PHOTON STRUCTURE FUNCTIONS: 1978 AND 2005*

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I describe the early days of the photon structure functions. In particular I discuss the parton model result of Walsh and Zerwas (1973), leading order QCD calculation of Witten (1976) and next-to-leading QCD calculation of Bardeen and myself (1978). A very brief summary of the progress made from 1978 til 2005 is also given.

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1. Introduction

I have been asked to review the early days of the photon structure functions and to summarize the present status of this field. I have worked actively on deep inelastic proton and photon structure functions [1] from 1977 to 1981, but, after summarizing the status of perturbative QCD at the Photon-Lepton Symposium in Bonn in 1981, I moved to study technicolour models, flavour physics, weak, CP-violating and rare decays of K- and B-mesons, supersymmetry, extra dimensions, little Higgs and petite unification. Returning in 2005 to photon structure functions, after being decoupled from this field for 24 years, was a very interesting experience. One should realize that in 1981 no experimental data on photon structure function F_2^γ were available, although the theory had reached already a rather advanced stage. I felt like a person returning from a long journey to the earth. My main question was whether predictions for F_2^γ, in which I took part mainly in the second half of 1978, have been confirmed by the future data. I will give the answer to this question at the end of this writing.

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2. Simple parton model result

The story begins in the early 1970's, when Brodsky, Kinoshita, Terezawa (1971), Walsh (1971), Walsh and Zerwas (February 1973) and Kingsley (May 1973) analyzed the deep inelastic scattering of a highly virtual photon ($Q^2 \gg p^2$) on a real ($p^2 \approx 0$) photon target as seen in Fig. 1. If the photon behaved only as a vector meson, the corresponding structure function $F_2^\gamma(x,Q^2)$ would exhibit Q^2 and Bjorken x dependences similar to the one of the proton structure function $F_2^p(x,Q^2)$. But as pointed out by Walsh and Zerwas [2], at very large Q^2 the point-like contribution to $F_2^\gamma(x,Q^2)$, represented by the box diagram in Fig. 1, should dominate over the hadronic component. Evaluating this box diagram they found

$$F_2^\gamma(x,Q^2) = \tilde{F}_2^\gamma(x)\log\frac{Q^2}{\Lambda^2} + \dots\,, \tag{2.1}$$

where

$$\tilde{F}_2^\gamma(x) = \frac{\sum e_i^4}{16\pi^2}\, x(1-2x+2x^2) \tag{2.2}$$

with e_i being quark charges. I have introduced a scale Λ for convenience and the non-logarithmic terms in (2.1) can be found in the original paper.

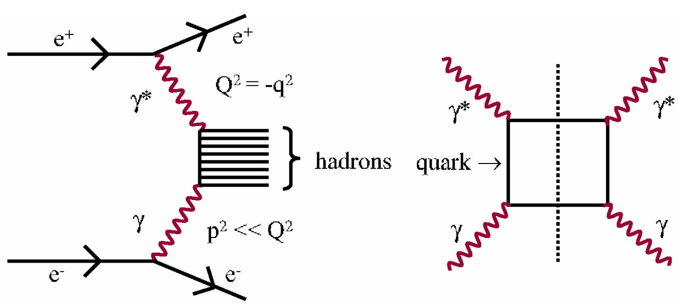

Fig. 1. The basic process and the simple Parton Model.

The most important results in (2.1) and (2.2) are:

- $F_2^\gamma(x,Q^2)$ *increases* at fixed x logarithmically with Q^2 as opposed to the corresponding *decrease* (at not too small x) observed for $F_2^p(x,Q^2)$.
- The x-dependence of $F_2^\gamma(x,Q^2)$ at large Q^2 is fully calculable as opposed to $F_2^p(x,Q^2)$, where only the Q^2 dependence (scaling violations) can be predicted within renormalization group improved theory but the actual shape of $F_2^p(x,Q^2)$ at a given Q^2 can only be found by means of non-perturbative methods or directly from the data.

3. Master formula for $F_2^\gamma(x, Q^2)$ in QCD

The simple parton model result of Walsh and Zerwas has been generalized by Witten (1976) [3] to QCD in the leading logarithmic approximation (LO). The NLO corrections to Witten's result have been calculated two years later by Bardeen and myself (1978) [4] but the complete NNLO contributions have been presented for the first time only at this conference [5], that is 27 years later.

The master formula for the moments of $F_2^\gamma(x, Q^2)$ in QCD reads as follows [4]:

$$\int_0^1 dx x^{n-2} F_2^\gamma(x, Q^2)$$
$$= \alpha^2 \left[\frac{4\pi}{\beta_0 \alpha_s(Q^2)} a_n + b_n + \sum_{i=1} r_n^{(i)} (\alpha_s(Q^2))^i + \sum_{i=1} h_n^{(i)} (\alpha_s(Q^2))^{d_n^{(i)}} \right], \quad (3.1)$$

where $n = 2, 3, \ldots,$ α is the QED structure constant, $\alpha_s(Q^2)$ is the QCD structure constant and β_0 the famous LO β-function coefficient for which Gross, Politzer and Wilczek received the Nobel-Prize last year [6]. It should be stressed that

- a_n, b_n and $r_n^{(i)}$ can be calculated in perturbation theory and correspond to LO, NLO and NNLO for $i = 1$, respectively.
- The coefficients $h_n^{(i)}$ in the last sum are incalculable in perturbative QCD. They can be calculated in the vector dominance model or in principle by means of lattice methods. They can also be extracted from the data.
- As $d_n^{(i)} \geq 0$, for very large Q^2 the first two terms dominate, except for one of $n = 2$ for which $d_2^{(i)} = 0$ and the Q^2 independent piece from the last sum in (3.1) has to be added to b_2.

4. More on Witten's analysis

Let us recall the basic formula for the moments of the proton structure function:

$$\int_0^1 dx x^{n-2} F_2^p(x, Q^2) = \sum_{i=NS,\psi,G} C_n^i \left(\frac{Q^2}{\mu^2}, g^2 \right) \langle p | O_i^n | p \rangle \, . \qquad (4.1)$$

Here C_n^i denote the Wilson coefficients (calculable in RG improved theory) of the local operators O_{NS}^n, O_ψ^n and O_G^n (quark non-singlet, quark singlet, gluon) and $\langle p|O_i^n|p\rangle$ the corresponding non-perturbative matrix elements between the proton states.

As pointed out by Witten [3], in the case of F_2^γ, an additional operator, O_γ^n, the analog of the gluon operator O_G^n with the non-Abelian field strength tensor $G_{\alpha\beta}$ replaced by the electromagnetic tensor $F_{\alpha\beta}$, has to be taken into account. The reason is that, although the Wilson coefficients C_n^γ are $\mathcal{O}(\alpha)$, the matrix elements $\langle\gamma|O_\gamma^n|\gamma\rangle$ are $\mathcal{O}(1)$. Therefore, the O_γ^n contribution to F_2^γ is of the same order in α as the contribution of quark and gluon operators. The latter have Wilson coefficients $\mathcal{O}(1)$, but matrix elements in photon states $\mathcal{O}(\alpha)$.

Explicitly:

$$\begin{aligned}&\int_0^1 dx x^{n-2} F_2^\gamma(x,Q^2)\\&= \sum_{i=\mathrm{NS},\psi,G} C_n^i\left(\frac{Q^2}{\mu^2},g^2\right)\langle\gamma|O_i^n|\gamma\rangle + C_n^\gamma\left(\frac{Q^2}{\mu^2},g^2,\alpha\right)\langle\gamma|O_\gamma^n|\gamma\rangle\end{aligned} \tag{4.2}$$

with $\langle\gamma|O_\gamma^n|\gamma\rangle = 1$.

Including only LO contributions Witten found

$$\int_0^1 dx x^{n-2} F_2^\gamma(x,Q^2) = C_n^\gamma\left(\frac{Q^2}{\mu^2},g^2,\alpha\right) = \alpha^2\frac{4\pi}{\beta_0\alpha_\mathrm{s}(Q^2)}a_n = \alpha^2 a_n \log\frac{Q^2}{\Lambda^2}\,. \tag{4.3}$$

Thus for very large Q^2 the logarithmic behaviour in Q^2 found by Walsh and Zerwas in the simple parton model remains valid in QCD as expected. However, what Witten found is that the coefficients a_n of the leading logarithm differ from the moments of $\tilde{F}_2^\gamma(x)$ in (2.2). The anomalous behaviour of $F_2^\gamma(x,Q^2)$ in QCD can be traced back to the non-vanishing anomalous dimension matrix in the space of the operators O_{NS}^n, O_ψ^n and O_G^n, known already from proton deep inelastic scattering and from the anomalous dimension K_G^n, describing the mixing of O_γ^n and O_G^n, under renormalization. Setting all these anomalous dimensions to zero and keeping only the mixing of O_{NS}^n and O_ψ^n with O_γ^n, Witten's result reduces to the one of Walsh and Zerwas: the *anomalous behaviour* of photon structure functions originates in certain non-vanishing *anomalous* dimensions. Witten's result has been re-derived by Llewellyn Smith (1978) using a diagrammatic method and by DeWitt *et al.* (1979) and Brodsky *et al.* (1978) in the framework of the Altarelli–Parisi approach (DGLAP).

In summary the most important two results of Witten's paper are the following ones:

- The coefficient of $\log Q^2$ is calculable in QCD. That is $F_2^\gamma(x, Q^2)$ is calculable in QCD at large Q^2.
- It differs from the free parton model result by finite calculable factors. In particular for x approaching 1 the increase of $F_2^\gamma(x, Q^2)$ with x, observed also in QCD at moderate x, turns into a decrease as shown in Fig. 2.

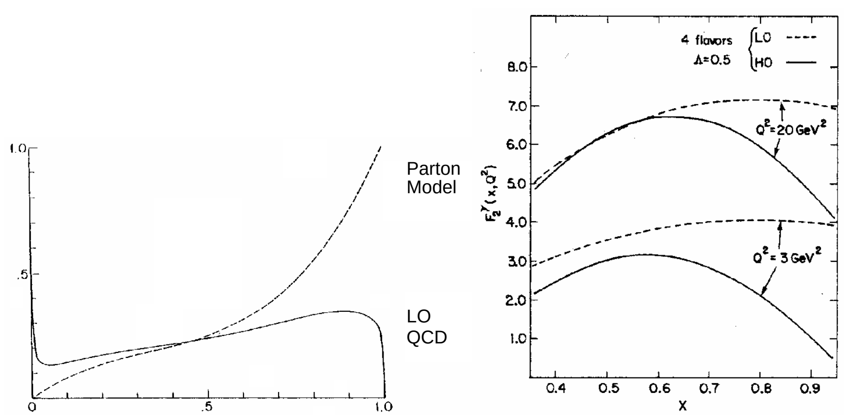

Fig. 2. F_2^γ in the Parton Model [2], QCD at LO [3] and NLO [4].

5. Our 1978 NLO analysis

My own work in this field has been triggered by the visit of Jonathan Rosner to Fermilab in the summer of 1978. Jonathan wanted to know more about Witten's paper and I explained it to him in the manner presented above. But on 24th of September 1978 I realized that Witten's result could be generalized to NLO without doing a single Feynman diagram calculation. One had only to translate the elements of the NLO analysis for $F_2^p(x, Q^2)$ into those for $F_2^\gamma(x, Q^2)$. A complete NLO analysis of $F_2^p(x, Q^2)$ has been presented for the first time in June 1978 in a work in collaboration with Bardeen, Duke and Muta [7]. It included the earlier result for two-loop anomalous dimension matrix involving $O^n_{\rm NS}$, O^n_ψ and O^n_G of Floratos, Ross and Sachrajda (1977, 1978). Thus in the fall of 1978 it was indeed possible without too much work to generalize Witten's result to NLO. I was truly delighted, when Bill Bardeen agreed to join me in our second project [4].

The main virtues of our work, that provided the constants b_n in (3.1) in QCD were the following ones:

- Having b_n at hand, a meaningful determination of $\alpha_s^{\overline{\rm MS}}$ or $\Lambda_{\overline{\rm MS}}$ [7] from $F_2^\gamma(x,Q^2)$ became possible. In view of the calculability of $F_2^\gamma(x,Q^2)$ at large Q^2, the prospects for the determination of $\alpha_s^{\overline{\rm MS}}$ from $F_2^\gamma(x,Q^2)$ appeared at least in principle better than from $F_2^p(x,Q^2)$. We will return to it at the end of this writing.

- Improved accuracy for the prediction of the shape of $F_2^\gamma(x,Q^2)$ in QCD.

- Identification of large NLO corrections for large n or equivalently large x, that made the turn over in the x-dependence of $F_2^\gamma(x,Q^2)$ stronger than at LO as seen in Fig 2.

- First comments on the divergent behaviour of the point-like component for $x \to 0$ that has been analyzed by many other authors in the 1980's.

In the following years our result has been confirmed by many groups, in particular by Duke and Owens [8], Glück, Grassie and Reya [9] and others, but in 1991 Fontannaz and Pilon [10] and independently Glück, Reya and Vogt [11] spotted a small error in our translation from the mixing (O_G^n, O_G^n) relevant for $F_2^p(x,Q^2)$ to the mixing (O_G^n, O_γ^n) relevant for $F_2^\gamma(x,Q^2)$. This was a very stupid error but fortunately without essentially any numerical consequence for $F_2^\gamma(x,Q^2)$. A visible impact on the gluon distribution in the photon has been however identified.

6. Photon structure functions 1978–2005

There is certainly no space to describe here in an adequate manner the developments after 1978. Selected reviews can be found in [12–14]. Let me then list only a few points:

On the theoretical side:

- The evolution equations for quark and gluon distributions in the photon have been studied by Brodsky *et al.* (1978), Glück, Grassie and Reya (1983), Rossi (1983), Drees (1983) and others. See additional references below.

- The issue of the singular behaviour for $x \to 0$ has been addressed in several papers by Duke and Owens, Bardeen, Glück, Grassie and Reya, Rossi, Antoniadis, Grunberg and others. In particular the non-perturbative component has been used as a regulator.

- As stressed in particular by Glück, Reya and Vogt (1991) [11], the Mellin n-moment technique for the study of the Q^2-evolution of $F_2^\gamma(x,Q^2)$ and of quark and gluon distributions in the photon is technically superior to the Bjorken-x space technique developed earlier by Glück and Reya, Rossi, Drees, Da Luz Vieira, Storrow and others. In particular α_s-counting problems can be straightforwardly avoided.

- A large number of parametrizations of parton distributions including heavy flavours have been proposed. Compilations of these parametrizations (more than 25 in total) can be found in [12, 13]. Here the group of Marysia Krawczyk is among the leading groups. The most recent efforts in these directions can be found in [15–18], where numerous references to earlier literature can be found.

- The NNLO corrections have been recently completed [5]. I am told that they are small but we have to wait until the numerical analysis of these corrections has been published.

While definitely a significant progress on the study of the implications of the Witten's and our calculations has been done since 1978, the main progress in this field has been done by experimentalists. After the first measurements of $F_2^\gamma(x,Q^2)$ by the CELLO collaboration in 1983 there was a dramatic progress in collecting data made by PLUTO, JADE, TASSO, TOPAZ, AMY, DELPHI, L3, ALEPH, OPAL and TPC/2. The relevant references can be found in [15–18]. As a result of these efforts $F_2^\gamma(x,Q^2)$ and the quark distributions $q^\gamma(x,Q^2)$ are quite well known at present, while the gluon distribution $G^\gamma(x,Q^2)$ is still poorly known. The ranges in x and Q^2 explored so far are very impressive

$$0.001 \le x \le 0.9 \quad \text{and} \quad 1.9\ \text{GeV}^2 \le Q^2 \le 780\ \text{GeV}^2\,.$$

7. Conclusions

Let us finally ask whether the $\log Q^2$ and x-dependences of the point-like component predicted in 1970's have been confirmed by the data. The answer is given in Fig. 3. Indeed the increase of $F_2^\gamma(x,Q^2)$ with increasing x and Q^2 is clearly visible. A more detailed analysis also shows that for $x \to 1$ the turn over predicted by QCD becomes visible but more data is required to expose this feature clearly.

On the other hand the present data on $F_2^\gamma(x,Q^2)$ allow already now a rather precise determination of $\alpha_s^{\overline{\rm MS}}$. One finds [19]

$$\alpha_s^{\overline{\rm MS}}(M_Z) = 0.1183 \pm 0.0050(\text{exp}) \pm 0.0028(\text{theory}) \tag{7.1}$$

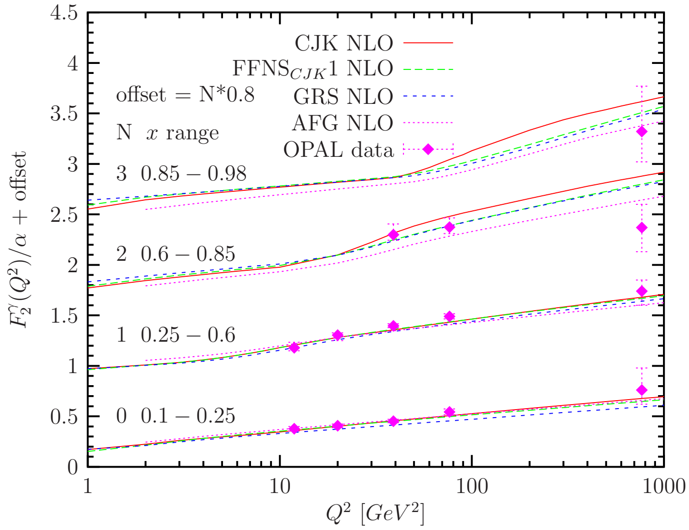

Fig. 3. F_2^γ as a function of Q^2 for different x compared with experiment [15].

in a very good agreement with the world average $\alpha_s^{\overline{\rm MS}}(M_Z)=0.1182\pm0.0027$. The accuracy could be further improved at the NLC and a photon collider.

In summary, the photon structure functions predicted by QCD agree well with the data, even if further studies are clearly desirable. It became a mature field and definitely there is more to come in the next 25 years. As I am now returning back to flavour physics, weak decays and CP violation I wish all explorers of the physics of photon structure functions good luck!

I would like to thank Marysia Krawczyk for inviting me to this so well organized and very interesting symposium. The six days I spent in Warsaw, in particular the piano concertos in the Warsaw philharmony, will remain in my memory forever.

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