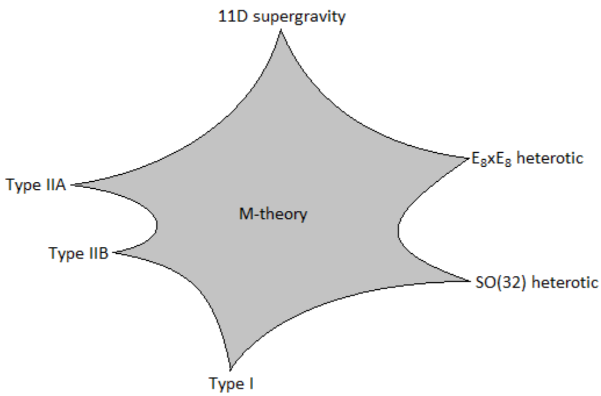

Fig. 2. Sketch of the relationship between M-theory, the five ten-dimensional superstring theories, and eleven-dimensional supergravity theory.

For an *explicit* description of the new physics phase replacing the Big Bang, we will use the IIB matrix model of Kawai and collaborators [24, 25], which has been proposed as a nonperturbative formulation of type-IIB superstring theory and, thereby, of M-theory (the assumption is that all theories of Fig. 2 belong to the same universality class).

4.2. IIB matrix model

The IIB matrix model, also known as IKKT model [24, 25], has a finite number of $N \times N$ traceless Hermitian matrices: ten bosonic matrices A^{μ} and essentially eight fermionic (Majorana–Weyl) matrices Ψ_{α}.

The partition function Z of the Lorentzian IIB matrix model is defined by the following "path" integral [24–27]:

$$Z = \int \mathrm{d}A\, \mathrm{d}\Psi \ \exp\left[i\, S/\ell^4\right] = \int \mathrm{d}A \ \exp\left[i\, S_{\text{eff}}/\ell^4\right] , \tag{4.1a}$$

$$S = -\mathrm{Tr}\left(\frac{1}{4}\,[A^\mu,\, A^\nu]\,[A^\rho,\, A^\sigma]\,\widetilde{\eta}_{\mu\rho}\,\widetilde{\eta}_{\nu\sigma} + \tfrac{1}{2}\overline{\Psi}_\beta\,\widetilde{\Gamma}^\mu_{\beta\alpha}\,\widetilde{\eta}_{\mu\nu}\,[A^\nu,\, \Psi_\alpha]\right) , \tag{4.1b}$$

$$\widetilde{\eta}_{\mu\nu} = [\mathrm{diag}\,(-1,\, 1,\, \ldots\, ,\, 1)]_{\mu\nu}\, , \quad \text{for} \quad \mu,\nu \in \{0,\, 1,\, \ldots\, ,\, 9\}\, , \tag{4.1c}$$

where the action (4.1b) contains only Yang–Mills-type commutators, defined by $[X,\, Y] \equiv X\cdot Y - Y\cdot X$ for square matrices X and Y. The fermionic matrices have been integrated out in the last step of (4.1a) and the effective action S_{eff} includes a complicated high-order term in addition to the bosonic quartic term. Expectation values of further observables will be discussed later.

We have two technical remarks. First, we speak of the "Lorentzian" IIB matrix model, because the coupling constants $\widetilde{\eta}_{\mu\nu}$ in (4.1c) resemble the Lorentzian metric of ten-dimensional Minkowski spacetime. Second, a model length scale, ℓ, has been introduced in (4.1a), so that A^μ from (4.1b) has the dimension of length and Ψ_α the dimension of $(\text{length})^{3/2}$.

Now, the matrices A^μ and Ψ_α in (4.1a) are merely *integration variables*. Moreover, there is *no obvious small dimensionless parameter* to motivate a saddle-point approximation. Hence, the following conceptual question arises; *where is the classical spacetime?*

Recently, we have suggested to revisit an old idea, the large-N master field of Witten [28–30], for a possible origin of classical spacetime in the context of the IIB matrix model [31]. Let us, first, recall the meaning of this mysterious master field (a name coined by Coleman [29]) and, then, present the final result (with technical details moved to the Appendices).

4.3. Large-N master field

Consider the following bosonic observable:

$$w^{\mu_1\,\cdots\,\mu_m} \equiv \mathrm{Tr}\,\left(A^{\mu_1}\ldots\, A^{\mu_m}\right)\, , \tag{4.2}$$

which is invariant under a global gauge transformation

$$A'^{\,\mu} = \Omega\, A^{\mu}\, \Omega^{\dagger}\, , \qquad \Omega \in \mathrm{SU}(N)\, . \tag{4.3}$$

Arbitrary strings of these w observables have expectation values

$$\langle w^{\mu_1\,\cdots\,\mu_m}\, w^{\nu_1\,\cdots\,\nu_n}\, \ldots\rangle = \frac{1}{Z}\int \mathrm{d}A\, \left(w^{\mu_1\,\cdots\,\mu_m}\, w^{\nu_1\,\cdots\,\nu_n}\, \ldots\right) \exp\left[i\, S_{\text{eff}}/\ell^4\right]\, , \tag{4.4}$$

where the normalization factor $1/Z$ gives $\langle\, 1\,\rangle = 1$.

For a string of two identical w observables, the following factorization property holds to leading order in N:

$$\langle\, w^{\mu_1\,\cdots\,\mu_m}\; w^{\mu_1\,\cdots\,\mu_m}\,\rangle \stackrel{N}{=} \langle w^{\mu_1\,\cdots\,\mu_m}\rangle\;\langle w^{\mu_1\,\cdots\,\mu_m}\rangle\;, \tag{4.5}$$

without sums over repeated indices. Similar large-N factorization properties hold for *all* expectation values (4.4),

$$\langle\, w^{\mu_1\,\cdots\,\mu_m}\; w^{\nu_1\,\cdots\,\nu_n}\;\ldots\; w^{\omega_1\,\cdots\,\omega_z}\,\rangle \stackrel{N}{=} \langle\, w^{\,\mu_1\,\cdots\,\mu_m}\,\rangle\;\langle\, w^{\,\nu_1\,\cdots\,\nu_n}\,\rangle\;\ldots\;\langle\, w^{\,\omega_1\,\cdots\,\omega_z}\,\rangle\;, \tag{4.6}$$

with a product of expectation values on the right-hand side.

The leading-order equality (4.5) states that the expectation value of the square of w equals the square of the expectation value of w, which is a truly remarkable result for a statistical (quantum) theory. Indeed, according to Witten [28], the factorizations (4.5) and (4.6) imply that the path integrals (4.4) are *saturated by a single configuration*, the so-called master field $\widehat{A}^{\,\mu}$ (from now on, the caret will denote a master-field variable).

Considering one w observable for simplicity and working to leading order in N, we then have for its expectation value:

$$\langle\, w^{\mu_1\,\cdots\,\mu_m}\,\rangle \stackrel{N}{=} \mathrm{Tr}\,\left(\widehat{A}^{\,\mu_1}\,\ldots\,\widehat{A}^{\,\mu_m}\right) \equiv \widehat{w}^{\,\mu_1\,\cdots\,\mu_m}\,. \tag{4.7}$$

Similarly, we have for the other expectation values (4.4):

$$\langle\, w^{\mu_1\,\cdots\,\mu_m}\; w^{\nu_1\,\cdots\,\nu_n}\;\ldots\; w^{\omega_1\,\cdots\,\omega_z}\,\rangle \stackrel{N}{=} \widehat{w}^{\,\mu_1\,\cdots\,\mu_m}\;\widehat{w}^{\,\nu_1\,\cdots\,\nu_n}\;\ldots\;\widehat{w}^{\,\omega_1\,\cdots\,\omega_z}\,, \tag{4.8}$$

with a product of real numbers on the right-hand side. Hence, we do not have to perform the path integrals on the right-hand side of (4.4): we "only" need ten traceless Hermitian matrices $\widehat{A}^{\,\mu}$ to get *all* these expectation values from the simple procedure of replacing each $A^{\,\mu}$ in the observables by $\widehat{A}^{\,\mu}$ (the quotation marks are there, because the ten matrices are, of course, very large as N increases towards infinity).

4.4. Emergent classical spacetime

Now, the meaning of the suggestion in the last paragraph of Section 4.2 is clear: classical spacetime may reside in the matrices $\widehat{A}^{\,\mu}$ of the IIB-matrix-model master field. The heuristics is as follows:

— The expectation values $\langle w^{\mu_1\,\cdots\,\mu_m}\; w^{\nu_1\,\cdots\,\nu_n}\;\ldots\; w^{\omega_1\,\cdots\,\omega_z}\rangle$ from (4.4) correspond to an infinity of real numbers and these real numbers contain a large part of the *information content* of the IIB matrix model (but, of course, not all the information).

— That *same* information is carried by the master-field matrices $\widehat{A}^{\mu}$, which reproduce, to leading order in N, the very same real numbers as the products $\widehat{w}^{\mu_1 \cdots \mu_m}\, \widehat{w}^{\nu_1 \cdots \nu_n} \ldots \widehat{w}^{\omega_1 \cdots \omega_z}$, where each real number $\widehat{w}$ entering these products is the observable w evaluated for the master-field matrices $\widehat{A}^{\mu}$.

— From these master-field matrices $\widehat{A}^{\mu}$, it appears indeed feasible to *extract* the points and metric of an emergent classical spacetime (recall that the original matrices A^{μ} were merely integration variables).

It is certainly satisfying to have a heuristic understanding, but we would like to proceed further.

Let us, first, *assume* that the matrices $\widehat{A}^{\mu}$ of the Lorentzian-IIB-matrix-model master field are known and that they are approximately band-diagonal (as suggested by the numerical results of Refs. [26, 27]). Then, it is possible [31] to extract a discrete set of spacetime points $\{\widehat{x}_k^{\mu}\}$ and the emergent inverse metric $g^{\mu\nu}(x)$ for a continuous (interpolating) spacetime coordinate x^{μ}; toy-model calculations have been presented in Refs. [32, 33]. The metric $g_{\mu\nu}(x)$ is simply obtained as matrix inverse of $g^{\mu\nu}(x)$.

It is even possible [34] that the large-N master field of the Lorentzian-IIB-matrix model gives the regularized-big-bang metric (3.1). In that case, the final result is that the effective length parameter b of the emergent regularized-big-bang metric (3.1) is calculated in terms of the IIB-matrix-model length scale ℓ:

$$b_{\text{eff}} \sim \ell \overset{?}{\sim} l_{\text{Planck}} \equiv \sqrt{\hbar\, G/c^3} \approx 1.62 \times 10^{-35}\,\text{m}\,. \tag{4.9}$$

The argument for the connection of ℓ and l_{Planck} has been presented in Sec. IV of Ref. [34], under the assumption that Einstein gravity is recovered from the IIB matrix model.

Technical details are collected in four Appendices. Specifically, Appendix A discusses how precisely the spacetime points are extracted from the matrices $\widehat{A}^{\mu}$ of the Lorentzian-IIB-matrix-model master field, under the assumption that these matrices are known. Appendix B then shows how the emergent spacetime metric is obtained from the distributions of the extracted spacetime points. Appendix C discusses different emergent spacetimes, which result from different assumptions for the properties of the master-field matrices. (Discussed as well, in the second subsection of Appendix C, is the issue of "topology without topology", namely obtaining an effective nontrivial topology from strong gravitational fields [35].) Appendix D presents an alternative mechanism for getting a Lorentzian signature of the emergent spacetime metric.

At this moment, we should mention that there are also other approaches to extracting a classical spacetime, examples being noncommutative geometry [36, 37] and entanglement [38, 39].

5. Conclusion

It is conceivable that a new physics phase replaces the Friedmann Big Bang singularity suggested by our current theories, general relativity and the Standard Model of elementary particles. For an explicit calculation, we have turned to the IIB matrix model, which has been proposed as a nonperturbative formulation of type-IIB superstring theory (M-theory).

The crucial insight is that the emergent classical spacetime may reside in the matrices $\widehat{A}^{\mu}$ of the large-N master field of the IIB matrix model. The master-field matrices $\widehat{A}^{\mu}$ can, in principle, produce the regularized-big-bang metric (3.1) with length parameter $b \sim \ell$, where ℓ is the length scale of the matrix model.

The outstanding task, now, is to *calculate* the exact matrices $\widehat{A}^{\mu}$ of the IIB-matrix-model master field (see also the last subsection of Appendix D for further comments). As the exact matrices $\widehat{A}^{\mu}$ will be hard to obtain, it perhaps makes sense to first look for a reliable approximation of them.

It is a pleasure to thank Z.L. Wang for discussions on cosmological singularity theorems and the Conference Organizers for bringing about this interesting online meeting.

Note Added: Following up on the remarks in the very last paragraph of Appendix D, we have started with the calculation of the master-field matrices [40, 41].

Appendix A

Extraction of the spacetime points

Aoki *et al.* [25] have argued that the *eigenvalues* of the bosonic matrices A^{μ} of model (4.1) can be interpreted as *spacetime coordinates*, so that the model has a ten-dimensional $\mathcal{N}=2$ spacetime supersymmetry. This supersymmetry, incidentally, implies the existence of a graviton, as long as there are massless particles in the spectrum.

Here, we will turn to the eigenvalues of the *master-field* matrices $\widehat{A}^{\mu}$. Assume that the matrices $\widehat{A}^{\mu}$ of the Lorentzian-IIB-matrix-model master field are known and that they are approximately band-diagonal with width

$\Delta N < N$ (as suggested by the numerical results of Refs. [26, 27]). Then, make a particular global gauge transformation [26],

$$\underline{\widehat{A}}^{\mu} = \underline{\Omega}\, \widehat{A}^{\mu}\, \underline{\Omega}^{\dagger}, \qquad \underline{\Omega} \in \mathrm{SU}(N)\,, \tag{A.1}$$

so that the transformed 0-component matrix is diagonal (the component 0 is singled out by the "Lorentzian" coupling constants $\widetilde{\eta}_{\mu\nu}$) and has ordered eigenvalues $\widehat{\alpha}_i \in \mathbb{R}$,

$$\underline{\widehat{A}}^{0} = \mathrm{diag}\,(\widehat{\alpha}_1,\, \widehat{\alpha}_2,\, \ldots\,,\, \widehat{\alpha}_{N-1},\, \widehat{\alpha}_N)\;, \tag{A.2a}$$

$$\widehat{\alpha}_1 \leq \widehat{\alpha}_2 \leq \;\ldots\; \leq \widehat{\alpha}_{N-1} \leq \widehat{\alpha}_N\,, \tag{A.2b}$$

$$\sum_{i=1}^{N} \widehat{\alpha}_i = 0\,. \tag{A.2c}$$

The ordering (A.2b) will turn out to be crucial for the time coordinate $\widehat{t}$ to be obtained later.

A relatively simple procedure [31] *approximates* the eigenvalues of the spatial matrices $\underline{\widehat{A}}^{m}$ but still manages to *order them along the diagonal*, in sync with the temporal eigenvalues $\widehat{\alpha}_i$ from (A.2). This procedure corresponds, in fact, to a type of coarse graining of some of the information contained in the master field.

We start from the following trivial observations. If M is an $N \times N$ Hermitian matrix, then any $n \times n$ block centered on the diagonal of M is also Hermitian, which holds for any value of n with $1 \leq n \leq N$. If the matrix M is, moreover, band-diagonal with width $\Delta N < N$, then the eigenvalues of the $n \times n$ blocks on the diagonal approximate the original eigenvalues of M, provided $n \gtrsim \Delta N$.

Now, let K and n be divisors of N, so that

$$N = K\, n\,, \tag{A.3}$$

where both K and n are positive integers. Then, consider, in each of the ten matrices $\underline{\widehat{A}}^{\mu}$, the K adjacent blocks of size $n \times n$ centered on the diagonal.

From (A.2), we already know the diagonalized $n \times n$ blocks of $\underline{\widehat{A}}^{0}$ with eigenvalues $\widehat{\alpha}_i$. This allows us to define the following time coordinate $\widehat{t}\,(\sigma)$ for $\sigma \in (0,\, 1]$ from the averages of the $\widehat{\alpha}_i$'s in the blocks (labeled k):

$$\widehat{x}^{0}\;(k/K) \equiv \widetilde{c}\,\widehat{t}\;(k/K) = \frac{1}{n}\sum_{j=1}^{n} \widehat{\alpha}_{(k-1)\,n+j}\,, \tag{A.4}$$

with $k \in \{1, \ldots, K\}$ and a velocity $\widetilde{c}$ to be set to unity later. The time coordinates from (A.4) are ordered,

$$\widehat{t}\,(1/K) \,\leq\, \widehat{t}\,(2/K) \,\leq\, \ldots \,\leq\, \widehat{t}\,(1-1/K) \,\leq\, \widehat{t}(1)\,, \tag{A.5}$$

precisely because the $\widehat{\alpha}_i$ are ordered according to (A.2b). Observe that this ordering property is the defining characteristic of what makes a physical time.

Next, let us obtain the eigenvalues of the $n \times n$ blocks of the nine spatial matrices $\underline{\widehat{A}}^{m}$ and denote these real eigenvalues by $(\widehat{\beta}^{m})_i$, with a label $i \in \{1, \ldots, N\}$ respecting the order of the n-dimensional blocks. Define, just as for the time coordinate in (A.4), the following nine spatial coordinates $\widehat{x}^{m}(\sigma)$ for $\sigma \in (0, 1]$ from the respective averages:

$$\widehat{x}^{m}\,(k/K) = \frac{1}{n}\,\sum_{j=1}^{n}\,\left[\,\widehat{\beta}^{m}\,\right]_{(k-1)\,n+j}\,, \tag{A.6}$$

with $k \in \{1, \ldots, K\}$.

If the master-field matrices $\underline{\widehat{A}}^{\mu}$ are approximately *band-diagonal* with width ΔN and if the eigenvalues of the spatial $n \times n$ blocks (with $n \gtrsim \Delta N$) show significant *scattering* [32], then the expressions (A.4) and (A.6) may provide suitable spacetime points. In a somewhat different notation, these points are denoted

$$\widehat{x}^{\mu}_{k} = \left(\,\widehat{x}^{0}_{k},\, \widehat{x}^{m}_{k}\,\right) \equiv \left(\,\widehat{x}^{0}\,(k/K)\,,\, \widehat{x}^{m}\,(k/K)\,\right)\,, \tag{A.7}$$

where k runs over $\{1, \ldots, K\}$ with $K = N/n$ from (A.3). Each of these coordinates $\widehat{x}^{\mu}_{k}$ has the dimension of length, which traces back to the dimension of the bosonic matrix variable A^{μ}, as mentioned in the second technical remark of the new paragraph starting a few lines below (4.1).

There are alternative procedures for the extraction of spacetime points, one of which is discussed in Appendix B of Ref. [32]. That alternative procedure randomly selects *one* eigenvalue from *each* $n \times n$ block of the gauge-transformed master-field matrices $\underline{\widehat{A}}^{\mu}$ and gives the following extracted points (denoted by a tilde instead of a caret):

$$\widetilde{x}^{0}_{k} \;=\; \widehat{\alpha}_{(k-1)\,n+\mathrm{rand}[1,\,n]}\,, \tag{A.8a}$$

$$\widetilde{x}^{m}_{k} \;=\; \left[\,\widehat{\beta}^{m}\,\right]_{(k-1)\,n+\mathrm{rand}[1,\,n]}\,, \tag{A.8b}$$

where k runs over $\{1, \ldots, K\}$ and "rand[1, n]" is a uniform pseudorandom integer from the set $\{1, 2, \ldots, n\}$. For the moment, we focus our attention on the averaging procedure from (A.4) and (A.6).

To summarize, with $N = K\,n$ and $n \gtrsim \Delta N$, the extracted spacetime points $\widehat{x}_k^{\,\mu}$, for $k \in \{1, \ldots, K\}$, are obtained as *averaged eigenvalues* of the $n \times n$ blocks along the diagonals of the gauge-transformed master-field matrices $\widehat{\underline{A}}^{\,\mu}$ from (A.1)–(A.2).

Appendix B

Extraction of the spacetime metric

The discrete set of points $\{\widehat{x}_k^{\,\mu}\}$ from (A.7) effectively builds a spacetime manifold with continuous (interpolating) coordinates x^μ if there is also an emerging metric $g_{\mu\nu}(x)$.

From the effective action of a low-energy scalar degree of freedom σ "propagating" over the discrete spacetime points $\widehat{x}_k^{\,\mu}$ (see below for details), the following expression for the emergent *inverse* metric has been obtained [25, 31]:

$$g^{\mu\nu}(x) \;\sim\; \int\limits_{\mathbb{R}^D} \mathrm{d}^D y\; \rho_{\mathrm{av}}(y)\; (x-y)^\mu\,(x-y)^\nu\; f(x-y)\; r(x,\,y)\,, \tag{B.1a}$$

$$\rho_{\mathrm{av}}(y) \;\equiv\; \langle\langle\, \rho(y) \,\rangle\rangle\,, \tag{B.1b}$$

with continuous spacetime coordinates x^μ having the dimension of length and spacetime dimension $D = 1 + 9 = 10$ for the original model (4.1).

The quantities that enter the integral (B.1) are the density function

$$\rho(x) \;\equiv\; \sum_{k=1}^{K} \delta^{(D)}\left(x - \widehat{x}_k\right)\,, \tag{B.2}$$

the density correlation function $r(x,\,y)$ defined by

$$\langle\langle\, \rho(x)\,\rho(y) \,\rangle\rangle \;\equiv\; \langle\langle\, \rho(x) \,\rangle\rangle\; \langle\langle\, \rho(y) \,\rangle\rangle\; r(x,\,y)\,, \tag{B.3}$$

and a localized real function $f(x)$ from the scalar effective action,

$$S_{\mathrm{eff}}[\sigma] \sim \sum_{k,\,l} \frac{1}{2}\, f\left(\widehat{x}_k - \widehat{x}_l\right)\; \left(\sigma_k - \sigma_l\right)^2\,, \tag{B.4}$$

where σ_k is the real field value at the point $\widehat{x}_k$ (the real scalar degree of freedom σ arises from a perturbation of the master field $\widehat{\underline{A}}^{\,\mu}$ and has the dimension of length; see Appendix A in Ref. [31] for a toy-model calculation). For the extraction procedure of Appendix A, the quantity $\langle\langle\, \rho(y) \,\rangle\rangle$ in (B.1b) results from averaging over different block sizes n and block positions along the diagonals in the master-field matrices $\widehat{\underline{A}}^{\,\mu}$ (possibly with smaller blocks at the end or beginning of the diagonal, for a fixed large value of N).

Very briefly, expression (B.1) is obtained as follows from the effective action (B.4). Define the continuous field $\sigma(x)$ as having $\sigma(\widehat{x}_k) = \sigma_k$ and write (B.4) in terms of $\sigma(x)$. Next, average over different block structures in the master-field matrices (see above) and make appropriate Taylor expansions of $\sigma(x)$. The continuous field $\sigma(x)$ is then found to have a standard local kinetic term $(1/2)\, \partial_\mu \sigma(x)\, \partial_\nu \sigma(x)\, g^{\mu\nu}(x)$ in the action, with the inverse metric $g^{\mu\nu}(x)$ as given by the expression (B.1). See Sec. 4.2 of Ref. [25] for further details.

As $r(x,\, y)$ is dimensionless and $f(x)$ has dimension $1/(\text{length})^2$, the inverse metric $g^{\mu\nu}(x)$ from (B.1) is seen to be dimensionless. The metric $g_{\mu\nu}$ is obtained as the matrix inverse of $g^{\mu\nu}$. See Sec. II B of Ref. [34] for some heuristic remarks on expression (B.1) for the emergent inverse metric.

Note that, in principle, the origin of expression (B.1) need not be the IIB matrix model but can be an entirely different theory, as long as the emerging inverse metric is given by a multiple integral with the same basic structure. But, for the moment, we only discuss a IIB-matrix-model origin.

To summarize, the emergent metric, in the context of the IIB matrix model, is obtained from *correlations* of the extracted spacetime points and the master-field perturbations.

Appendix C

Various emergent spacetimes

Preliminaries

The obvious question, now, is which spacetime and metric *do* we get from the steps outlined in Appendices A and B. We do not know for sure, as we do not have the exact IIB-matrix-model master field. But, awaiting the final result on the master field, we can already investigate what properties the master field *would* need to have in order to be able to produce certain desired metrics. It is far from obvious that these desired metrics can be obtained from expression (B.1), but it appears indeed feasible. First results are presented in the rest of this appendix.

Emergent Minkowski and Robertson–Walker metrics

We restrict ourselves to four "large" spacetime dimensions [26, 27], setting

$$D = 1 + 3 = 4\,, \tag{C.1}$$

and use length units that normalize the Lorentzian-IIB-matrix-model length scale,

$$\ell = 1\,. \tag{C.2}$$

In Ref. [33], we have then shown that it is possible to choose appropriate functions $\rho_{\text{av}}(y)$, $f(x-y)$, and $r(x,y)$ in (B.1) for $D=4$, so that the Minkowski metric is obtained, as given by (2.3) for $a^2(t)=1$. Furthermore, it is possible to deform the chosen function $\rho_{\text{av}}(y)$, so that the spatially flat ($k=0$) Robertson–Walker metric (2.3) is obtained.

The question was raised in Ref. [33] whether or not it would also be possible, in principle, to obtain a Robertson–Walker universe with positive ($k=+1$) or negative ($k=-1$) spatial curvature. This is perhaps not excluded as the following result suggests. It has been shown, in fact, that the $k=1$ Robertson–Walker universe need not really have an underlying $\mathbb{R}\times S^3$ topology but can result from strong gravitational fields over Minkowski spacetime with an $\mathbb{R}^4$ topology (see Ref. [35], which builds upon unpublished work from 1992 by M. Veltman and the present author).

Emergent regularized-big-bang metric

In order to get an inverse metric whose component g^{00} *diverges* at $t=0$, it is necessary to *relax* the convergence properties of the y^0 integral in (B.1) by a suitable change in the *Ansätze* for $\rho_{\text{av}}(y)$, $f(x-y)$, and $r(x,y)$.

In this way, we are able to obtain the following inverse metric [34]:

$$g^{\mu\nu}_{(\text{eff})} \sim \begin{cases} -\dfrac{t^2+c_{-2}}{t^2}\,, & \text{for}\;\; \mu=\nu=0\,, \\ 1+c_2\,t^2+c_4\,t^4+\dots\,, & \text{for}\;\; \mu=\nu=m\in\{1,\,2,\,3\}\,, \\ 0\,, & \text{otherwise}\,, \end{cases} \tag{C.3}$$

with real dimensionless coefficients c_n in $g^{mm}_{(\text{eff})}$ resulting from the requirement that t^n terms, for $n>0$, vanish in $g^{00}_{(\text{eff})}$. The matrix inverse of (C.3) gives the following Lorentzian metric:

$$g_{\mu\nu}^{(\text{eff})} \sim \begin{cases} -\dfrac{t^2}{t^2+c_{-2}}\,, & \text{for}\;\; \mu=\nu=0\,, \\ \dfrac{1}{1+c_2\,t^2+c_4\,t^4+\dots}\,, & \text{for}\;\; \mu=\nu=m\in\{1,\,2,\,3\}\,, \\ 0\,, & \text{otherwise}\,, \end{cases} \tag{C.4}$$

which has, for $c_{-2}>0$, a vanishing determinant at $t=0$ and is, therefore, degenerate.

The emergent degenerate metric (C.4) has, in fact, the structure of the RWK metric (3.1), with the following effective parameters:

$$b^2_{\text{eff}} \sim c_{-2}\,\ell^2\,, \qquad a^2_{\text{eff}}(t) \sim 1-c_2\;(t/\ell)^2+\dots\,, \tag{C.5}$$

where the IIB-matrix-model length scale ℓ has been restored and where the leading coefficients c_{-2} and c_2 have been calculated [34]. By choosing the *Ansatz* parameters appropriately, we can get $c_{-2} > 0$ and $c_2 < 0$ in (C.5), so that the emergent classical spacetime corresponds to the spacetime of a nonsingular cosmic bounce at $t = 0$, as obtained in (3.3a) from Einstein's gravitational field equation with a homogeneous relativistic perfect fluid. Incidentally, a possibly odd functional behavior of $a(t)$, as mentioned in point 2 of Section 3.3, could result from a consistency condition involving the fermionic degrees of freedom, which are present in the original matrix model (4.1).

As a final remark, we note that the *origin* of the spacetime defect in the regularized-big-bang (RWK) metric appears to be due to *long-range tails* [34] of certain correlation functions entering the multiple-integral expression (B.1) for the emergent inverse metric.

Appendix D

More on the Lorentzian signature

Alternative mechanism

Up till now, we have considered the Lorentzian-IIB-matrix model, which has two characteristics:

1. the Feynman phase factor $\exp\left[i\,S/\ell^4\,\right]$ in the "path" integral (4.1a);
2. the "Lorentzian" coupling constants $\widetilde{\eta}_{\mu\nu}$ from (4.1c) entering the action (4.1b).

With an assumed master field of this Lorentzian matrix model, we obtained the spacetime points from expressions (A.4) and (A.6) in Appendix A and the inverse metric from expression (B.1) in Appendix B.

Several Lorentzian inverse metrics were found in Appendix C, where the used *Ansätze* [33, 34] relied on the available "Lorentzian" coupling constants $\widetilde{\eta}_{\mu\nu}$. Specifically, the metrics obtained were the Minkowski metric, the Robertson–Walker metric, and the regularized-big-bang (RWK) metric.

There is, however, another way [31] to obtain Lorentzian inverse metrics, namely by making an appropriately *odd Ansatz* for the correlations functions entering expression (B.1), so that the resulting matrix has an *off-diagonal* structure with one real eigenvalue having a different sign than the others.

With this appropriately odd *Ansatz*, it is, in principle, also possible to get a *Lorentzian* emergent inverse metric from the *Euclidean* matrix model, which has a weight factor $\exp\left[-S/\ell^4\,\right]$ in the path integral and nonnegative coupling constants $\widetilde{\delta}_{\mu\nu}$ in the action. The spacetime points are extracted from the Euclidean master field (without need for a particular gauge transformation) by expression (A.6), where m now runs over $\{1, \ldots, D\}$.

Toy-model calculation

We present, here, the full details of a Euclidean toy-model calculation, which appeared as a parenthetical remark in the last paragraph of Appendix B in Ref. [31].

We start the calculation with the multiple integral (B.1), for spacetime dimension $D = 4$ and model length scale $\ell = 1$, and write in the integrand

$$f(x-y)\, r(x,\, y) = f(x-y)\, \widetilde{r}(y-x)\, \overline{r}(x,\, y) = h(y-x)\, \overline{r}(x,\, y)\,, \tag{D.1}$$

where the new function $\overline{r}(x,\, y)$ has a more complicated dependence on x and y than the combination $x - y$.

The $D = 4$ multiple integral (B.1), having y^0 replaced by y^4, is then evaluated at the spacetime point

$$x^\mu = 0\,, \tag{D.2a}$$

with the replacement (D.1) in the integrand, two further simplifications

$$\rho_{\text{av}}(y) = 1\,, \qquad \overline{r}(x,\, y) = 1\,, \tag{D.2b}$$

and symmetric cutoffs on the integrals

$$\int_{-1}^{1} \mathrm{d}y^1 \ldots \int_{-1}^{1} \mathrm{d}y^4\,. \tag{D.2c}$$

The only nontrivial contribution to the integrand of (B.1) then comes from the correlation function h as defined by (D.1).

From (B.1), (D.1), and (D.2), we get the following expression for the emergent inverse metric at $x^\mu = 0$:

$$g^{\mu\nu}_{\text{test,E4}}(0) \sim \int_{-1}^{1} \mathrm{d}y^1 \int_{-1}^{1} \mathrm{d}y^2 \int_{-1}^{1} \mathrm{d}y^3 \int_{-1}^{1} \mathrm{d}y^4 \;\; y^\mu\, y^\nu\, h(y)\,. \tag{D.3}$$

Next, let us make an appropriate *Ansatz* for the correlation function h in (D.3):

$$h(y) = 1 - \gamma\, \left(y^1\, y^2 + y^1\, y^3 + y^1\, y^4 + y^2\, y^3 + y^2\, y^4 + y^3\, y^4\right)\,, \tag{D.4}$$

where γ multiplies monomials that are odd in two coordinates and even in the two others. Note that the *Ansatz* (D.4) treats *all* coordinates y^1, y^2, y^3, and y^4 *equally*, matching the structure of the coupling constants $\widetilde{\delta}_{\mu\nu}$ of the Euclidean matrix model.

The integrals of (D.3) with *Ansatz* function (D.4) are trivial and we obtain

$$g_{\gamma}^{\mu\nu}(0) \sim \frac{16}{9} \begin{pmatrix} 3 & -\gamma & -\gamma & -\gamma \\ -\gamma & 3 & -\gamma & -\gamma \\ -\gamma & -\gamma & 3 & -\gamma \\ -\gamma & -\gamma & -\gamma & 3 \end{pmatrix}, \tag{D.5a}$$

where the matrix on the right-hand side has the following eigenvalues and corresponding (normalized) eigenvectors:

$$\mathcal{E}_{\gamma} = \frac{16}{9} \left\{(3-3\,\gamma)\,,\,(3+\gamma)\,,\,(3+\gamma)\,,\,(3+\gamma)\right\}\,, \tag{D.5b}$$

$$\mathcal{V}_{\gamma} = \left\{ \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \right\}. \tag{D.5c}$$

From the eigenvalues (D.5b), we get the following signatures:

$$(+---)\,, \quad \text{for } \gamma \in (-\infty,\,-3)\,, \tag{D.6a}$$

$$(++++)\,, \quad \text{for } \gamma \in (-3,\,1)\,, \tag{D.6b}$$

$$(-+++)\,, \quad \text{for } \gamma \in (1,\,\infty)\,. \tag{D.6c}$$

Hence, we find Lorentzian signatures for parameter values γ sufficiently far away from zero, $\gamma > 1$ or $\gamma < -3$. Note that, for the Lorentzian cases, the time direction corresponds to the first eigenvector of (D.5c) with four equal components.

Outlook

The above toy-model calculation has shown that it is, in principle, possible to get a Lorentzian emergent inverse metric from the Euclidean-IIB-matrix model, provided the correlation functions have an appropriate structure. This observation, if applicable, would remove the need for working with the possibly more difficult Lorentzian-IIB-matrix model [26, 27] and we could return to the original Euclidean-IIB-matrix model [24, 25].

In this respect, it is worth mentioning that, from earlier work by Greensite and Halpern [30], we have obtained [31] an *algebraic equation* for the Euclidean bosonic master field. It remains to solve this equation ...

REFERENCES

[1] A. Einstein, «Die Grundlage der allgemeinen Relativitätstheorie», *Ann. Phys. (Leipzig)* **49**, 769 (1916).

[2] A.A. Friedmann, «Über die Krümmung des Raumes», *Z. Phys.* **10**, 377 (1922); «Über die Möglichkeit einer Welt mit konstanter negativer Krümmung des Raumes», *Z. Phys.* **21**, 326 (1924).

[3] G. Lemaître, «Un univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques», *Ann. Soc. Sci. Bruxelles* **A47**, 49 (1927).

[4] H.P. Robertson, «Kinematics and World-Structure», *Astrophys. J.* **82**, 284 (1935); «Kinematics and World-Structure II.», *Astrophys. J.* **83**, 187 (1936); «Kinematics and World-Structure III.», *Astrophys. J.* **83**, 257 (1936).

[5] A.G. Walker, «On Milne's Theory of World-Structure», *Proc. Lond. Math. Soc.* **42**, 90 (1937).

[6] F.R. Klinkhamer, «Regularized big bang singularity», *Phys. Rev. D* **100**, 023536 (2019), `arXiv:1903.10450 [gr-qc]`.

[7] F.R. Klinkhamer, «More on the regularized big bang singularity», *Phys. Rev. D* **101**, 064029 (2020), `arXiv:1907.06547 [gr-qc]`.

[8] F.R. Klinkhamer, Z.L. Wang, «Nonsingular bouncing cosmology from general relativity», *Phys. Rev. D* **100**, 083534 (2019), `arXiv:1904.09961 [gr-qc]`.

[9] F.R. Klinkhamer, Z.L. Wang, «Nonsingular bouncing cosmology from general relativity: Scalar metric perturbations», *Phys. Rev. D* **101**, 064061 (2020), `arXiv:1911.06173 [gr-qc]`.

[10] Z.L. Wang, «Spacetime Defects and Bouncing Cosmology», Ph.D. Thesis, KIT, Karlsruhe, Germany, 2020; available at: `https://publikationen.bibliothek.kit.edu/1000126191`

[11] F.R. Klinkhamer, «Skyrmion spacetime defect», *Phys. Rev. D* **90**, 024007 (2014), `arXiv:1402.7048 [gr-qc]`.

[12] F.R. Klinkhamer, F. Sorba, «Comparison of spacetime defects which are homeomorphic but not diffeomorphic», *J. Math. Phys. (N.Y.)* **55**, 112503 (2014), `arXiv:1404.2901 [hep-th]`.

[13] M. Günther, «Skyrmion Spacetime Defect, Degenerate Metric, and Negative Gravitational Mass», Master Thesis, KIT, Karlsruhe, Germany, 2017; available at: `https://www.itp.kit.edu/en/publications/diploma`

[14] F.R. Klinkhamer, «On a soliton-type spacetime defect», *J. Phys.: Conf. Ser.* **1275**, 012012 (2019), `arXiv:1811.01078 [gr-qc]`.

[15] S.W. Hawking, «Occurrence of Singularities in Open Universes», *Phys. Rev. Lett.* **15**, 689 (1965).

[16] S.W. Hawking, «The occurrence of singularities in cosmology. III. Causality and singularities», *Proc. R. Soc. Lond. A* **300**, 187 (1967).

[17] S.W. Hawking, R. Penrose, «The singularities of gravitational collapse and cosmology», *Proc. R. Soc. Lond. A* **314**, 529 (1970).

[18] R.M. Wald, «General Relativity», *Chicago University Press*, Chicago, USA 1984.

[19] G.T. Horowitz, «Topology change in classical and quantum gravity», *Class. Quantum Grav.* **8**, 587 (1991).

[20] L. Boyle, K. Finn, N. Turok, «CPT-Symmetric Universe», *Phys. Rev. Lett.* **121**, 251301 (2018), arXiv:1803.08928 [hep-ph].

[21] E. Witten, «String theory dynamics in various dimensions», *Nucl. Phys. B* **443**, 85 (1995), arXiv:hep-th/9503124.

[22] P. Horava, E. Witten, «Heterotic and type I string dynamics from eleven dimensions», *Nucl. Phys. B* **460**, 506 (1996), arXiv:hep-th/9510209.

[23] M.J. Duff, «M theory (The Theory Formerly Known as Strings)», *Int. J. Mod. Phys. A* **11**, 5623 (1996), arXiv:hep-th/9608117.

[24] N. Ishibashi, H. Kawai, Y. Kitazawa, A. Tsuchiya, «A large-N reduced model as superstring», *Nucl. Phys. B* **498**, 467 (1997), arXiv:hep-th/9612115.

[25] H. Aoki S. Iso, H. Kawai, Y. Kitazawa, A. Tsuchiya, T. Tada, «IIB Matrix Model», *Prog. Theor. Phys. Suppl.* **134**, 47 (1999), arXiv:hep-th/9908038.

[26] S.W. Kim, J. Nishimura, A. Tsuchiya, «Expanding (3+1)-Dimensional Universe from a Lorentzian Matrix Model for Superstring Theory in (9+1) Dimensions», *Phys. Rev. Lett.* **108**, 011601 (2012), arXiv:1108.1540 [hep-th].

[27] J. Nishimura, A. Tsuchiya, «Complex Langevin analysis of the space-time structure in the Lorentzian type IIB matrix model», *J. High Energy Phys.* **1906**, 077 (2019), arXiv:1904.05919 [hep-th].

[28] E. Witten, «The $1/N$ expansion in atomic and particle physics», in: G. 't Hooft *et al.* (Eds.) «Recent Developments in Gauge Theories. Cargese 1979», *Plenum Press*, New York, USA 1980, pp. 403–419.

[29] S. Coleman, «$1/N$», in: «Aspects of Symmetry: Selected Erice Lectures», *Cambridge University Press*, Cambridge, UK 1985, pp. 351–402.

[30] J. Greensite, M.B. Halpern, «Quenched master fields», *Nucl. Phys. B* **211**, 343 (1983).

[31] F.R. Klinkhamer, «IIB matrix model: Emergent spacetime from the master field», *Prog. Theor. Exp. Phys.* **2021**, 013B04 (2021), arXiv:2007.08485 [hep-th].

[32] F.R. Klinkhamer, «IIB matrix model: Extracting the spacetime points», arXiv:2008.01058 [hep-th].

[33] F.R. Klinkhamer, «IIB matrix model: Extracting the spacetime metric», arXiv:2008.11699 [hep-th].

[34] F.R. Klinkhamer, «IIB matrix model and regularized big bang», *Prog. Theor. Exp. Phys.* **2021**, 063B05 (2021), arXiv:2009.06525 [hep-th].

[35] F.R. Klinkhamer, «Gravity without curved spacetime: A simple calculation», arXiv:1208.3168 [gr-qc].

[36] A. Connes, «A short survey of noncommutative geometry», *J. Math. Phys.* **41**, 3832 (2000), arXiv:hep-th/0003006.

[37] H.C. Steinacker, «On the quantum structure of space-time, gravity, and higher spin in matrix models», *Class. Quantum Grav.* **37**, 113001 (2020), arXiv:1911.03162 [hep-th].

[38] M. van Raamsdonk, «Spacetime from bits», *Science* **370**, 198 (2020).

[39] S.R. Das, A. Kaushal, G. Mandal, S.P. Trivedi, «Bulk entanglement entropy and matrices», *J. Phys. A* **53**, 444002 (2020), arXiv:2004.00613 [hep-th].

[40] F.R. Klinkhamer, «A first look at the bosonic master-field equation of the IIB matrix model», arXiv:2105.05831 [hep-th].

[41] F.R. Klinkhamer, «Solutions of the exact bosonic master-field equation from a supersymmetric matrix model», arXiv:2106.07632 [hep-th].