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MASS MATRICES FOR QUARKS AND LEPTONS IN TRIANGULAR FORM

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We assume that all quark and lepton mass matrices have upper triangular form. Using all available experimental data on quark and lepton masses and mixing angles we make a fit in which we determine mass matrices elements. There are too many free parameters and our solutions are not unique. We look for solutions with small non diagonal mixing matrix elements. In order to reduce the number of free parameters we assume that the matrix element $(M)_{13}$ vanishes in all mass matrices. Such universal assumption was drawn from considering different numerical solutions. The lepton sector, due to large mixing angles and very small errors for charged lepton masses, is more restrictive then quark sector. We present the solution in this case. The absolute values of neutrino masses are not fixed. Another possibility of reducing number of free parameters was considered by us before. With the additional assumption motivated by SU(5) symmetry which connects mixing in right handed down quarks with left handed charged leptons we get a solution in which observed Cabibbo–Kobayashi–Maskawa mixing for quarks comes mainly from non diagonal terms in up quark mass matrix.

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The understanding of masses and mixing for quarks and leptons, after discovery that neutrinos are massive [1], is one of the most interesting and hot problems [2] in the standard model. In this paper we shall consider rather unusual idea of mass matrices in triangular form. Such idea was proposed in [3]. In that paper, taking into account very specific additional assumptions, mass matrices for up and down quarks, neutrinos and charged leptons were given together with mixing matrices for left and right handed components of fields. We considered model which neglects CP violation as well as the one that took it into account in quark sector. Neutrinos, contrary to now popular assumption of being Majorana particles, were treated as Dirac particles.

In this paper we also assume that the neutrino is a Dirac particle. We believe that because there is no positive result of double β decay experiment it is still a viable hypothesis. It could be worthwhile (smaller number of parameters) to study consequences of this assumption. We have shown in [3], first considering real matrices, that inclusion of CP violation for quarks was not a problem. In this paper we will consider real matrices neglecting CP violation because we believe after experiences from [3] that it will be not difficult to include it. In addition $(M)_{13}$ element of lepton mixing matrix seems to be very small (if at all different from zero) and we do not have any information about CP-violating phase.

How we can justify the assumption of using mass matrices in triangular form. We believe that left handed SU(2) doublets and not right handed singlets are responsible for mixing. After spontaneous symmetry breaking our basic objects that we have in Lagrangian are mass matrices for up and down quarks, neutrinos and charged leptons. When we make a decomposition of mass matrix into diagonal matrix and two unitary matrices, taking into account that we have strongly hierarchical mass spectrum for up and down quarks as well for charged leptons and not so strongly for neutrinos, we will find out that matrix elements above diagonal influence mostly mixing for left handed components and below diagonal right handed components. If we want to have mixing determined by left handed components (active in interactions) we assume that in our mass matrices matrix elements responsible for mixing of right handed (below diagonal) components vanish. In this way we end up with upper triangular mass matrices that (with hierarchical mass spectrum) give very asymmetrical mixing much stronger for left handed then for right handed components. In this paper we continue to use the assumption of triangular form for quark and lepton mass matrices but we do not take into account all other specific assumptions we made in [3]. The masses of quarks and leptons and the corresponding errors are very different spanning many orders of magnitude. To cope with this and the fact that for neutrinos we do not know masses but only mass squared differences we will use, as in [3] χ^2 function (the least square method). Knowing measured mixing matrices for left components and their errors and masses of quarks and leptons with corresponding errors we will try to calculate mass matrices for quarks and leptons assumed in upper triangular form.

In general we have too many free parameters and we cannot find solution that is unique. We give examples of solutions with relatively small non diagonal matrix elements in mass matrices. The question is if we can reduce the number of free parameters by assuming that some of them vanish. It is not enough to calculate formally number of free parameters. In the case of neutrinos and charged leptons it is not possible to get solution (with small χ^2 value) when neutrino mass matrix is diagonal and whole mixing is in charged lepton sector in spite of the fact that formally we have enough free parameters to get a solution. We can get analogous solution for up and down quarks. We discuss in detail the different cases for quarks and leptons when we have enough free parameters and could not find the solution with small χ^2 value. It seems that lepton sector is more restrictive then quark sector. The mixing for leptons is much stronger then for quarks and charged lepton masses are known with high accuracy (specially electron and muon masses) in comparison with up and down quark masses. As a result of our numerical analysis we have found out that when in addition to the assumption of upper triangular form of mass matrices we assume that in neutrino and charged leptons mass matrix matrix element $(M)_{13}$ vanishes we can find a solution. This assumption is generalized and is made for all multiplets of up and down quarks neutrinos and charged leptons so they are all treated in the same way. Such assumption can be justified by the fact that measured mixing matrices for quarks and leptons have matrix element $(U)_{13}$ very small (smaller then the others, in the case of neutrinos it is not well known but very close to zero or even zero) so we expect that mixing of first and third generation comes as a result of mixing of other generations. Absolute values of neutrino masses cannot be fixed in this way.

At the end we will give rather exotic example of solution in which number of free parameters is reduced by 3 by a relation between different mixing matrices. We thought at the beginning that it could be an alternative to the assumption of vanishing mass matrix elements. We will show that it is possible to get solutions for triangle mass matrices when we assume connection between left and right handed mixing matrices. It is suggested by SU(5) symmetry condition that right handed components of down quarks transform in the same way as left handed components of charged leptons (belonging to the same SU(5) multiplet). The errors for electron and muon masses are very small so the mixing of charged leptons is rather small and that has a consequence that the contribution of mixing to CKM matrix from down quarks is not big and comes mostly from the mixing of up quarks. With our additional assumption for all mass matrices $(M)_{13} = 0$ we cannot find such solution.

Let us introduce the notation for quarks. We will follow the notation used in [1] and repeat introductory formulas. The SU(3)×SU(2)×U(1) gauge invariant Yukawa interactions for quarks are given by

$$-L_{\mathrm{Y}} = \bar{Q}_i (Y_u)_{ij} u_{\mathrm{R}j} H^\dagger + \bar{Q}_i (Y_d)_{ij} d_{\mathrm{R}j} H + \mathrm{h.c.}\,, \tag{1}$$

where Q_i denote the SU(2) doublets of left-handed quarks and $u_{\rm R}, d_{\rm R}$ are the right-handed up and down-type quarks, respectively. The Yukawa couplings Y_u and Y_d are 3×3 matrices (i, j are the generation indices) and H is the SU(2) doublet Higgs field. After the electroweak symmetry breaking, these Yukawa interactions lead to the following quark mass terms

$$\begin{aligned} -L_m &= \bar{u}_{\rm Li}(M_u)_{ij}u_{{\rm R}j} + \bar{d}_{\rm Li}(M_d)_{ij}d_{{\rm R}j} + {\rm h.c.}\,, \\ (M_u)_{ij} &= (Y_u)_{ij}v\,, \\ (M_d)_{ij} &= (Y_d)_{ij}v\,, \end{aligned} \tag{2}$$

where v is vacuum expectation value of the neutral component of the Higgs field H.

Mass matrices M_u and M_d (considered by us as basic objects) can be diagonalized using two unitary matrices U and V

$$M_u = U_u M_u^{\rm D} V_u^\dagger\,, \tag{3}$$

$$M_d = U_d M_d^{\rm D} V_d^\dagger\,. \tag{4}$$

The diagonal matrix elements (of $M_u^{\rm D}$ and $M_d^{\rm D}$) correspond to the experimentally observed mass eigenvalues. The matrices U and V describe mixing between states with definite flavor u, d and with definite mass u', d' for left handed and right handed components

$$\begin{aligned} u_{\rm L} &= U_u u'_{\rm L}\,, \\ u_{\rm R} &= V_u u'_{\rm R}\,, \\ d_{\rm L} &= U_d d'_{\rm L}\,, \\ d_{\rm R} &= V_d d'_{\rm R}\,. \end{aligned} \tag{5}$$

The generation mixing in the charged weak current after expressing in terms of fields with definite mass is described by the Cabibbo–Kobayashi–Maskawa (CKM) matrix [4] which consists of two unitary matrices

$$U_{\rm CKM} = U_u^\dagger U_d\,. \tag{6}$$

In the similar way, assuming that neutrinos are traditional Dirac particles, we have mixing for the charged weak current in the lepton sector described by Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [5]

$$U_{\rm MNS} = U_l^\dagger U_\nu\,. \tag{7}$$

The relations for neutrino mass M_ν and charged leptons M_l are:

$$M_\nu = U_\nu M_\nu^{\rm D} V_\nu^\dagger\,, \tag{8}$$

$$M_l = U_l M_l^{\rm D} V_l^\dagger\,. \tag{9}$$

As before, diagonal matrix elements of $M_\nu^{\rm D}$ and $M_l^{\rm D}$ correspond to the experimentally observed mass eigenvalues.

We have got experimental data for CKM and PMNS matrices. We will use for CKM matrix and the errors of matrix elements values given in [6]. The values and the errors for up and down quark masses are taken from the [7]. The parameters of PMNS matrix and corresponding errors are taken from recent updating of the fit [8] taking into account recent results from KamLAND and MINOS experiments given in [9]. Mass squared differences of neutrino masses are taken from the same fit [8] and masses of charged leptons and their errors are taken from [7]. The masses of charged leptons are known with extremely high accuracy in comparison with quark and neutrino ones. For neutrinos only mass squared differences are known. That was the reason why in [3], in order to fit mass matrices for quarks and leptons in triangular form to experimental data, we used numerical minimalization of χ^2 function taking into account experimental errors. The corresponding modified function was used for PMNS matrix and mass values for neutrinos and charged leptons. Minimalization of χ^2 gives most probable value of parameters or could give numerical solution to equations (as it was used in [3]). When there are too many free parameters we can get one of many possible solutions. In [3] we have also assumed that we have upper or lower triangular matrices for up, down, neutrinos and charged leptons and that matrices diagonalizing right handed components are the same in the weak isotpic spin multiplets.

In the present paper we relax these additional assumptions. The left handed components of all quarks and leptons take part in charged weak current interactions. We want to have a situation that mixing in active left handed components determines the whole mixing. We assume that mass matrices for up, down quarks, neutrinos and charged leptons are upper triangular matrices. When we have upper triangular mass matrices with hierarchical diagonal elements decomposition into two unitary matrices and diagonal matrix we have very simple situation. Mixing matrix elements for left handed components U are determined to the first order by the ratios of non diagonal and corresponding diagonal mass matrix elements $((U)_{12} \simeq (M)_{12}/(M)_{22}$, $((U)_{23} \simeq (M)_{23}/(M)_{33}$, $((U)_{13} \simeq (M)_{13}/(M)_{33})$ and matrix elements for right handed components V are determined by left handed mixing elements multiplied by corresponding ratios of diagonal masses $((M)_{11}/(M)_{22}$, $(M)_{11}/(M)_{33}$, $(M)_{22}/(M)_{33})$. Ortogonality of matrices U and V must, of course, be taken into account. When we have hierarchical masses the mixing in V is determined by U and much smaller. All that will clearly be seen in our example of the first solutions. In [3] we used artificial errors for electron and muon masses. We want to remind that masses of electron and muon are known with extremely high accuracy in comparison with all other particles. It seems that error in the mass of electron is comparable with the expected naively mass of heaviest neutrino. One can expect that these small errors can give some limitations on mixing in charged mass matrices because when we diagonalize them you have to end up with the values within small errors. We will try to see what are the consequences of small errors in masses of charged leptons in comparison with quarks.

Following [3] for quarks we will minimalize the function

$$\chi^2 = \sum_{ij} \frac{\left((U_u^\dagger U_d)_{ij} - U^{\rm exp}_{{\rm CKM}ij}\right)^2}{\left(\Delta U^{\rm exp}_{{\rm CKM}ij}\right)^2} + \sum_i \frac{\left(m^{\rm D}_{di} - m^{\rm exp}_{di}\right)^2}{\left(\Delta m^{\rm exp}_{di}\right)^2} + \sum_i \frac{\left(m^{\rm D}_{ui} - m^{\rm exp}_{ui}\right)^2}{\left(\Delta m^{\rm exp}_{ui}\right)^2} . \qquad (10)$$

There are 6 free parameters (3 diagonal and 3 non diagonal) for upper triangular matrix M_d and 6 parameters for upper triangular matrix M_u (in principle we could also use lower triangular matrix for M_u like in [3]). As we mentioned before the values $m^{\rm exp}_{di} = (m_d, m_s, m_b)$, $m^{\rm exp}_{ui} = (m_u, m_c, m_t)$ and corresponding errors $\Delta m^{\rm exp}_{di}$, $\Delta m^{\rm exp}_{ui}$ are taken from the Particle Data Group [7] and $U^{\rm exp}_{\rm CKM}$ and corresponding errors from [6]. The values of $U^{\rm exp}_{\rm CKM}$ are expressed in terms of angles to have unitarity satisfied to high degree and then we use calculated in this way matrix elements. For neutrinos and charged leptons we use the same procedure with obvious modifications. In the χ^2 function we fit $U_{\rm CKM}$ or $U_{\rm PMNS}$ matrix elements and masses of quarks or leptons using 12 free parameters (2 mass matrices, 3 diagonal and 3 non diagonal elements). There are more parameters we want to determine then independent fitted quantities so we cannot uniquely determine them. Anyhow, we do not understand the mass scales and big mass ratios for different generations of quarks. So it could happen that just because of big difference in mass scales non diagonal matrix elements are small in comparison with diagonal ones (for quantum systems very small mixing between very different energy levels) giving rather small mixing in $U_{\rm CKM}$ matrix. We will look for example of this type of solution.

We will start with $U_{\rm CKM}$ matrix and up and down quark mass matrices and we consider the fit with relatively small parameters (elements of mass matrices are given in MeV)

$$M_d = \begin{pmatrix} 5.11523 & 20.03 & 10.5861 \\ 0 & 92.9391 & 172.911 \\ 0 & 0 & 4196.42 \end{pmatrix} , \qquad (11)$$

$$U_d = \begin{pmatrix} 0.97741 & 0.21134 & 0.00252 \\ -0.21126 & 0.97656 & 0.04119 \\ 0.00624 & -0.04079 & 0.99915 \end{pmatrix}, \tag{12}$$

$$M_d^{\mathrm{D}} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 95 & 0 \\ 0 & 0 & 4200 \end{pmatrix}, \tag{13}$$

$$V_d^{\dagger} = \begin{pmatrix} 0.99993 & -0.01138 & 7.43 \times 10^{-6} \\ 0.01138 & 0.99993 & -0.00092 \\ 3.08 \times 10^{-6} & 0.00924 & 1 \end{pmatrix} \tag{14}$$

and

$$M_u = \begin{pmatrix} 2.2503 & -20.37 & -105.96 \\ 0 & 1249.83 & 111.654 \\ 0 & 0 & 172500 \end{pmatrix}, \tag{15}$$

$$U_u = \begin{pmatrix} 0.99987 & -0.01630 & -0.00061 \\ 0.01630 & 0.99987 & -0.00064 \\ 0.00062 & 0.00064 & 1 \end{pmatrix}, \tag{16}$$

$$M_u^{\mathrm{D}} = \begin{pmatrix} 2.25 & 0 & \\ 0 & 1250 & 0 \\ 0 & 0 & 172500 \end{pmatrix}, \tag{17}$$

$$V_u^{\dagger} = \begin{pmatrix} 1 & 0.00003 & 8.15 \times 10^{-9} \\ -0.00003 & 1 & 4.62 \times 10^{-6} \\ -8.01 \times 10^{-9} & -4.62 \times 10^{-6} & 1 \end{pmatrix}. \tag{18}$$

The χ^2 is practically zero (we get the number of the order of 10^{-14}) so we can treat these parameters as a numerical solution of Eqs. (3), (4) and Eq. (6). We see that relations between matrix elements of U and V mentioned before and mass matrix elements are satisfied. The obtained solution is not so different in character from that obtained with different additional assumptions in [3]. One can easily give (because we have 3 free parameters) examples with bigger non diagonal matrix elements giving the same U_{CKM} mixing matrix.

When we look for small mixing parameters in case of neutrinos and charged lepton mass matrices we get (in eV)

$$M_\nu = \begin{pmatrix} 0.00265 & 0.00529 & 0.00327 \\ 0 & 0.01050 & 0.03329 \\ 0 & 0 & 0.03502 \end{pmatrix} \tag{19}$$

$$U_\nu = \begin{pmatrix} 0.82188 & 0.56353 & 0.083388 \\ -0.44863 & 0.55008 & 0.70437 \\ 0.35107 & -0.61632 & 0.70491 \end{pmatrix}, \tag{20}$$

$$M_\nu^{\rm D} = \begin{pmatrix} 0.00221 & 0 & 0 \\ 0 & 0.00899 & 0 \\ 0 & 0 & 0.04904 \end{pmatrix}, \tag{21}$$

$$V_\nu^\dagger = \begin{pmatrix} 0.98611 & -0.16461 & 0.02214 \\ 0.16603 & 0.97333 & -0.15828 \\ 0.00451 & 0.15975 & 0.98715 \end{pmatrix}, \tag{22}$$

and (in MeV)

$$M_l = \begin{pmatrix} 0.510999 & -0.03867 & -0.07211 \\ 0 & 105.658 & -0.71477 \\ 0 & 0 & 1776.99 \end{pmatrix}, \tag{23}$$

$$U_l = \begin{pmatrix} 1 & -0.00037 & -0.00004 \\ 0.00037 & 1 & -0.00040 \\ 0.00004 & 0.00040 & 1 \end{pmatrix}, \tag{24}$$

$$M_l^{\rm D} = \begin{pmatrix} 0.510999 & 0 & 0 \\ 0 & 105.658 & 0 \\ 0 & 0 & 1776.99 \end{pmatrix}, \tag{25}$$

$$V_l^\dagger = \begin{pmatrix} 1 & 1.77\times 10^{-6} & 1.17\times 10^{-8} \\ 1.77\times 10^{-6} & 1 & 0.00002 \\ -1.17\times 10^{-8} & -0.00002 & 1 \end{pmatrix}. \tag{26}$$

These are just examples of solutions with small non diagonal terms in mass matrices M_u and M_l. We have relatively small mixing in U_u and V_u coming from M_u and in U_l and V_l connected with M_l. The mixing in $U_{\rm CKM}$ and $U_{\rm PMNS}$ comes mainly from U_d and U_ν connected with non diagonal terms of matrices M_d and M_ν. For the quarks we have 3 free parameters and for leptons 4 (we do not know values of masses of neutrinos only mass squared differences) so we can find many very different solutions. In the CKM matrix $U_{\rm CKM} = U_u^\dagger U_d$ mixing can come from both matrices (the solution where most of the mixing in $U_{\rm CKM}$ comes from U_u is possible. That it is not the case for leptons. We do not have solutions where in $U_{\rm PMNS}$ mixing matrix most of the mixing comes from charged lepton mixing matrix U_l. The mixing in the lepton case is much stronger then for quarks and quark masses are known with much less accuracy then masses of charged leptons.

In the case of leptons situation is a bit different because of very small errors in masses of e and μ leptons. The diagonal masses are nearly equal to the measured charged lepton masses and non diagonal masses are really tiny. The biggest error is for τ lepton mass. We have the strongest dependence on only one parameter namely element $(M_l)_{23}$ of charged lepton mass matrix. One can numerically try to find out how big this element could be still having solutions to our equations. We will give an example of the solution for leptons where we have relatively strong mixing because of "big" non diagonal matrix elements in m_l mass matrix for charged leptons.

$$M_\nu = \begin{pmatrix} 0.00971 & 0.00278 & -0.00038 \\ 0 & 0.01493 & 0.02908 \\ 0 & 0 & 0.03942 \end{pmatrix}, \tag{27}$$

$$M_l = \begin{pmatrix} 0.51403 & -11.3797 & -9.29737 \\ 0 & 105.647 & -190.258 \\ 0 & 0 & 1766.71 \end{pmatrix}. \tag{28}$$

In this case U_ν and U_l are given by

$$U_\nu = \begin{pmatrix} 0.86365 & 0.50407 & 0.00323 \\ -0.39334 & 0.66988 & 0.62972 \\ 0.31527 & -0.54513 & 0.77681 \end{pmatrix}, \tag{29}$$

$$U_l = \begin{pmatrix} 0.99411 & -0.10826 & -0.00519 \\ 0.10708 & 0.98843 & -0.10744 \\ 0.01676 & 0.10625 & 0.99420 \end{pmatrix}. \tag{30}$$

This is the solution that contrary to solution given in Eqs. (19)–(26) corresponding to small non diagonal mass matrix elements gives relatively big non diagonal matrix elements for M_l. The mixing for charged leptons is still rather small in comparison with that for neutrinos. This makes that situation for leptons is different from that for quarks. In the case where non diagonal terms in charged lepton mass are relatively big the mixing in charged lepton mass matrix U_l is small and is only a small correction to U_ν being mainly responsible for the value of U_{PMNS}. Comparing Eq. (27) with Eq. (19) we see that there is some increase in diagonal matrix elements in M_ν and that corresponds to bigger neutrino masses $m_{1\nu} = 9.1\,\mathrm{meV}$, $m_{2\nu} = 12.6\,\mathrm{meV}$, $m_{3\nu} = 49.8\,\mathrm{meV}$ in comparison with $m_{1\nu} = 2.2\,\mathrm{meV}$, $m_{2\nu} = 9.0\,\mathrm{meV}$, $m_{3\nu} = 49.0\,\mathrm{meV}$ in Eq. (21). Contrary to quarks, in the lepton case we have rather small range of non diagonal terms in charged lepton mass matrix. The mixing given by U_l and V_l is small and the values of U_{PMNS} are dominated by non diagonal terms in M_ν and mixing matrix U_ν.

Up to now we have considered examples of solutions with too many free parameters which we want to determine. One can restrict the freedom of possible solutions by some additional conditions *e.g.* as in [3] when the equality of V matrices in the same weak isospin multiplet was assumed. We will present mass matrices when we limit the number of parameters in mass matrices by putting 3 of them equal to zero. We will start with extreme cases when all vanishing mass matrix elements are in up or down quark mass matrix. In the case of quarks we get for diagonal M_u:

$$M_d = \begin{pmatrix} 5.13392 & 21.5375 & 16.0237 \\ 0 & 92.6034 & 175.388 \\ 0 & 0 & 4196.3 \end{pmatrix} \tag{31}$$

and $U_d = U_{\rm CKM}$, whereas for diagonal M_d we have:

$$M_u = \begin{pmatrix} 2.31044 & -284.247 & 996.199 \\ 0 & 1218.37 & -7167.79 \\ 0 & 0 & 172348 \end{pmatrix} \tag{32}$$

and, of course, $U_u^\dagger = U_{\rm CKM}$.

These are extreme cases when mixing in $U_{\rm CKM}$ matrix either comes from non diagonal terms in down quark mass matrix or from non diagonal terms in up quark matrix. We can also numerically study solutions when three zero matrix elements are distributed among two such matrices. It is easy to check that in spite of the fact that formally we have enough free parameters (3 zero matrix elements in M_u and M_d matrices) such solutions do not exist (χ^2 is not small). When $(M_d)_{13} = 0$, $(M_u)_{12} = 0$, $(M_u)_{13} = 0$ or we change indexes u and d namely $(M_u)_{13} = 0$, $(M_d)_{12} = 0$, $(M_d)_{13} = 0$ there are no solutions ($\chi^2 > 700$ or $\chi^2 > 1400$). There is also no solution when $(M_d)_{23} = 0$, $(M_u)_{13} = 0$, $(M_u)_{23} = 0$. That means that when we have number of free parameters equal to the number of experimental data we not always can find a solution. On the other hand when $(M_d)_{13} = 0$, $(M_u)_{13} = 0$ and $(M_u)_{23} = 0$ or $(M_d)_{13} = 0$, $(M_u)_{12} = 0$, $(M_u)_{23} = 0$ we can easily find a solution.

In the case of leptons when charged lepton mass matrix is diagonal (non diagonal elements are put equal to zero) we get:

$$M_\nu = \begin{pmatrix} 0.00385 & 0.00510 & 0.00328 \\ 0 & 0.01100 & 0.03323 \\ 0 & 0 & 0.03509 \end{pmatrix} \tag{33}$$

and $U_{\rm PMNS} = U_\nu$. It is also possible to have solution for M_ν with $m_{1\nu}$ close to zero.

We get

$$M_\nu = \begin{pmatrix} 10^{-12} & 0.00549 & 0.00329 \\ 0 & 0.01005 & 0.03337 \\ 0 & 0 & 0.03494 \end{pmatrix}. \tag{34}$$

We want to stress that with diagonal mass matrix for charged leptons we still have one free parameter (we do not know absolute values of neutrino masses) and the smallest neutrino mass equal to zero is not excluded.

In the case of leptons contrary to quarks, where we could have solution with diagonal M_d (Eq. (32)) but it is not possible (at least with high probability $\chi^2 > 170$) to have a satisfactory solution that neutrino mass matrix is diagonal and the whole strong mixing that we have in U_{PMNS} comes from charged leptons. It is understandable from the discussion of mixing in charged lepton sector. With relatively small mixing in the charged lepton sector we cannot reproduce very strong mixing observed in U_{PMNS}. The question is if we can split three zeros between neutrino mass matrix and charged lepton mass matrix to have U_ν and U_l mixing matrices that reproduce strong mixing in U_{PMNS}. To reproduce U_{PMNS} with two big mixing angles we need nonzero matrix elements $(M_\nu)_{12}$ and $(M_\nu)_{23}$. Having only these non diagonal matrix elements different from zero with all the other non diagonal matrix elements in M_l equal to zero we get χ^2 bigger than 2. With $(M_\nu)_{12}$ and $(M_\nu)_{23}$ different from zero and $(M_l)_{12}$ or $(M_l)_{13}$ or $(M_l)_{23}$ we always get χ^2 bigger 2. With 3 additional zeros in mass matrices we still have 1 free parameter connected with lack of scale for neutrino masses. Calculating formally number of parameters we should have a solution. The next possibility is to add one more non zero non diagonal matrix element. If we want to treat neutrinos and charged leptons in the same way then the choice is obvious. We assume that in charged lepton mass matrix non diagonal matrix element $(M_l)_{23}$ is different from zero. In this case we can find a solution. It means that guided by numerical calculations we assumed that matrix elements $(M_\nu)_{13}$ and $(M_l)_{13}$ are equal to zero. We will also assume that for mass matrices of up and down quarks. It is the assumption corresponding for triangular matrices to that considered and advertised for quarks by Fritzsch, idea of having mixing only between neighbored flavors [10]. On the other hand, from experiment we know that $(U_{\text{CKM}})_{13}$ is small and in $(U_{\text{PMNS}})_{13}$ is small (not well known) and still consistent with zero so assuming that in basic mass matrices $(M)_{13}$ vanishes is not so unnatural. In some sense the situation is very simple. We ask if it is possible to reduce number of solutions by assuming that some mass matrix elements are equal to zero. If we want to add in a universal way (in the same way for up and down quarks, neutrinos and charged leptons) condition of vanishing mass matrix elements we do not get a solution with 2 zeros in every mass matrix. The next possibility is to consider one vanishing mass matrix element in every mass matrix. Discussion of numerical solutions in lepton sector has helped us to localize these vanishing mass matrix elements. We will present a solution with two additional zeros when matrix elements $(M_\nu)_{13}$ and $(M_l)_{13}$ are equal to zero in neutrino and charged lepton mass matrices. In this case we get:

$$M_\nu = \begin{pmatrix} 0.00508 & 0.00381 & 0 \\ 0 & 0.01125 & 0.02987 \\ 0 & 0 & 0.03845 \end{pmatrix}, \quad (35)$$

$$U_\nu = \begin{pmatrix} 0.8632 & 0.5047 & 0.01141 \\ -0.39654 & 0.6639 & 0.63403 \\ 0.31242 & -0.55183 & 0.77323 \end{pmatrix}, \quad (36)$$

$$M_\nu^{\mathrm{D}} = \begin{pmatrix} 0.00455 & 0 & 0 \\ 0 & 0.00983 & 0 \\ 0 & 0 & 0.0492 \end{pmatrix}, \quad (37)$$

$$V_\nu^\dagger = \begin{pmatrix} 0.96535 & -0.25834 & 0.03693 \\ 0.26096 & 0.95498 & -0.14109 \\ 0.00118 & 0.14584 & 0.98931 \end{pmatrix} \quad (38)$$

and (in MeV)

$$M_l = \begin{pmatrix} 0.51368 & -10.7738 & 0 \\ 0 & 105.655 & -180.361 \\ 0 & 0 & 1767.78 \end{pmatrix}, \quad (39)$$

$$U_l = \begin{pmatrix} 0.99479 & -0.10197 & 0.00004 \\ 0.10144 & 0.98961 & -0.10186 \\ 0.01035 & 0.10133 & 0.9948 \end{pmatrix}, \quad (40)$$

$$M_l^{\mathrm{D}} = \begin{pmatrix} 0.510999 & 0 & 0 \\ 0 & 105.658 & 0 \\ 0 & 0 & 1776.99 \end{pmatrix}, \quad (41)$$

$$V_l^\dagger = \begin{pmatrix} 1 & 0.0005 & 2.99\times10^{-6} \\ -0.0005 & 0.99998 & 0.00606 \\ 1.06\times10^{-8} & -0.00606 & 0.99998 \end{pmatrix} \quad (42)$$

we have got a solution in which mixing matrix elements with value 0.1 in charged lepton mixing matrix U_l give substantial contribution to lepton mixing matrix U_{PMNS}. They are smaller then neutrino mixing matrix elements but not as tiny as in examples of solutions given in the beginning (Eq. (24). This solution is not unique. It is possible to get solution (with accuracy 10^{-13}) with higher values of neutrino masses with slightly modified neutrino mass matrix and no change in lepton mass matrix parameters. One should remember that in the case of neutrinos we have one additional free parameter compared to quarks because we do not know absolute values of neutrino masses only mass squared differences. It is not possible to fix absolute values of neutrino masses in this fit. We can also find a solution with smallest neutrino mass equal to zero with $\chi^2 \simeq 0.24$. The value of χ^2 is higher then for solutions mentioned before, it cannot be excluded, but is highly unsatisfactory.

With the additional universal assumption $(M)_{13} = 0$ we can also look for solutions of this type for quarks. As we know from the previous discussion there is no problem with getting this type of solution. We know that it is possible to give a solution for quarks in which in addition to $(M_u)_{13} = 0$ also $(M_u)_{23} = 0$. In this case we get:

$$M_d = \begin{pmatrix} 5.04798 & 13.0481 & 0 \\ 0 & 94.18 & 176.118 \\ 0 & 0 & 4196.3 \end{pmatrix}, \tag{43}$$

$$U_d = \begin{pmatrix} 0.99047 & 0.13773 & 2.92 \times 10^{-6} \\ -0.13761 & 0.9896 & 0.04195 \\ 0.00578 & -0.04155 & 0.99912 \end{pmatrix}, \tag{44}$$

$$M_d^{\mathrm{D}} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 95 & 0 \\ 0 & 0 & 4200 \end{pmatrix}, \tag{45}$$

$$V_d^\dagger = \begin{pmatrix} 0.99997 & -0.00732 & 6.88 \times 10^{-6} \\ 0.00732 & 0.99997 & -0.00094 \\ 3.51 \times 10^{-9} & 0.00094 & 1 \end{pmatrix} \tag{46}$$

and

$$M_u = \begin{pmatrix} 2.25937 & -113.728 & 0 \\ 0 & 1244.82 & 0 \\ 0 & 0 & 172500 \end{pmatrix}, \tag{47}$$

$$U_u = \begin{pmatrix} 0.99585 & -0.09098 & 0 \\ 0.09098 & 0.99585 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{48}$$

$$M_u^{\mathrm{D}} = \begin{pmatrix} 2.25 & 0 & \\ 0 & 1250 & 0 \\ 0 & 0 & 172500 \end{pmatrix}, \tag{49}$$

$$V_u^\dagger = \begin{pmatrix} 1 & 0.00016 & 0 \\ -0.00016 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{50}$$

We see from this solution that mixing in $(U_{\mathrm{CKM}})_{12}$ is split, both mixing matrices U_d and U_u contribute to U_{CKM}. It is similar to neutrino and charged lepton case in Eqs. (36, 40). Matrix element $(U_{\mathrm{PMNS}})_{13}$ is reproduced by mixing in $(M_u)_{12}$ and $(M_d)_{23}$ in mass matrices for up and down quarks.

Unlike in the case of leptons mass matrix for up quark is not uniquely determined. It is possible to give examples of solutions in which matrix element $(M_u)_{23}$ is different from zero ($(M_d)_{23}$ also changes) but mixing in $(U_u)_{12}$ (that is bigger) is not strongly influenced. We have much stronger mass hierarchy for up and down quarks (with the mass errors relatively much bigger then in the case of charged leptons) and small mixing in $U_{\rm CKM}$ in comparison with $U_{\rm PMNS}$ so the lepton sector is somehow more restrictive in our numerical analysis.

We also want to give an example of solution in which number of free parameters is reduced by 3 (but still we have formally 4 free parameters) by the relation between different mixing matrices. We thought at the beginning that it could be an alternative to the assumption of vanishing mass matrix elements. On the other hand from the grand unification SU(5) symmetry ($\bar{5}$ representation) we have a hint about connection between right handed components of down quarks and left handed components of leptons. This components being in the same SU(5) multiplet when there is a mixing between families and in some way SU(5) structure is not completely lost in low energies [11] we would have the same mixing matrix for right handed down quarks and left handed charged leptons. We will now look what are the consequences of assumption $V_d = U_l$ (see also [12]). It is not clear how well this relation could be satisfied because of corrections so we will assume artificial error 0.001 (we know that mixing in U_l is rather weak). Minimalizing χ^2 function being sum of all terms corresponding to quarks and leptons and taking deviations of V_d from U_l with artificial error we can find solution

$$M_d = \begin{pmatrix} 4.95067 & 5.4794 & 9.94122 \\ 0 & 96.0835 & 84.3431 \\ 0 & 0 & 4199 \end{pmatrix}, \tag{51}$$

$$U_d = \begin{pmatrix} 0.99837 & 0.05049 & 0.00237 \\ -0.05709 & 0.99817 & 0.02009 \\ -0.00122 & -0.02020 & 0.99979 \end{pmatrix}, \tag{52}$$

$$M_d^{\rm D} = \begin{pmatrix} 4.94261 & 0 & 0 \\ 0 & 96.2204 & 0 \\ 0 & 0 & 4199.86 \end{pmatrix}, \tag{53}$$

$$V_d^\dagger = \begin{pmatrix} 1 & -0.00294 & -1.43\times 10^{-6} \\ 0.00294 & 1 & -0.00046 \\ 2.79\times 10^{-6} & 0.00046 & 1 \end{pmatrix} \tag{54}$$

and

$$M_u = \begin{pmatrix} 2.24369 & -214.598 & 993.738 \\ 0 & 1234.48 & -3748.62 \\ 0 & 0 & 172461 \end{pmatrix}, \tag{55}$$

$$U_u = \begin{pmatrix} 0.98522 & -0.17118 & 0.00576 \\ 0.17127 & 0.98498 & -0.02173 \\ -0.00195 & 0.02240 & 0.99975 \end{pmatrix}, \tag{56}$$

$$M_u^{\mathrm{D}} = \begin{pmatrix} 2.21053 & 0 & \\ 0 & 1252.68 & 0 \\ 0 & 0 & 172505 \end{pmatrix}, \tag{57}$$

$$V_u^\dagger = \begin{pmatrix} 1 & 0.00031 & -2.50\times 10^{-8} \\ -0.00031 & 1 & 0.00016 \\ 7.49\times 10^{-8} & -0.00016 & 1 \end{pmatrix}, \tag{58}$$

$$M_\nu = \begin{pmatrix} 0.00750 & 0.00437 & 0.00333 \\ 0 & 0.01358 & 0.03279 \\ 0 & 0 & 0.03556 \end{pmatrix}, \tag{59}$$

$$U_\nu = \begin{pmatrix} 0.82042 & 0.56532 & 0.08563 \\ -0.45100 & 0.54778 & 0.70465 \\ 0.35145 & -0.61673 & 0.70437 \end{pmatrix}, \tag{60}$$

$$M_\nu^{\mathrm{D}} = \begin{pmatrix} 0.00667 & 0 & 0 \\ 0 & 0.01098 & 0 \\ 0 & 0 & 0.04945 \end{pmatrix}, \tag{61}$$

$$V_\nu^\dagger = \begin{pmatrix} 0.92236 & -0.38066 & 0.06590 \\ 0.38611 & 0.90260 & -0.19034 \\ 0.01298 & 0.20101 & 0.97950 \end{pmatrix} \tag{62}$$

and in (MeV)

$$M_l = \begin{pmatrix} 0.51100 & 0.30913 & -0.03127 \\ 0 & 105.658 & 0.81566 \\ 0 & 0 & 1776.98 \end{pmatrix}, \tag{63}$$

$$U_l = \begin{pmatrix} 1 & 0.00293 & -0.00002 \\ -0.00293 & 1 & 0.00046 \\ 0.00002 & -0.00046 & 1 \end{pmatrix}, \tag{64}$$

$$M_l^{\mathrm{D}} = \begin{pmatrix} 0.510999 & 0 & 0 \\ 0 & 105.658 & 0 \\ 0 & 0 & 1776.98 \end{pmatrix}, \tag{65}$$

$$V_l^\dagger = \begin{pmatrix} 1 & -0.00001 & 6.58\times 10^{-9} \\ 0.00001 & 1 & -0.00003 \\ -6.19\times 10^{-9} & 0.00003 & 1 \end{pmatrix}. \tag{66}$$

We get for this solution $\chi^2 = 0.0086$ with formally 4 free parameters. This solution is not unexpected. Small mixing in U_l enforces by the condition $V_d = U_l$ small mixing in V_d and it means that mixing in $U_{\rm CKM}$ comes mainly from the mass matrix for up quarks. The obtained solution corresponds to the solution for M_u and M_ν matrices given in Eq. (32) and Eq. (33). The mass matrices given in Eq. (32) and Eq. (33) with diagonal matrices for down quarks and leptons satisfy SU(5) condition as unit matrices. It is difficult to find a solution with a small χ^2 value corresponding to the situation where most of the mixing in $U_{\rm CKM}$ comes from down quark and only small part from M_u mass matrix. It is not possible to find such solution with additional assumption that in all mass matrices matrix element $(M)_{13}$ vanishes.

We have tried to determine mass matrices for quarks and leptons assuming that they have triangular form. For all particles it is assumed that mass matrices are upper triangular matrices. It means that in the decomposition of given mass matrix into diagonal and two unitary matrices left handed components of fields determine the mixing matrices. For hierarchical mass spectrum mixing in left handed components is much stronger then in the right handed components. As an input we use the values of matrix elements for CKM and PMNS matrices, masses of quarks and charged leptons with known errors and neutrinos squared mass differences with corresponding errors. With these assumptions one cannot determine the elements of mass matrix uniquely using fit to experimental data. We have discussed examples of solutions for quarks and leptons with relatively small non diagonal matrix elements. It was stressed that the masses of charged leptons are known with extremely high accuracy (the error in the determination of the mass of electron is comparable with naively expected highest neutrino mass). Together with very strong mixing in lepton sector, lepton sector seems to be more restrictive then quark sector. Considering various numerical solutions for leptons we came to the conclusion that in order to restrict the number of possible solutions and treat neutrinos and charged leptons in the same way we have to assume dynamical condition $(M_\nu)_{13} = 0$ and $(M_l)_{13} = 0$. We have given the solutions under these assumptions. Unfortunately, that does not fix the absolute scale for neutrino masses. Then we have extended this assumption to up and down quarks. The experimental justification for this assumption is that in $U_{\rm CKM}$ and $U_{\rm PMNS}$ mixing matrices matrix element $(U)_{13}$ is much smaller then the other mixing matrix elements. We have also at the beginning as alternative to vanishing mass matrix elements considered other possibilities. With the additional assumption which connects mixing matrices for right handed down quarks and left handed charged leptons in the way suggested by SU(5) symmetry it is possible to find a solution for mass matrices with weak mixing for down quarks and charged leptons. The mixing is stronger for neutrinos and up quarks (mixing of left handed up quarks is mainly responsible for mixing in CKM matrix). With our additional assumption for all mass matrices $(M)_{13} = 0$ we cannot find such solution.

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