LATTICE QCD STUDY OF EXCITED HADRON RESONANCES*

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The spectrum of excited hadron resonances in QCD is studied using Monte Carlo path integration techniques formulated on a large $32^3 \times 256$ anisotropic space-time lattice. A large number of probe interpolating operators are used, and calculation of temporal correlations is accomplished using a stochastic method of treating the low-lying modes of quark propagation that exploits Laplacian Heaviside quark-field smearing. Plans to use an effective Hamiltonian to interpret the finite-volume energies and determine the masses and widths of the resonances are outlined. The construction of tetraquark operators is discussed.

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1. Introduction

In a series of papers [1–6], we have been working towards computing the finite-volume stationary-state energies of QCD using Markov-chain Monte Carlo integration of the QCD path integrals formulated on a space-time lattice. In this paper, we report on our progress using a large $32^3 \times 256$ anisotropic lattice for which the pion mass is around 240 MeV. A stochastic method [5] of dealing with the low-lying modes of quark propagation using Laplacian Heaviside quark-field smearing allows us to calculate all needed Wick contractions very efficiently. We are able to extract large numbers of

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levels in each symmetry channel. Overlap factors are used for level identification. Our plans for determining resonance masses and widths from the finite-volume energies are outlined below. New experimental findings motivate us to incorporate tetraquark operators into our correlation matrices. The construction of such operators is also described.

2. Excited-state energies from correlation matrices

The stationary-state energies in a particular symmetry sector can be extracted from a Hermitian correlation matrix $\mathcal{C}_{ij}(t) = \langle 0|\, O_i(t{+}t_0)\, \overline{O}_j(t_0)\, |0\rangle$, where the operators $\overline{O}_j$ act on the vacuum to create the states of interest at source time t_0 and are accompanied by conjugate operators O_i that can annihilate these states at a later time $t+t_0$. Estimates of $\mathcal{C}_{ij}(t)$ are obtained with the Monte Carlo method using the stochastic LapH method [5] which allows all needed quark-line diagrams to be computed.

Our single-hadron operators are assembled using basic building blocks which are gauge-covariantly-displaced, LapH-smeared quark fields, as described in Refs. [1, 5, 6]. Each of our single-hadron operators creates and annihilates a definite momentum. Group-theoretical projections are used to construct operators that transform according to the irreducible representations of the space group O_h^1, plus G-parity, when appropriate. In order to build up the necessary orbital and radial structures expected in the hadron excitations, we use a variety of spatially-extended configurations. Our two-hadron operators are combinations of single-hadron operators of definite momenta. Again, group-theoretical projections are employed to produce two-hadron operators that transform irreducibly under the symmetry operations of our system. This approach is efficient for creating large numbers of two-hadron operators, and generalizes to three or more hadrons.

In finite volume, all energies are discrete so that each correlator matrix element has a spectral representation of the form

$$\mathcal{C}_{ij}(t) = \sum_n Z_i^{(n)} Z_j^{(n)*}\; e^{-E_n t}\,, \qquad Z_j^{(n)} = \langle 0|O_j|n\rangle\,, \tag{1}$$

assuming temporal wrap-around (thermal) effects are negligible. We extract energies from our correlation matrices using a "single rotation" or "fixed coefficient" method [7]. QCD is a complicated interacting quantum field theory, so characterizing its stationary states in finite volume might not be simply done. Estimates of the overlap factors $Z_j^{(n)}$ are obtained to help identify the eigenstates. If the $Z_j^{(n)}$ factors for a particular level n are only appreciable for operators j that are quark–antiquark operators, we classify this level as predominantly single-hadron. If the $Z_j^{(n)}$ factors for a particular level n are only significant for operators j that are two-hadron operators, we classify this level as predominantly two-hadron. Levels with significant overlaps with both quark–antiquark and two-meson operators are considered admixtures of single- and two-meson states.

Here, we present results obtained using a set of 412 gauge-field configurations on a large $32^3\times256$ anisotropic lattice with a pion mass $m_\pi \sim 240$ MeV. An improved anisotropic clover fermion action and an improved gauge field action are used [8]. The spatial grid size is $a_{\rm s} \sim 0.12$ fm, whereas the temporal spacing is $a_{\rm t} \sim 0.035$ fm. In our operators, a stout-link [9] staple weight $\xi = 0.10$ is used with $n_\xi = 10$ iterations. For the cutoff in the LapH smearing, we use $\sigma_{\rm s}^2 = 0.33$, which translates into the number $N_{\rm v}$ of LapH eigenvectors retained being $N_{\rm v} = 264$ for our 32^3 lattice. We use Z_4 noise in all of our stochastic estimates of quark propagation. Our variance reduction procedure is described in Ref. [5]. On the 32^3 lattices, we use 8 widely-separated source times t_0.

Our results in the $I = 1,\ S = 0,\ T_{1u}^+$ channel of total zero momentum are shown in Fig. 1. This channel has odd parity, even G-parity, and contains the spin-1 and spin-3 mesons. We used 14 single-meson (quark–antiquark) operators, 23 isovector–isovector meson operators, 31 operators that combine an isovector with a light isoscalar (using only u, d quarks), 31 operators that combine an isovector with an $\overline{s}s$ isoscalar meson, and 9 kaon–antikaon operators. We obtained results for the lowest 50 energy levels using the $(32^3|240)$ ensemble from our 108×108 correlation matrix. In our single pivot rotation of the correlation matrices, we used $\tau_0 = 5$ and $\tau_D = 8$. This figure demonstrates that the extraction of a large number of energy levels in lattice QCD is now possible, opening up the study of excited states. Keep in mind that we have not included any three-meson operators in our correlation matrix.

Figure 1 shows a comparison of our quark–antiquark dominated finite-volume energies to the experimental masses of resonances that should occur in this channel. The finite-volume energies should agree with the experiment only to within the widths of the infinite-volume resonances. We believe we have extracted all meson resonances that are quark–antiquark excitations. One observes more levels in experiment, although the experimental observations are controversial in some cases. Keep in mind that resonances that are not quark–antiquark excitations, such as so-called molecular states, would not be identified by our quark–antiquark operator overlaps. To identify resonances of molecular type, additional techniques are necessary.

Extracting infinite-volume resonance masses and widths from finite-volume energies can, in principle, be done by a complicated procedure described in Refs. [10, 11]. We have used this procedure for determining the width of the ρ resonance [12]. However, the goal of our investigation of higher

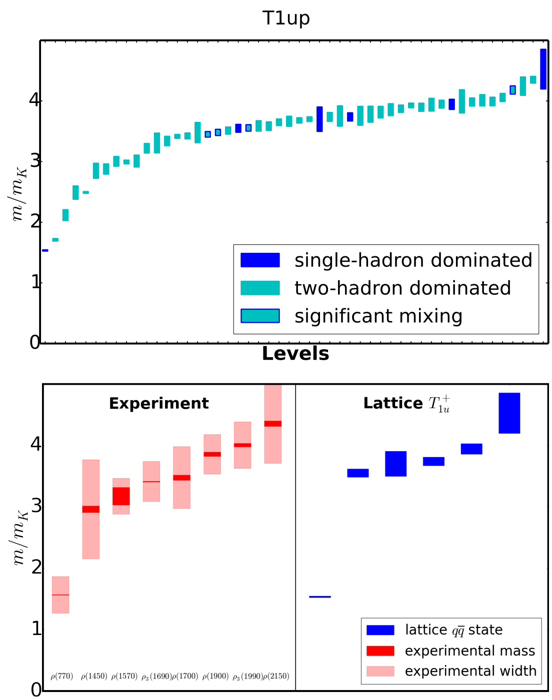

Fig. 1. (Top) Energies m as ratios of the kaon mass m_K for the first fifty states excited by our single- and two-hadron operators in the $I = 1$, $S = 0$, T_{1u}^+ zero-momentum channel. (Bottom) Comparison of the experimental spectrum of resonances with our finite-volume energies corresponding to quark–antiquark excitations. In the left-hand side, dark gray (dark red) boxes indicate the experimental masses, with the vertical heights showing the uncertainties in the mass measurements. The light gray (light red) boxes indicate the experimental widths of the resonances. In the right-hand side, our masses for the quark–antiquark excitations are shown by black (dark blue) boxes, whose heights indicate statistical uncertainties only. This T_{1u}^+ channel includes both ρ (spin 1) and ρ_3 (spin 3) states.

lying excited states is a first glimpse of the spectrum, so high precision is not necessary. We are working on developing more qualitative methods, such as the use of effective Hamiltonians [13], to help interpret our finite volume energies and extract information concerning excited resonances.

During the last few years, there has been a surge of new experimental findings in hadron spectroscopy. New resonances, collectively known as XYZ mesons, have been found, whose interpretations suggest new structures unlike those of conventional mesons and baryons. This has motivated us to design and incorporate *tetraquark* operators into our correlation matrices. As with our single meson and baryon operators, we assemble these operators using gauge-covariantly displaced LapH-smeared quark fields, but with a very different color structure. Schematic depictions of the tetraquark operators we will use are illustrated in Fig. 2. We plan to add such operators to our correlation matrices and obtain the overlap Z factors for such operators with respect to the QCD eigenstates.

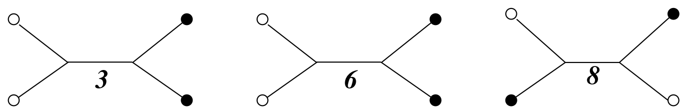

Fig. 2. Schematic depiction of the tetraquark operators we have designed. Again, we assemble these operators using gauge-covariantly displaced LapH-smeared quark fields, indicated by the circles with a connecting line which represents the displacement. Quark fields are displaced using stout-smeared link variables. The central displacement between the two diquarks can be a product of link variables in the 3-dimensional fundamental representation of SU(3) color, or in the 6-dimensional or 8-dimensional adjoint representation.

3. Conclusion

In this paper, our progress in computing the finite-volume stationary-state energies of QCD was described. Results in the zero-momentum bosonic $I = 1,\ S = 0,\ T_{1u}^{+}$ symmetry sector of QCD on a large $32^3 \times 256$ anisotropic lattice for $m_\pi \sim 240$ MeV using a correlation matrix of 108 operators were presented. All needed Wick contractions were efficiently evaluated using the stochastic LapH method. Issues related to level identification were discussed. Although a procedure is known for relating two-particle finite-volume energies to the infinite-volume S matrix, it is a very complicated method and is practical in only a handful of scattering cases. The need for simpler, more qualitative methods will be important for interpreting our higher-lying finite-volume excited-state energies. Techniques based on effective Hamiltonians are showing promise. Lastly, we described our efforts to design and implement extended tetraquark operators to study their role in the QCD resonance spectrum.

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