NUMERICAL STUDY OF THE BARYON SPECTRUM AND CHIRAL SYMMETRY RESTORATION*

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We study light baryons using a simple relativistic but non-covariant Coulomb Gauge QCD-inspired model. A variational basis is employed to compute the energies and wave functions of the baryon states, for different values of angular momentum and parity. Results are obtained for both the N and the Δ excitations. A special look is given to the high angular momentum states going up to $J = 13/2$. In this limit, we test the effect of chiral symmetry restoration on the baryonic spectrum.

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1. Introduction

Dynamical chiral symmetry breaking (χSB) is a fundamental aspect of low-energy QCD. It has been proposed, however, that for higher energies the effects of chiral symmetry breaking are no longer relevant, an effect called "insensitivity to chiral symmetry breaking" (IChSB). Indeed, when theoretically studying light–light and static–light mesons in the high-energy limit [1–3], we find that mesons of opposite parity become degenerate, a prediction consistent with that insensitivity. Baryons, it has been argued, would also display such degeneracy [4–7].

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In a previous paper [8], we argued that, when considering the space of highly excited light baryon states, the chiral charge operator

$$Q_5^\alpha = \int \mathrm{d}^3\boldsymbol{x} \Psi^\dagger(\boldsymbol{x})\gamma_5 \frac{\tau^\alpha}{2}\Psi(x) \tag{1}$$

becomes an asymptotic symmetry $\langle n_1|[Q_5^\alpha, H]|n_2\rangle \to 0$, producing a degeneracy in the baryonic spectrum $E(\sigma_1^P) = E(\sigma_2^{-P})$ with

$$Q_5^\alpha \left|\sigma_1^P\right\rangle \propto \left|\sigma_2^{-P}\right\rangle \qquad \text{and} \qquad Q_5^\alpha \left|\sigma_2^{-P}\right\rangle \propto \left|\sigma_1^P\right\rangle . \tag{2}$$

The energy splitting $|M_- - M_+|$ for high energy between the two states in a parity quartet we found in [8] to behave as

$$|M_- - M_+| \propto \frac{m(k)}{k} \tag{3}$$

for 3-quark baryons, and so in the chiral limit $m(k) = 0$ (which happens for $\langle k\rangle \to \infty$), they become degenerate very fast.

Experimentally, there are few results for the splittings of the parity doublets, which we show in Fig. 1. These results for the $I = 1/2$ are consistent with IChSB, but not so much are those for $I = 3/2$ as can be seen in Fig. 1, which are roughly inconclusive.

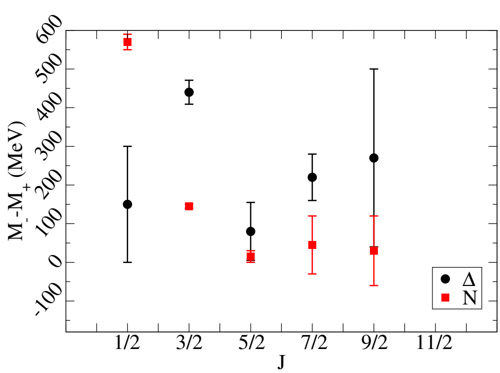

Fig. 1. Experimental data for the mass splittings $|M_- - M_+|$ as a function of J.

2. Chiral model

We study the light baryon spectrum using a simple Coulomb Gauge QCD model

$$\begin{aligned}\hat{H} \;&=\; \int \mathrm{d}^3\boldsymbol{x}\; \Psi(\boldsymbol{x})^\dagger(-i\boldsymbol{\alpha}\cdot\nabla + \beta m)\Psi(\boldsymbol{x}) \\ &\quad -\frac{1}{2}\int \mathrm{d}^3\boldsymbol{x}\;\mathrm{d}^3\boldsymbol{y}\;\Psi(\boldsymbol{x})^\dagger\frac{\lambda^a}{2}\Psi(\boldsymbol{x})V(|\boldsymbol{x}-\boldsymbol{y}|)\Psi(\boldsymbol{y})^\dagger\frac{\lambda^a}{2}\Psi(\boldsymbol{y})\,.\end{aligned} \tag{4}$$

The interaction here is chiral invariant, with the only term explicitly breaking chiral symmetry being the mass term. This theory has a non-trivial vacuum where quarks acquire a dynamical mass $m(k)$ [9], thereby breaking chiral symmetry and lifting the degeneracy of Eq. (3) for the light spectrum.

3. Baryons

To describe a baryon state $|\mathcal{B}\rangle$, we use the simple variational Ansatz

$$|\mathcal{B}\rangle = \sum_{csf} \frac{\epsilon^{c_1 c_2 c_3}}{\sqrt{6}} \int \prod_i \frac{\mathrm{d}^3 \boldsymbol{p}}{(2\pi)^3} F^{sf}_{\mathcal{B}}(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3) B^\dagger_1 B^\dagger_2 B^\dagger_3 |\Omega\rangle \,. \tag{5}$$

To calculate the eigenstates of the Hamiltonian, we first expand F^{sf} in a basis of states $|\Phi_i\rangle$, then calculate the matrix elements $\langle \Phi_i|\hat{H}|\Phi_j\rangle$, and diagonalize the resulting matrix. This matrix elements are computed by performing a nine-dimensional integral.

3.1. Building the qqq variational basis

We use Jacobi variables $\boldsymbol{p}_\rho = \frac{\boldsymbol{p}_1 - \boldsymbol{p}_2}{\sqrt{2}}$ and $\boldsymbol{p}_\lambda = \frac{\boldsymbol{p}_1 + \boldsymbol{p}_2 - 2\boldsymbol{p}_3}{\sqrt{6}}$ to describe the momenta of the three quarks on the CM frame $\boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p}_3 = 0$.

We start with the initial basis

$$|\phi_i\rangle = C^{JM}_{LM_l S M_S} C^{LM_l}_{l_\rho m_\rho l_\lambda m_\lambda} |\varphi^\alpha_{n_\rho l_\rho m_\rho}\rangle |\varphi^\alpha_{n_\lambda l_\lambda m_\lambda}\rangle |S S_{12} M_S\rangle |I I_{12} I_z\rangle \,, \tag{6}$$

which, although orthonormal, has functions that are not symmetric under quark/antiquark exchange. To construct an orthonormal basis containing only states $|\Phi_i\rangle$ with the correct symmetry, we want that, for all the exchange operators P_{ij}, the states should obey $P_{ij}|\Phi\rangle = |\Phi\rangle$. Since all P_{ij} can be written as a function of P_{12} and P_{23}, we just need to find the common eigenstates of both operators with eigenvalue $+1$. For P_{12} this is easy, just note that $P_{12}|\phi_i\rangle = (-1)^{l_\rho + S_{12} + I_{12}}|\phi_i\rangle$. For P_{23}, we construct the matrix elements of this operator and diagonalize it. This is done by blocks of constant L, S and $N = 2n_\rho + l_\rho + 2n_\lambda + l_\lambda$. At the end, we construct a orthonormal basis with the correct exchange symmetries.

3.2. Extrapolation

We truncate our basis by considering only states with $N \leq N_{\max}$. We discover that, going until $N_{\max} = 11$ or $N_{\max} = 12$, the results are not quite converged yet. But given that the number of states increase with $N^4_{\max}$ and, with more complex states, we also need to increase the numerical integration precision, we cannot improve this situation much further given our time and hardware constraints. To solve this, we extrapolate the energies $E(N_{\max}) = E_\infty + \frac{a}{N_{\max}}$ and take the resulting E_∞ as the final value.

4. Results

We compare our model predicted baryon masses with the experimental ones in Fig. 2. As can be seen there, while we get a behavior roughly similar to the experimental, the results do not accurately describe experiment. This is not surprising, given the simplicity of our model. Anyway, we want to tackle the question of whether or not the high energy spectrum is insensitive to χSB, and this model should suffice for that.

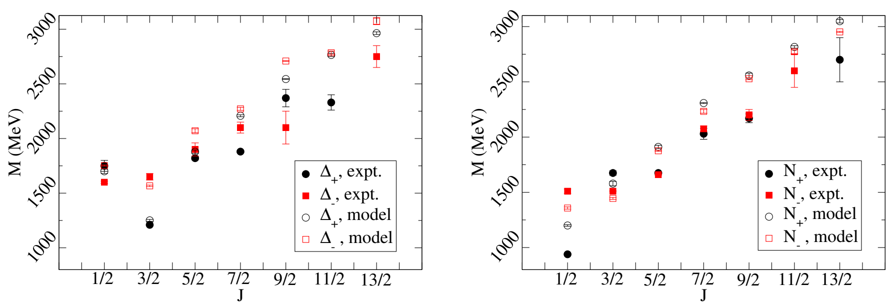

Fig. 2. Comparison of the model results to the experimental ones.

Looking at the left-hand side of Fig. 3, we see the energy splittings $|M_- - M_+|$ for the first ten radial states of the Δ ($I = J = 3/2$). There, we see that the effects of spontaneous χSB do not vanish for high radial excitations, but the identification of partners is problematic.

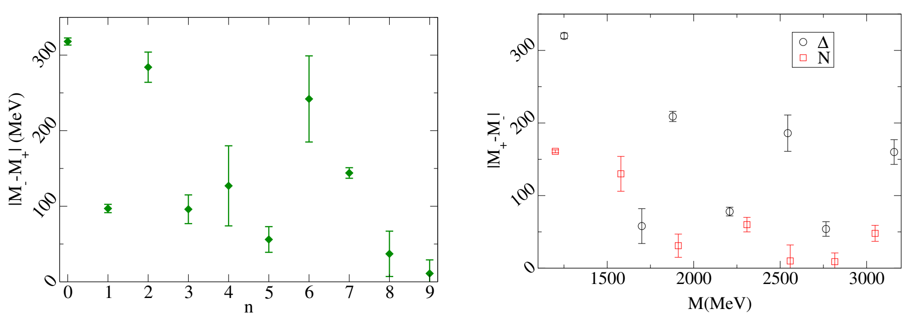

Fig. 3. Left: Splittings for the first radial states of the Δ, $J = 3/2$ case. Right: Parity splittings for different angular N and Δ states.

More interesting is to look at the right-hand side of Fig. 3, where we plot the splittings of the radial ground states as a function of the energy. For the N states, there is a trend of decreasing splittings with increasing energy. For the Δ sector, the behavior is more complex, with some states having a similar behavior to $I = 1/2$, while for others, this clearly does not happen. The anomalous behavior happens for $J = 1/2 + 2n$.

Looking at Fig. 4, where the momentum probability density for the $I = 3/2$ states is shown, we can start to see what is happening to the anomalous Δ states. We see there that for states with $J = 3/2 + 2n$, the momentum probability densities for the opposite parity states seem to follow a similar trend, while for $J = 1/2 + 2n$, this does not happen, with the densities for the negative parities states vanishing for $k \to 0$, while the positive parity states have the maximum at $k = 0$, and are thus sensitive to a larger quark mass.

This state of affairs is confirmed by Fig. 5. There we see the probability density for p_ρ and p_λ. As can be seen, when $J = 11/2$, all the four combinations of parity and isospin have a similar behavior, while for $J = 13/2$, the case with $I = 3/2$ and $P = -$ has a behavior that is distinct from the other three, with larger $\boldsymbol{p}_\rho$ than $\boldsymbol{p}_\lambda$.

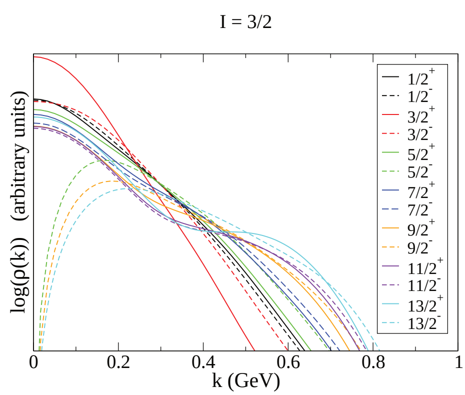

Fig. 4. Logarithm of the momentum space probability density.

5. Conclusions

We have constructed a numerical method to calculate baryonic states using a relativistic and chiral quantum field model, and calculated several states. Our model needs to be improved in order to give a more precise description of the light baryon spectrum. Nevertheless, it has given some interesting results for the IChSB. For angular excitations of nucleons, we have observed a behavior that is mostly consistent with it. For the Δ sector however, this was only observed for those states with $J = 3/2 + 2n$.

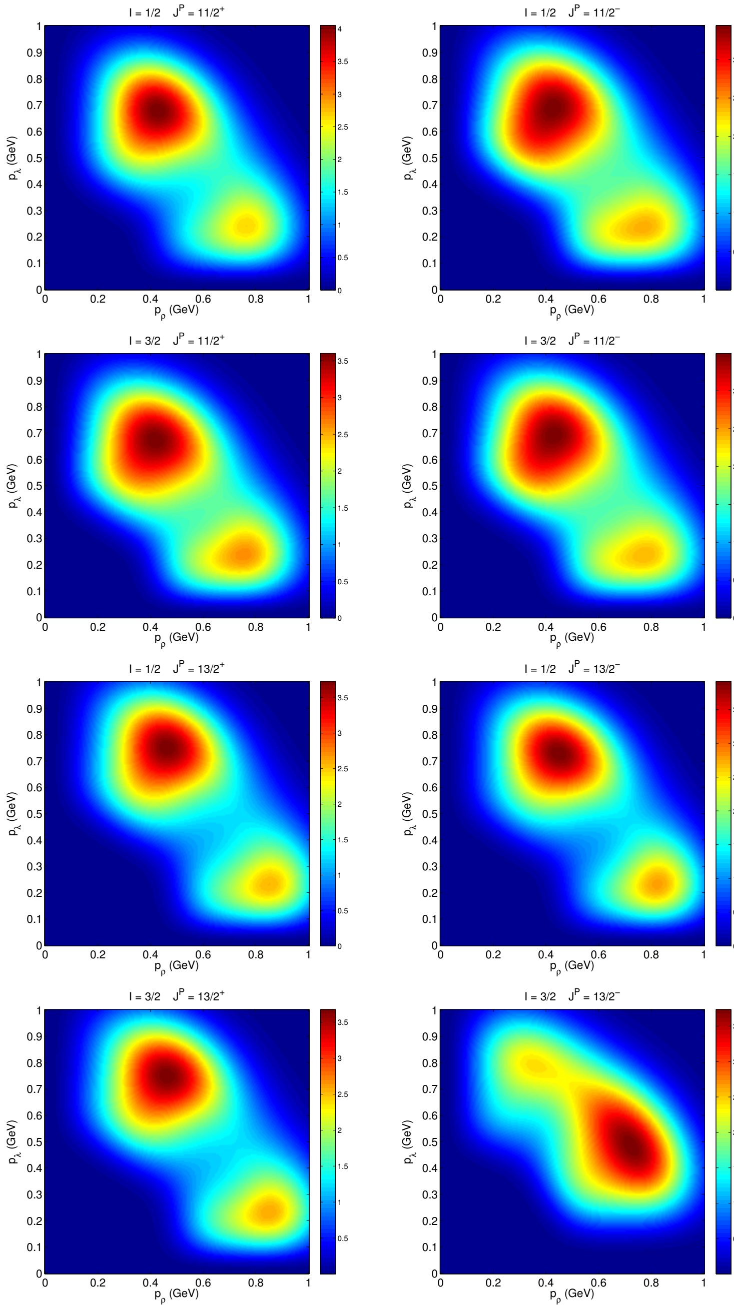

Fig. 5. Density probability for $J = 11/2$ and $J = 13/2$ (all isospin-parity combinations).

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