ON THE VOLKOV–AKULOV MODEL

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Dedicated to the memory of Professor Jan Rzewuski

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For the classical Volkov–Akulov Model the energy-momentum tensor, the supercurrent as well as the equations of motion are given. The quantal approach based on these quantities is preliminarily discussed with emphasis upon the phenomenon of spontaneously broken supersymmetry.

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1. Introduction

It is a puzzling fact that the spontaneous breaking of symmetry can occur only for scalar and spinor fields [1, 2]. For massless, free, real, scalar quantum field $\phi^0(x)$ the locally conserved translationally covariant current

$$j^0_\mu(x) = \partial_\mu \phi^0(x)$$

is related to the mapping

$$\phi^0(x) \to \phi^0(x) + a\,,$$

where a is a real constant, which can not be unitarily implemented in the Hilbert space. For the massless, free, Majorana spinor quantum field $\lambda^0(x)$ the translationally covariant current [2]

$$J^0_{\mu\alpha} = (\gamma_\mu \lambda^0)_\alpha \qquad \alpha = 1,2,3,4 \quad \mu = 0,1,2,3 \tag{1.1}$$

corresponds to the supertransformation

$$\lambda^0_\alpha(x) \to \lambda^0_\alpha(x) + \eta_\alpha \qquad \alpha = 1,2,3,4\,, \tag{1.2}$$

where η_α are anticommuting constants. This will be shown in Section 5. Notice that the current $\partial_\mu \lambda^0_\alpha$ does not give rise to any symmetry. As we shall see in case (1.1) the formal supercharge

$$\overline{Q}^0_\alpha = \int \overline{J}^0_{0\alpha} d^3x = -\int \lambda^0_\alpha d^3x$$

satisfies

$$\{Q^0_\alpha, \overline{Q}^0_\beta\} = -(\gamma_0)_{\alpha\beta} V\,, \tag{1.3}$$

where V = infinite constant, representing the volume corresponding to the 3-dimensional space. It is evident from (1.3) that in this case the supercharge is not at all related to the energy-momentum operator. Both fields, $\phi^0(x)$ as well as $\lambda^0(x)$, are Goldstone fields. It is obvious that the supersymmetry induced by (1.1) must be degenerate as the theory consists of only one free spinor field, free of any scalar boundstates. It is easy to see that for a massless, free, real tensor quantum field $F^0_{\mu\nu} = F^0_{\nu\mu}$ neither the current

$$j^0_{\mu\nu} = F^0_{\mu\nu}\,,$$

nor

$$j^0_{\mu\lambda\nu} = \partial_\mu F^0_{\lambda\nu}\,,$$

give rise to a symmetry; the latter could be related to the mapping $F^0_{\mu\nu} \to F^0_{\mu\nu} + \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ are constants, were it not that $\partial^\mu F^0_{\mu\nu} = 0$ as we keep the assumption that the metric in the Hilbert space has to be positive definite. There exist, however, translationally non-covariant currents giving rise to such symmetries (D. Buchholz — private information).

Let us return to the observation that the anticommutator of a would-be spinor charge in a theory of one free spinor field does not yield the energy-momentum operator. To prove or disprove this assertion for the case of quantum interacting fields is difficult as one does not know in general the quantization and renormalization procedure. There are several papers [3] concerning this subject, all of them unsatisfactory or inconclusive in my opinion. We shall exhibit the difficulties looking at the model of Volkov and Akulov [4, 5]. This model consists of one Majorana field and displays supersymmetric covariance. We shall follow the ideas presented in my earlier papers [2]; to be frank the way of presentation there in is intricate; nevertheless the main ideas seem to be suitable for application to the Volkov–Akulov model.

2. General

Any supersymmetric local, locally conserved current can be uniquely decomposed in two local components,

$$J_{\alpha\mu} = s_{\alpha\mu} + [(\gamma_\mu)t]_\alpha \,, \tag{2.1}$$

where

$$(\gamma^\mu s_\mu)_\alpha = 0 \,, \tag{2.2}$$

and $\mu = 0,1,2,3$ is a vector index. It can be shown [2] that $s_{\alpha\mu}$ regarded as a quantum field does not break the supersymmetry. The necessary condition for the spontaneous breaking is $t_\alpha \neq 0$. The sufficient condition is that $t_\alpha \Omega$, where Ω is the unique vacuum, carries a massless one particle contribution. Thus there must exist massless one-particle states and we may apply the results due to Buchholz [6] concerning the collision theory of massless particles according to which t_α can be represented as follows

$$t_\alpha = t_\alpha^{\mathbf{ex}} + r_\alpha^{\mathbf{ex}} \qquad \mathrm{ex} = \text{either "in" or "out"}\,, \tag{2.3}$$

where $t_\alpha^{\mathbf{ex}}$ is either an incoming or outgoing free field of zero mass and r_α does not contribute to the massless one particle states. The fields t_α^{in} or t_α^{out} (of course, $t_\alpha^{\mathrm{in}}\Omega = t_\alpha^{\mathrm{out}}\Omega$) are linked to the Goldstone massless field. Although $t_\alpha^{\mathbf{ex}}$ is local, but it is not local with respect to t_α; however, for any localized polynomial P in the fields local with respect to $J_{\alpha\mu}$ or t_α the amplitude $(\psi(\vec{k}), P\Omega)$, where $\psi(\vec{k})$ is a one particle state of momentum $\vec{k}$ and vanishing mass, is smooth in $\vec{k}$, therefore, the integral

$$\int d^3x \; f_R(\vec{x}) \int \frac{d^3k}{2|\vec{k}|} \; e^{i\vec{k}\vec{x}}(\Omega, t^{\mathrm{ex}}\psi(\vec{k}))(\psi(\vec{k}), P\Omega)$$

always exists. Here $f_R(\vec{x})$ is the standard smearing function concentrated in a ball of radius R.

Notice that

$$\gamma_\mu \partial^\mu t_\alpha^{\mathbf{ex}} = 0 \,. \tag{2.4}$$

The part responsible for the spontaneous breaking of supersymmetry reads

$$t_\alpha = \tfrac{1}{4}(\gamma^\mu J_{\alpha\mu}) \,. \tag{2.5}$$

Notice that although the supercharge

$$Q_\alpha = \lim_{R\to\infty} \int j_{0\alpha}(x) f_R(\vec{x}) \alpha(x_0) d^4x$$

does not exist as an operator, the expressions

$$\{Q_\beta, \lambda_\gamma\} \equiv F_{\beta\gamma}, \tag{2.6}$$

as well as

$$[Q_\alpha, F_{\beta\gamma}] \equiv G_{\alpha\beta\gamma}, \tag{2.7}$$

are well defined fields (anti)local with respect to $j_{\mu\alpha}$ as well as λ_β. Here $\alpha(x_0)$ is the standard smearing function and λ_β any spinor field, antilocal with respect to $j_{\mu\alpha}$. Also

$$[\{Q_\alpha, \overline{Q}_\beta\}, \lambda_\gamma] = -(\gamma_0)_{\beta\delta}(G_{\alpha\delta\gamma} + G_{\delta\alpha\gamma}) \tag{2.8}$$

is a well defined field. It is, however, not obvious that $\{Q_\alpha, \overline{Q}_\beta\}$ behaves like a global, conserved charge and that

$$[\{Q^{\text{ex}}_\alpha, \overline{Q}^{\text{ex}}_\beta\}, \lambda^{\text{ex}}_\gamma] = [\{Q_\alpha, \overline{Q}_\beta\}, \lambda^{\text{ex}}_\gamma] = -(\gamma_0)_{\beta\delta}(G^{\text{ex}}_{\alpha\delta\gamma} + G^{\text{ex}}_{\delta\alpha\gamma}). \tag{2.9}$$

Should it be so the r.h.s. has to vanish according to (1.3).

3. Model of Volkov and Akulov and its supersymmetric invariance

The Lagrangean is

$$L = -\frac{1}{a^2}\det W \qquad [L] = cm^{-4} \tag{3.1}$$

with

$$W_\mu{}^\nu = g_\mu{}^\nu - ia^2(\overline{\lambda}\gamma_\mu\partial^\nu\lambda) \equiv g_\mu{}^\nu + T_\mu{}^\nu. \tag{3.2}$$

Here $\lambda_\alpha(x)$ $\alpha = 1,2,3,4$ is a Majorana spinor field, whose components anticommute with each other [1], a is a coupling constant of dimension cm^2.

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[1] $\lambda = \lambda^*$, $\overline{\lambda} = \lambda\gamma_0$, $\{\lambda_\alpha(x), \lambda_\beta(y)\} = 0$, λ^*-complex conjugate of λ.

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \qquad g_{\mu\nu} = diag(1,-1,-1,-1)$$

$$\gamma_0^T = -\gamma^0 \quad \gamma_j^T = \gamma_j \;\; j = 1,2,3 \;\; e.g.\; \gamma_0 = \begin{pmatrix} 0 & 1i \\ -1i & 0 \end{pmatrix} \; \gamma_j = \begin{pmatrix} 0 & i\sigma_j \\ i\sigma_j & 0 \end{pmatrix}$$

Hence

$$\gamma_0\gamma_\lambda = (\gamma_0\gamma_\lambda)^T, \quad \overline{\lambda}\gamma_\mu\lambda = 0, \quad \partial^\mu\overline{\lambda}\gamma_\nu\partial^\lambda\lambda = -\partial^\lambda\overline{\lambda}\gamma_\nu\partial^\mu\lambda$$

Further relations are collected in Appendix C.

The Lagrangean, written in extension reads

$$L = -\frac{1}{a^2}\big(1 + A + \tfrac{1}{2}A^2 - \tfrac{1}{2}B + \tfrac{1}{3}C - \tfrac{1}{2}AB + \tfrac{1}{6}A^3 - \tfrac{1}{4}D + \tfrac{1}{3}AC + \tfrac{1}{8}B^2 - \tfrac{1}{4}A^2B + \tfrac{1}{24}A^4\big)\,, \tag{3.3}$$

where

$$A = \mathrm{Tr}\,(T) \qquad B = \mathrm{Tr}\,(T^2) \qquad C = \mathrm{Tr}\,(T^3) \qquad D = \mathrm{Tr}\,(T^4)\,, \tag{3.4}$$

(see Appendix A).

This Lagrangean is invariant under the following supersymmetric transformation (see *e.g.* [7])

$$\delta\lambda_\alpha = \epsilon_\alpha - ia^2(\bar\epsilon\gamma_\mu\lambda)\lambda'^{\,\mu}_\alpha\,, \tag{3.5}$$

where ϵ_α $\alpha = 1, 2, 3, 4$ are anticommutating constants and $\lambda'^{\,\mu}_\alpha \equiv \partial^\mu\lambda_\alpha$. This amounts in quantum language to

$$\{Q_\alpha, \lambda_\beta\} = (\gamma_0)_{\alpha\beta} - ia^2\ \text{"}(\gamma_\mu\lambda)_\alpha\lambda'^{\,\mu}_\beta\text{"} \tag{3.6}$$

with

$$\bar\epsilon_\beta\{Q_\beta, \lambda_\alpha\} = \delta\lambda_\alpha\,,$$

where ϵ_β is a constant spinor and Q_α a spinorial charge. Notice that the expression in quotation marks on the r.h.s. of (3.6) is not well defined. Examining (3.6) in the tree approximation we conclude that the supersymmetry is spontaneously broken and, therefore, in quantum theory, Q_α can not exist as an operator. After some formal manipulations (see *e.g.* [7]) one gets from (3.6)

$$[\{Q_\alpha, \overline{Q}_\beta\}, \lambda_\gamma] = 2ia^2(\gamma_\mu)_{\alpha\beta}\partial^\mu\lambda_\gamma\,. \tag{3.7}$$

The latter relation shows that in the classical case or in the naive quantum approach the action of $\{Q_\alpha, \overline{Q}_\beta\}$ upon the field is equivalent to that of $(-2a^2(\gamma_\mu)_{\alpha\beta}P^\mu)$, where P^μ, $\mu = 0, 1, 2, 3$, is the energy-momentum operator. This sounds reasonable as in supersymmetric theories one expects $\{Q_\alpha, \overline{Q}_\beta\}$ to be proportional to $(\gamma_\mu)_{\alpha\beta}P^\mu$ with the proviso that proportionality factor is negative. What in our case is peculiar is that the right hand side of (3.7) depends quadratically upon a; this dependence on a could be removed by replacing

$$Q_\alpha \qquad \textit{by} \qquad \hat{Q}_\alpha \equiv \frac{1}{a}Q_\alpha\,;$$

then

$$\delta\hat\lambda_\alpha = \frac{1}{a}\epsilon_\alpha - ia(\bar\epsilon\gamma_\mu\lambda)\lambda'^{\,\mu}_\alpha\,. \tag{3.8}$$

Notice, however, that if we agree to keep the correspondence with the case of free fields, which results when we let the coupling constant a tend to zero then the definition (3.5), not (3.8), is the proper one. For $a \to 0$ the Lagrangean reduces to

$$\lim_{a \to 0}(L + \frac{1}{a^2}) = i(\overline{\lambda}^0 \gamma_\mu \partial^\mu \lambda^0)\,. \tag{3.9}$$

For such a field the spontaneously broken supersymmetric transformation amounts to

$$\delta\lambda^0 = \epsilon_\alpha\,,$$

which coincides with (3.5) for $a \to 0$. In this limit the right hand of (3.7) vanishes which would coincide with the action of (1.3). Should we treat (3.7) seriously as an relation valid for the quantal case we would face a difficulty in reconciling the asymptotic limit of (3.7) with (1.3). Obviously, the r.h.s. of (3.7) tends in asymptotic limit towards

$$2ia^2(\gamma_\mu)_{\alpha\beta}\partial^\mu\lambda_\gamma^{\mathrm{ex}} = -2a^2(\gamma_\mu)_{\alpha\beta}[P^\mu, \lambda_\gamma^{\mathrm{ex}}]\,.$$

As far as the l.h.s. is concerned a careful investigation is needed (see Section 2) as it does not necessarily tend towards

$$[\{Q_\alpha, \overline{Q}_\beta\}, \lambda_\gamma^{\mathrm{ex}}]$$

in spite of that it is well defined. There is, however, no good reason to accept (3.7) as a relation valid in quantum field theory as it was obtained without taking into account the quantal structure of the theory and as well as renormalization effects.

4. Energy–momentum tensor

From (3.1) follows (see Appendix B)

$$Lg_\mu{}^\nu = \frac{\partial L}{\partial T_\nu{}^\mu} + \frac{\partial L}{\partial T_\lambda{}^\mu} T_\lambda{}^\nu = \frac{\partial L}{\partial T_\nu{}^\mu} + \frac{\partial L}{\partial \lambda'_\alpha{}^\mu}\lambda'_\alpha{}^\nu\,. \tag{4.1}$$

We define

$$\theta_\mu{}^\nu \equiv -Lg_\mu{}^\nu + \frac{\partial L}{\partial \lambda'_\alpha{}^\mu}\lambda'_\alpha{}^\nu = -\frac{\partial L}{\partial T_\nu{}^\mu}\,. \tag{4.2}$$

This quantity is locally conserved, *viz.*

$$\partial^\mu\theta_\mu{}^\nu = -\partial^\nu L + \frac{\partial L}{\lambda_\alpha}\lambda'_\alpha{}^\nu + \frac{\partial L}{\lambda'_\alpha{}^\mu}\lambda'_\alpha{}^{\nu\mu} = -\partial^\nu L + \partial^\nu L = 0\,. \tag{4.3}$$

Here we used the formal equations of motion

$$\partial^\mu \frac{\partial L}{\partial \lambda'^\mu} = \frac{\partial L}{\partial \lambda_\alpha} . \tag{4.4}$$

as well as the relation

$$\partial^\nu L = \frac{\partial L}{\partial \lambda_\alpha} \lambda'^\nu_\alpha + \frac{\partial L}{\partial \lambda'^\mu_\alpha} \partial \lambda'^{\mu\nu}_\alpha .$$

Consequently

$$\partial^\mu \left(\frac{\partial L}{\partial T_\nu{}^\mu} \right) = 0 . \tag{4.5}$$

The quantity (4.2) can be viewed as nonsymmetrized energy-momentum tensor. The symmetrized tensor reads

$$\theta^{sup}_{\lambda\rho} = \theta_{\lambda\rho} + \tfrac{a^2}{2}\theta_\mu{}^\kappa(\bar{\lambda}\gamma_\kappa \sigma_{\lambda\rho}\lambda'^\mu + \tfrac{a^2}{2}\partial^\mu[\theta_\lambda{}^\kappa(\bar{\lambda}\gamma_\kappa\sigma_{\rho\mu}\lambda) + \theta_\rho{}^\kappa(\bar{\lambda}\gamma_\kappa\sigma_{\mu\lambda}\lambda)] . \tag{4.6}$$

From (3.3) and (4.2) follows that

$$\begin{aligned}\theta_\mu{}^\nu = -\frac{\partial L}{\partial T_\mu{}^\nu} &= +\tfrac{1}{a^2}(1 + A - \tfrac{1}{2}B + \tfrac{1}{2}A^2 + \tfrac{1}{3}C - \tfrac{1}{2}AB + \tfrac{1}{6}A^3)g_\mu{}^\nu \\ &\quad - \tfrac{1}{a^2}(T - T^2 + AT + T^3 - AT^2 - \tfrac{1}{2}BT + \tfrac{1}{2}A^2T)_\mu{}^\nu .\end{aligned} \tag{4.7}$$

The trace can be found from (4.2), *viz.*

$$\theta_\mu{}^\mu = -4L + \frac{\partial L}{\partial \lambda'_\alpha{}^\mu}\lambda'_\alpha{}^\mu = -\frac{\partial L}{\partial T_\mu{}^\mu} .$$

To evaluate it further notice that

$$\frac{\partial L}{\partial \lambda_\alpha}\lambda_\alpha = \frac{\partial L}{\partial T_\mu{}^\nu}\frac{\partial T_\mu{}^\nu}{\partial \lambda_\alpha}\lambda_\alpha = ia^2\frac{\partial L}{\partial T_\mu{}^\nu}(\gamma_0\gamma_\mu\partial^\nu\lambda)_\alpha\lambda_\alpha = \frac{\partial L}{\partial T_\mu{}^\nu}T_\mu{}^\nu ,$$

as well as

$$\frac{\partial L}{\partial \lambda'_\alpha{}^\sigma}\lambda'_\alpha{}^\sigma = -ia^2\frac{\partial L}{\partial T_\mu{}^\sigma}(\bar{\lambda}\gamma_\mu)_\alpha\lambda'_\alpha{}^\sigma = \frac{\partial L}{T_\mu{}^\nu}T_\mu{}^\nu .$$

It will be shown in the next section that $\frac{\partial L}{\partial \lambda_\alpha} = 0$. This relation together with both formulae above implies

$$\frac{\partial L}{\partial \lambda_\alpha}\lambda_\alpha = \frac{\partial L}{\partial \lambda'_\alpha{}^\sigma}\lambda'_\alpha{}^\sigma = 0 . \tag{4.8}$$

Thus,

$$\theta_\mu{}^\mu = -4L\,. \tag{4.9}$$

This relation can not be immediately checked by inserting formula (4.7) on the l.h.s. of (4.9) and formula (3.3) on the r.h.s. of it since there are some identities which have to be taken into account and which will be considered in Section 7. After these identities are exploited we get

$$\theta_\mu{}^\mu = +\tfrac{1}{a^2}(4 + 3A - B + \tfrac{1}{2}A^2 + \tfrac{1}{3}C)\,.$$

For a free field λ^0 we get from (4.7) by taking the limit $a \to 0$

$$\theta^{0\nu}_\mu = \lim_{a\to 0}(\theta_\mu{}^\nu - \tfrac{1}{a^2}g_\mu{}^\nu) = -i(\overline{\lambda}^0\gamma_\mu\partial^\mu\lambda^0)g_{\mu\nu} + i(\overline{\lambda}^0\gamma_\mu\partial^\nu\lambda^0)\,.$$

Because of the free field equation of motion

$$\gamma_\mu\partial^\mu\lambda^0 = 0 \tag{4.11}$$

this reduces to

$$\theta^{0\nu}_\mu = +i(\overline{\lambda}^0\gamma_\mu\partial^\nu\lambda^0) = (\theta^{0\nu}_\mu)^+\,. \tag{4.12}$$

Moreover, because of (4.11)

$$\theta^{0\mu}_\mu = 0\,. \tag{4.13}$$

The symmetrized energy momentum tensor for the free field reads

$$(\theta^0)^{sym}_{\mu\nu} = +\tfrac{i}{2}(\overline{\lambda}^0\gamma_\mu\partial_\nu\lambda^0) + \tfrac{i}{2}(\overline{\lambda}^0\gamma_\nu\partial_\mu\lambda^0)\,.$$

We except that in the classical as well as quantal theory we should get formally

$$\int d^3x\theta_{0\mu}(x) \sim P_\mu\,, \tag{4.14}$$

where P_μ is the energy-momentum vector. We expect also that in quantal case

$$[P_\mu, \lambda_\alpha(x)] = -i\partial_\mu\lambda_\alpha(x)\,. \tag{4.15}$$

To show (4.15) in general case is not possible as long as we do not know the proper commutation relations for the field. Therefore we shall try in our future work to use (4.15) as an assumption to extract the proper commutation rules for the interacting field λ.

For a free field λ^0 which satisfies the quantal equal time anticommutation relations

$$\{\lambda_\alpha(x), \lambda_\beta(y)\}_{x_0=y_0} = \delta(\vec{x}-\vec{y})\delta_{\alpha\beta}\,, \tag{4.16a}$$

$$\{\partial^0\lambda_\alpha(x), \lambda_\beta(y)\}_{x_0=y_0} = [(\gamma^j\partial_j - im)\gamma_0]_{\alpha\beta}\delta(\vec{x}-\vec{y})\,, \tag{4.16b}$$

and $\theta^0{}_{\mu\nu}$ as defined by (4.12) this is really so.

5. Supercurrent

There is another locally conserved current. This current is a Rarita–Schwinger spinor. It reads[2]

$$\overline{J}_{\mu\alpha} \equiv i\frac{\partial L}{\partial \lambda'_\alpha{}^\mu} = -a^2\theta_\mu{}^\rho(\overline{\lambda}\gamma_\rho)_\alpha\,. \tag{5.1}$$

To show that (5.1) is locally conserved notice that on one hand

$$\begin{aligned}\partial^\mu\overline{J}_{\mu\alpha} &= -a^2\theta_\mu{}^\rho(\partial^\mu\overline{\lambda}\gamma_\rho)_\alpha = -a^2\theta_\mu{}^\rho(\gamma_0\gamma_\rho\partial^\mu\lambda)_\alpha \\ &= i\frac{\partial L}{\partial T_\rho{}^\mu}(-ia^2(\gamma_0\gamma_\rho\partial^\mu\lambda))_\alpha = -i\frac{\partial L}{\partial T_\rho{}^\mu}\frac{\partial T_\rho{}^\mu}{\partial\lambda_\alpha} = -i\frac{\partial L}{\partial\lambda_\alpha}\end{aligned} \tag{5.2}$$

on the other — using the equations of motion (4.4) —

$$\partial^\mu\overline{J}_{\mu\alpha} = i\partial_\mu\left(\frac{\partial L}{\partial\lambda'_\alpha{}^\mu}\right) = i\frac{\partial L}{\partial\lambda_\alpha}\,. \tag{5.3}$$

By comparing (5.2) and (5.3) we get

$$\partial^\mu\overline{J}_{\mu\alpha} = 0 \tag{5.4}$$

as well as the surprising result

$$\frac{\partial L}{\partial\lambda_\alpha} = 0\,. \tag{5.5}$$

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$$-\frac{\partial L}{\partial\lambda'_\alpha{}^\mu} = -\frac{\partial L}{\partial T_\lambda{}^\kappa}\frac{\partial T_\lambda{}^\kappa}{\partial\lambda'_\alpha{}^\mu} = ia^2\frac{\partial L}{\partial T_\lambda{}^\mu}(\overline{\lambda}\gamma_\lambda)_\alpha = -ia^2\theta_\mu{}^\lambda(\overline{\lambda}\gamma_\lambda)_\alpha$$

The current

$$J_{\mu\alpha} = (\bar{J}_\mu \gamma_0)_\alpha = a^2 \theta_\mu{}^\rho (\gamma_\rho \lambda)_\alpha \tag{5.6}$$

is, of course, also conserved.

The supercurrent (5.1) or (5.6) should be a good candidate to reproduce the supersymmetry transformation (3.5). This can not be seen immediately when $a \neq 0$; in case $a \to 0$, however, this is an easy task. Relations (5.1) and (3.9) yield (see (1.1))

$$\bar{J}^0_{\mu\alpha} = i \frac{\partial L}{\partial \lambda^{0\mu'}_\alpha} = -(\bar{\lambda}_0 \gamma_\mu)_\alpha , \tag{5.7}$$

$$J^0_{\mu\alpha} = (\gamma_\mu \lambda^0)_\alpha . \tag{5.8}$$

The same result one gets by taking the limit

$$\lim_{a \to 0} [-a^2 \theta_\mu{}^\rho (\bar{\lambda} \gamma_\rho)_\alpha] = -g_\mu{}^\rho (\bar{\lambda}^0 \gamma_\rho)_\alpha .$$

Let us regard the current and the field as quantized quantities. We have then perfectly well defined and Poincaré covariant expressions

$$\bar{\epsilon}_\alpha \{Q_\alpha, \lambda^0{}_\beta(y)\} = \bar{\epsilon}_\alpha (\gamma_0)_{\alpha\gamma} \int d^3x \, \{\lambda^0_\gamma(x), \lambda_\beta(y)\}_{x_0 = y_0} = \epsilon_\beta , \tag{5.9}$$

where we used the quantum equal time commutation relations (4.16) and ϵ_α are anticommuting constants. From (5.9) one gets also immediately (1.3).

We see from (5.9) and (1.3) that the free field supersymmetric current (5.7) and (5.8) is spontaneously broken and λ_α is the Goldstone field. We may expect that also the current (5.1) or (5.6) is spontaneously broken. This is, however, only a guess as we are not able, without quantizing the model, to show explicitly that this is really so. The breaking of the current (5.6) comes from the term

$$t_\alpha = \tfrac{1}{4} (\gamma^\mu J_\mu)_\alpha = \tfrac{a^2}{4} \theta_\mu{}^\rho (\gamma^\mu \gamma_\rho \lambda)_\alpha . \tag{5.10}$$

To justify our choice of the supercurrent (5.1) let us evaluate the Noether current. Let us examine

$$\delta L - \partial^\mu \Big(\frac{\partial L}{\partial \lambda'_\alpha{}^\mu} \delta \lambda_\alpha \Big) = 0 , \tag{5.11}$$

where $\delta\lambda_\alpha$ is given by (3.5). It can be shown that

$$\frac{\partial L}{\partial \lambda'_\alpha{}^\mu} \delta \lambda_\alpha = ia \left(\frac{\partial L}{\partial T_\nu{}^\mu} - \frac{\partial L}{\partial \lambda'_\beta{}^\mu} \lambda'_\beta{}^\nu \right) (\bar{\epsilon} \gamma_\nu \lambda) . \tag{5.12}$$

This expression does not vanish and causes a change in the Lagrangean by a divergence. The same effect as the variation (3.5) has the variation of the variables x, *viz.*

$$\delta x_\mu = -ia(\bar{\epsilon}\gamma_\mu\lambda)\,. \tag{5.13}$$

Thus

$$\delta L = -ia(\bar{\epsilon}\gamma_\mu\lambda)\partial^\mu L - ia(\bar{\epsilon}\gamma_\mu\partial^\mu\lambda)L\,,$$

the last term coming from the change of the Jacobian. This expression can be written in the form

$$\begin{aligned}\delta L &= -ia\partial^\mu[(\bar{\epsilon}\gamma_\mu\lambda)L] \\ &= -ia\partial^\mu\left[\left(\frac{\partial L}{T_\nu{}^\mu} + \frac{\partial L}{\partial\lambda'^{\ \nu}_\alpha}\,\partial\lambda'^\nu_\alpha\right)(\bar{\epsilon}\gamma_\nu\lambda)\right]\,,\end{aligned} \tag{5.14}$$

where we used the relation (4.1). Let us insert (5.11) and (5.13) in (5.10); we get

$$-2ia\partial^\mu\left(\frac{\partial L}{\partial T_\nu{}^\mu}\left(\gamma_0\gamma_\nu{}^\lambda\right)_\alpha\right) = 0\,,$$

or

$$\bar{J}_{\mu\alpha} = +2ia\theta_\mu{}^\nu(\bar{\lambda}\gamma_\nu)_\alpha\,;$$

up to the factor $(-\frac{2}{a}i)$ this coincides with (5.1).

To show that after quantization the charge induced by (5.6) yields the transformation (3.5) is a forbidding task; it will be the subject of our further investigation.

6. Equation of motion

The formal equations of motion are the Euler-Lagrange equations given by (4.4)

$$\partial^\mu\frac{\partial L}{\partial\lambda'^{\ \mu}_\alpha} = \frac{\partial L}{\partial_\alpha}\,.$$

Notice, however, that in addition we have (5.5). Hence the equations of motion reduce to

$$\frac{\partial L}{\partial T_\mu{}^\nu}(\gamma_0\gamma_\mu\partial^\nu\lambda)_\alpha = 0\,,$$

or

$$\theta_\nu{}^\mu(R_\mu{}^\nu)_\alpha = 0 \tag{6.1}$$

with

$$(R_\mu{}^\nu)_\alpha \equiv (\gamma_0\gamma_\mu\partial^\nu\lambda)_\alpha\,. \tag{6.2}$$

Let us replace $\theta_\nu{}^\mu$ in (6.1) by (4.7). We get

$$(1 + A - \tfrac{1}{2}B + \tfrac{1}{2}A^2 + \tfrac{1}{3}C - \tfrac{1}{2}AB + \tfrac{1}{6}A^3)(R_\mu{}^\mu)_\alpha$$
$$- \mathrm{Tr}\,[(T - T^2 + AT + T^3 - AT^2 - \frac{1}{2}BT + \frac{1}{2}A^2T)R_\alpha] = 0\,. \quad (6.3)$$

As we shall see this expression can be considerably simplified. To execute this simplification, however, we have to do some manipulations performed exactly on (6.3).

7. Some useful identities

The considerations of this and next sections are based on purely classical considerations, as in quantum field theory the components of the field do not in general commutate with each other and a product of fields at one point is not a well defined quantity. Nevertheless, we hope that this classical considerations will still be of some use in finding a suitable starting point for attacking the problem of quantization of the model.

The identities found here will simplify considerably the expressions of the equations of motion, of the energy momentum tensor as well as of the supercurrent.

Notice that A, B, C, D given by (3.4) involve one, two, three, four components of the field λ at the same point x resp. Since these components anticommute with each other every monomial of degree higher than four in components of λ at the same Minkowski point must vanish. For the derivatives of λ this is also true but for monomials of degree higher than sixteen.

Let us multiply (6.3) by $P_4 \neq 0$ a polynomial in $\lambda_\alpha(x)$ of 4-th order. Then we get

$$P_4(x)(R_\mu{}^\mu)_\alpha(x) = P_4 \mathrm{Tr}\,(R_\alpha) = 0\,. \quad (7.1)$$

Multiplying (6.3) by P_3 we get

$$P_3 \mathrm{Tr}\, R_\alpha = P_3 \mathrm{Tr}\,(TR_\alpha) \quad (7.2)$$

as

$$P_3 A R_\mu{}^\mu = 0$$

according to (7.1). Multiplying (6.3) by P_2 we get

$$P_2(1 + A)(R_\mu{}^\mu)_\alpha = P_2 \mathrm{Tr}\,[(T - T^2 + AT)R_\alpha]\,. \quad (7.3)$$

According to (7.2)

$$P_2 A(R_\mu{}^\mu)_\alpha = P_2 A \mathrm{Tr}\,(TR_\alpha) = P_2 \mathrm{Tr}\,(ATR_\alpha)\,.$$

Consequently (7.3) yields

$$P_2 \mathrm{Tr}\, R_\alpha = P_2 \mathrm{Tr}\, [(T - T^2) R_\alpha] \,. \tag{7.4}$$

Multiplying (6.3) by P_1 and making use of (7.1), (7.2) and (7.4) we get

$$P_1(1 + A - \frac{1}{2}B + \frac{1}{2}A^2)(R_\mu{}^\mu)_\alpha$$
$$= P_1 \mathrm{Tr}\, [(T - T^2 + AT + T^3 - AT^2 - \frac{1}{2}BT + \frac{1}{2}A^2 T) R_\alpha] \,. \tag{7.5}$$

Since

$$P_1 A (R_\mu{}^\mu)_\alpha = P_1 \mathrm{Tr}\, [(AT - AT^2) R_\alpha]$$
$$-\tfrac{1}{2} P_1 B (R_\mu{}^\mu)_\alpha = -\tfrac{1}{2} P_1 \mathrm{Tr}\, (BTR_\alpha)$$
$$\tfrac{1}{2} P_1 A^2 (R_\mu{}^\mu)_\alpha = \tfrac{1}{2} P_1 \mathrm{Tr}\, (A^2 T R_\alpha)$$

some of the term in (7.5) cancel and we get

$$P_1(\mathrm{Tr}\, R_\alpha) = P_1 \mathrm{Tr}\, [(T - T^2 + T^3) R_\alpha] \,. \tag{7.6}$$

We may comprise formulae (7.1), (7.2), (7.4) and (7.6) in one formula

$$P_j \mathrm{Tr}\, [(1 - T + T^2 + T^3) R_\alpha] = 0 \qquad j = 1, 2, 3, 4 \,. \tag{7.7}$$

From (7.6) follows when we there replace P_1 by λ_α that

$$A - B + C - D = 0 \tag{7.8}$$

as

$$\lambda_\alpha (R_\mu{}^\mu)_\alpha = \bar{\lambda} \gamma_\mu \partial^\mu \lambda = A \quad etc.$$

There are further identities implied by (7.8). We have

$$2A(A - B + C - D) = 2A^2 - 2AB + 2AC = 0 \tag{7.9}$$

as well as

$$(A - B + C - D)^2 = A^2 + B^2 - 2AB + 2AC = 0 \,. \tag{7.10}$$

Subtracting (7.9) from (7.10) we get

$$A^2 = B^2 \,. \tag{7.11}$$

The relation

$$B(A - B + C - D) = BA - B^2 = 0$$

together with (7.11) yields

$$A^2 = B^2 = AB \,. \tag{7.12}$$

Let us insert (7.12) in (7.10); we get

$$AC = 0 \,. \tag{7.13}$$

Multiplying (7.12) by A we get

$$A^3 = 0 \,. \tag{7.14}$$

and

$$A^2 B = 0 \,. \tag{7.15}$$

8. Simplified expressions for the equations of motions, energy-momentum tensor and the supercurrent

The first step in simplifying (6.3) is to use (7.12) and (7.14). We get

$$\mathrm{Tr}\,[(1 + A - \tfrac{1}{2}B + \tfrac{1}{3}C - T + T^2 - AT - T^3 + AT^2 + \tfrac{1}{2}BT)R_\alpha] = 0 \quad (8.1)$$

since

$$\tfrac{1}{2}A^2\mathrm{Tr}\,(TR)_\alpha = \tfrac{1}{2}B^2\mathrm{Tr}\,(TR)_\alpha = 0\,.$$

Formula (8.1) can be rewritten as follows

$$\mathrm{Tr}\,\{[(1 - T + T^2 - T^3) + A(1 - T + T^2) - \tfrac{1}{2}B(1 - T) + \tfrac{1}{3}C]R_\alpha\} = 0$$

or using (7.7)

$$\mathrm{Tr}\,\{[(1 - T + T^2 - T^3) + AT^3 + \tfrac{1}{2}BT^2 + \tfrac{1}{3}CT]R_\alpha\} = 0\,. \quad (8.2)$$

Now from $AD = 0$ follows

$$\frac{\partial AD}{\partial\lambda_\alpha} = \frac{\partial AD}{\partial T_\mu{}^\nu}(R_\mu{}^\nu)_\alpha = \mathrm{Tr}\,[(D + 4AT^3)R_\alpha] = 4\mathrm{Tr}\,(AT^3R_\alpha) = 0\,, \quad (8.3)$$

and from $BC = 0$

$$\frac{\partial BC}{\partial\lambda_\alpha} = \mathrm{Tr}\,[(2CT + 3BT^2)R_\alpha] = 0\,. \quad (8.4)$$

The justification of the formulae (8.3) and (8.4) is given in Appendix D. In view of (8.3) and (8.4) relation (8.2) reduces to

$$\mathrm{Tr}\,[(1 - T + T^2 - T^3)R_\alpha] = 0\,. \quad (8.5)$$

Equations (8.5) are just the simplified equations of motion. From $\mathrm{Tr}\,(T^j) = 0$ for $j \geq 5$ follows that

$$\frac{\partial\mathrm{Tr}\,(T^j)}{\partial\lambda_\alpha} = j\mathrm{Tr}\,(T^{j-1}R_\alpha) = 0\,. \quad (8.6)$$

By virtue of (8.6), equations (8.5) can be also written formally

$$\mathrm{Tr}\,\left(\frac{1}{1+T}R_\alpha\right) = 0\,. \quad (8.7)$$

For $a \to 0$ we get the free field equation (4.11).

As far as the energy momentum tensor is concerned the simplification amounts to

$$\begin{aligned}\theta_\mu{}^\nu = &+ \tfrac{1}{a^2}(1 + A - \tfrac{1}{2}B + \tfrac{1}{3}C)g_\mu{}^\nu \\ &- \tfrac{1}{a^2}[T - T^2 + T^3 + A(T - T^2) - \tfrac{1}{2}BT]_\mu{}^\nu \,. \end{aligned} \tag{8.8}$$

Relation (8.7) can be used in finding the formal expression *e.g.* for the incoming field. We have

$$\lambda^{in}(x) = \lambda(x) - \int \theta(x-y)S(x-y)\gamma^\mu\partial_\mu\lambda(y)\,dy\,,$$

where

$$S(x) = -\gamma_\nu\partial^\mu\Delta(x)$$

with $\Delta(x)$ denoting the Pauli–Jordan distribution. From (8.7) we get

$$\gamma^\mu\partial_\mu\lambda = \mathrm{Tr}\,[(T - T^2 + T^3 - T^4)\gamma_0 R]_\alpha \equiv r_\alpha\,.$$

Consequently

$$\lambda^{\mathrm{in}}(x) = \lambda - \int \theta(x-y)S(x-y)\mathrm{Tr}\,[(T - T^2 + T^3 - T^4)\gamma_0 R]_\alpha(y)dy\,.$$

To get a simplified expression for the supercurrent (5.1) we have to make use of (8.8); after inserting it in (5.1) we obtain

$$\begin{aligned}\overline{J}_{\mu\alpha} = &-(1 + A - \tfrac{1}{2}B + \tfrac{1}{3}C(\overline{\lambda}\gamma_\mu)_\alpha \\ &+ [T - T^2 + T^3 + A(T - T^2) - \tfrac{1}{2}BT]_\mu{}^\nu(\overline{\lambda}\gamma_\nu)_\alpha\,.\end{aligned}$$

Let us denote

$$1 + A - \tfrac{1}{2}B + \tfrac{1}{3}C - \tfrac{1}{4}D \equiv V = ((1 + \mathrm{Tr}\,[\ln(1+T)])\,.$$

Then

$$\overline{J}_{\mu\alpha} = -V(1 - T + T^2 - T^3 + T^4)_\mu{}^\nu(\overline{\lambda}\gamma_\nu)_\alpha \tag{8.9}$$

or formally

$$J_{\mu\alpha} = \left(\frac{V}{1+T}\right)_\mu^{\;\nu}(\gamma_\nu\lambda)_\alpha\,. \tag{8.10}$$

According to our remarks in Section 2 (see also (5.9)) only

$$\tfrac{1}{4}(\gamma^\mu J_\mu)_\alpha = t_\alpha \tag{8.11}$$

contributes on the mass shell and therefore taking for granted that (8.10) is in some sense valid in quantum theory

$$t_\alpha = \frac{1}{4}\left(\frac{V}{1+T}\right)_\mu^{\ \nu}(\gamma^\mu\gamma_\nu\lambda)_\alpha \tag{8.12}$$

is responsible for the appearance of the Goldstone particle.

Appendix A

$$L = -\frac{1}{a^2}\det W$$

$$W_\mu^\nu = g_\mu{}^\nu - ia^2(\bar{\lambda}\gamma_\mu\partial^\nu\lambda) = g_\mu{}^\nu + T_\mu{}^\nu$$

$$\det W = \epsilon^{\lambda_0\lambda_1\lambda_2\lambda_3} W_{\lambda_0}{}^0 W_{\lambda_1}{}^1 W_{\lambda_2}{}^2 W_{\lambda_3}{}^3$$

$$= \epsilon^{\lambda_0\lambda_1\lambda_2\lambda_3}(g_{\lambda_0}{}^0 + T_{\lambda_0}{}^0)(g_{\lambda_1}{}^1 + T_{\lambda_1}{}^1)(g_{\lambda_2}{}^2 + T_{\lambda_2}{}^2)(g_{\lambda_3}{}^3 + T_{\lambda_3}{}^3)$$

(a) $$= 1 + \epsilon^{\lambda_0 123}T_{\lambda_0}{}^0 + \epsilon^{0\lambda_1 23}T_{\lambda_1}{}^1 + \epsilon^{01\lambda_2 3}T_{\lambda_2}{}^2 + \epsilon^{012\lambda_3}T_{\lambda_3}{}^3$$

(b) $$+ \underbrace{\epsilon^{\lambda_0\lambda_1 23}T_{\lambda_0}{}^0 T_{\lambda_1}{}^1 + \epsilon^{\lambda_0 1\lambda_2 3}T_{\lambda_0}{}^0 T_{\lambda_2}{}^2}_{\binom{4}{2}=6\text{ choices}} + \cdots\cdots$$

(c) $$+ \underbrace{\epsilon^{\lambda_0\lambda_1\lambda_2 3}T_{\lambda_0}{}^0 T_{\lambda_1}{}^1 T_{\lambda_2}{}^2 + \epsilon^{\lambda_0\lambda_1 2\lambda_3}T_{\lambda_0}{}^0 T_{\lambda_1}{}^1 T_{\lambda_3}{}^3}_{\binom{4}{3}=4\text{ choices}} + \cdots$$

(d) $$+ \epsilon^{\lambda_0\lambda_1\lambda_2\lambda_3}T_{\lambda_0}{}^0 T_{\lambda_1}{}^1 T_{\lambda_2}{}^2 T_{\lambda_3}{}^3$$

ad (a) $$\epsilon^{\lambda_0 123}T_{\lambda_0}{}^0 + \cdots = T_\mu{}^\mu = A$$

ad (b) $$\epsilon^{\lambda_0\lambda_1 23}T_{\lambda_0}{}^0 T_{\lambda_1}{}^1 + \epsilon^{\lambda_0 1\lambda_2 3}T_{\lambda_0}{}^0 T_{\lambda_2}{}^2 + \cdots$$

$$= \sum_{\mu<\nu}(T_\mu{}^\mu T_\nu{}^\nu - T_\nu{}^\mu T_\mu{}^\nu) = \sum_{\mu\leq\nu}(T_\mu{}^\mu T_\nu{}^\nu - T_\nu{}^\mu T_\mu{}^\nu)$$

$$= \tfrac{1}{2}\sum_{\mu,\nu}(T_\mu{}^\mu T_\nu{}^\nu - T_\nu{}^\mu T_\mu{}^\nu) = \tfrac{1}{2}(A^2 - B)$$

ad (c) $$\epsilon^{\lambda_0\lambda_1\lambda_2 3}T_{\lambda_0}{}^0 T_{\lambda_1}{}^1 T_{\lambda_2}{}^2 + \epsilon^{\lambda_0\lambda_1 2\lambda_3}T_{\lambda_0}{}^0 T_{\lambda_1}{}^1 T_{\lambda_3}{}^3 + \cdots$$

$$= \sum_{\lambda<\mu\nu}(T_\lambda{}^\lambda T_\mu{}^\mu T_\nu{}^\nu + T_\mu{}^\lambda T_\nu{}^\mu T_\lambda{}^\nu + T_\nu{}^\lambda T_\lambda{}^\mu T_\mu{}^\nu$$

$$- T_\nu{}^\lambda T_\mu{}^\mu T_\lambda{}^\nu - T_\mu{}^\lambda T_\lambda{}^\mu T_\nu{}^\nu - T_\lambda{}^\lambda T_\nu{}^\mu T_\mu{}^\nu)$$

$$\sum_{\lambda\leq\mu\leq\nu}(\quad) = \tfrac{1}{6}\sum_{\lambda,\mu,\nu}(\quad) = \tfrac{1}{6}(A^3 + 2C - 3AB)$$

$$ad\ \text{(d)} \qquad \epsilon^{\lambda_0\lambda_1\lambda_2\lambda_3}T_{\lambda_0}{}^0T_{\lambda_1}{}^1T_{\lambda_2}{}^2T_{\lambda_3}{}^3$$

$$= \tfrac{1}{24}\epsilon^{\lambda_0\lambda_1\lambda_2\lambda_3}\epsilon_{\kappa_0\kappa_1\kappa_2\kappa_2\kappa_3}T_{\lambda_0}{}^{\kappa_0}T_{\lambda_1}{}^{\kappa_1}T_{\lambda_2}{}^{\kappa_2}T_{\lambda_3}{}^{\kappa_3}$$

$$= -\tfrac{1}{24}\begin{vmatrix} g_{\kappa_0}{}^{\lambda_0} & g_{\kappa_0}{}^{\lambda_1} & g_{\kappa_0}{}^{\lambda_2} & g_{\kappa_0}{}^{\lambda_3} \\ g_{\kappa_1}{}^{\lambda_0} & \cdots & & \\ g_{\kappa_2}{}^{\lambda_0} & \cdots & & \\ g_{\kappa_3}{}^{\lambda_0} & \cdots & & \end{vmatrix} T_{\lambda_0}{}^{\kappa_0}T_{\lambda_1}{}^{\kappa_1}T_{\lambda_2}{}^{\kappa_2}T_{\lambda_3}{}^{\kappa_3}$$

$$= \frac{1}{24}(-6D + 8AC + 3B^2 - 6A^2B - A^4)\,.$$

Appendix B

$$g_\mu{}^\nu \det W = \frac{\partial \det W}{\partial W_\kappa{}^\mu}W_\kappa{}^\nu\,;\ \text{hence}\ \det W = \frac{1}{4}\frac{\partial \det W}{\partial W_\kappa{}^\mu}W_\kappa{}^\mu$$

(each term in quadratic in $W_\kappa{}^\mu$).

$$g_\mu{}^\nu \det W = \frac{\partial \det W}{\partial T_\kappa{}^\mu}(g_\kappa{}^\nu + T_\kappa{}^\nu) = \frac{\partial \det W}{\partial T_\nu{}^\mu} + \frac{\partial \det W}{\partial T_\kappa{}^\mu}T_\kappa{}^\nu$$

$$g_\mu{}^\nu L = \frac{\partial L}{\partial T_\nu{}^\mu} + \frac{\partial L}{\partial T_\lambda{}^\mu}T_\lambda{}^\nu\,.$$

Notice that

$$\frac{\partial L}{\partial \lambda'_\alpha{}^\mu}\lambda'_\alpha{}^\nu = \frac{\partial L}{\partial T_\kappa{}^\lambda}\frac{\partial T_\kappa{}^\lambda}{\partial \lambda'_\alpha{}^\mu}\lambda'_\alpha{}^\nu = -ia^2\frac{\partial L}{\partial T_\kappa{}^\mu}(\bar\lambda\gamma_\kappa)_\alpha\lambda'_\alpha{}^\nu = \frac{\partial L}{\partial T_\kappa{}^\mu}T_\kappa{}^\nu\,.$$

Thus

$$g_\mu{}^\nu L = \frac{\partial L}{\partial T_\nu{}^\mu} + \frac{\partial L}{\partial \lambda'_\alpha{}^\mu}\lambda'_\alpha{}^\nu\,.$$

Appendix C

Hermiticity: The expression

$$(i\bar\lambda\gamma_\mu\partial^\mu\lambda)^+ = -i(\partial^\mu\lambda)_\beta(\gamma_0\gamma_\mu)_{\alpha\beta}\lambda_\alpha = i\bar\lambda\gamma_\mu\partial^\mu\lambda$$

is hermitean but

$$\left[i\frac{\partial(\bar\lambda\gamma_\mu\partial^\mu\lambda)}{\partial\lambda_\alpha}\right]^+ = [-i(\gamma_0\gamma_\mu\partial^\mu\lambda)_\alpha]^+ = i(\gamma_0\gamma_\mu\partial^\mu\lambda)_\alpha = -i\frac{\partial(\bar\lambda\gamma_\mu\partial^\mu\lambda)}{\partial\lambda_\alpha}$$

is antihermitean. Also

$$i\frac{\partial(\bar{\lambda}\gamma_\mu\partial^\mu\lambda)}{\lambda'^{\ \nu}_\alpha}$$

is anitihermitean.

Differentiation: The rule

$$\partial_\mu L = \frac{\partial L}{\partial\lambda_\alpha}\lambda_{\alpha,\mu} + \frac{\partial L}{\partial\lambda_{\alpha\nu}}\lambda_{\alpha,\mu\nu}$$

is correct. To see this let us examine a simple example. For

$$A = \lambda_\beta\lambda_\gamma\partial^\rho\lambda_\delta\,,$$

we have

$$\frac{\partial A}{\partial u} = \lambda'_\beta\lambda_\gamma\partial^\rho\lambda_\delta + \lambda_\beta\lambda'_\gamma\partial^\rho\lambda_\delta + \lambda_\beta\lambda_\gamma\partial^\rho\lambda'_\delta\,,$$

where $\lambda' = \frac{d\lambda}{du}$. Now

$$\frac{\partial A}{\partial\lambda_\alpha} = \delta_{\alpha\beta}\lambda_\gamma\partial^\rho\lambda^\delta - \lambda_\beta\delta_{\alpha\gamma}\partial^\rho\lambda_\delta\,,$$

$$\frac{\partial A}{\partial\lambda_{\alpha,\nu}} = \lambda_\beta\gamma_\lambda g_\rho{}^\nu\delta_{\alpha\delta}\,.$$

Thus

$$\frac{\partial A}{\partial\lambda_\alpha}\lambda'_\alpha + \frac{\partial A}{\partial\lambda_{\alpha,\nu}}\lambda'_{\alpha,\nu} = \frac{\partial A}{\partial u}\,.$$

Appendix D

(i) From Tr $(T^5) = 0$ (This vanishes as λ appears here 5 times) follows

$$\frac{\partial\mathrm{Tr}\,(T^5)}{\partial\lambda_\alpha} = \mathrm{Tr}\,(T^4 R_\alpha) = 0 \qquad (R_\mu{}^\nu)_\alpha \equiv ia^2(\gamma_0\gamma_\mu\partial^\mu\lambda)_\alpha\,. \qquad \text{(D.1)}$$

(ii) For $AD = 0$

$$\frac{\partial AD}{\partial\lambda_\alpha} = \frac{\partial AD}{\partial T_\mu{}^\nu}\frac{\partial T_\mu{}^\nu}{\partial\lambda_\alpha} = +ia^2\frac{\partial AD}{\partial T_\mu{}^\nu}(\gamma_0\gamma_\mu\partial^\nu\lambda)_\alpha = \frac{\partial AD}{\partial T_\mu{}^\nu}(R_\mu{}^\nu)_\alpha$$

$$= (g_\nu{}^\mu D + 4A(T^3)_\nu{}^\mu)(R_\mu{}^\nu)_\alpha = \mathrm{Tr}\,(R)_\alpha D + 4A\mathrm{Tr}\,(T^3R)_\alpha = 0\,. \qquad \text{(D.2)}$$

(iii) From $BC = 0$

$$\frac{\partial BC}{\partial \lambda_\alpha} = 2\mathrm{Tr}\,(TR)_\alpha C + 3B\mathrm{Tr}\,(T^2R)_\alpha = 0\,. \tag{D.3}$$

For better understanding of *(i)*, *(ii)*, *(iii)* we make the procedure plain on the following example.

Consider the case of $(T^3)_\mu{}^\sigma$ in a $s+1=d=2$-dimensional space. We have

$$(T^3)_\mu{}^\sigma = \lambda_\alpha R_\alpha \lambda_\beta R'_\beta \lambda_\gamma R'' = 0\,, \tag{D.4}$$

where

$$\begin{aligned} R_\alpha &= (R_\mu{}^\nu)_\alpha = (\gamma_0\gamma_\mu\partial^\nu\lambda)_\alpha\,,\\ R'_\alpha &= (R_\nu{}^\rho)_\alpha\,,\\ R''_\alpha &= (R_\rho{}^\sigma)_\alpha \end{aligned}$$

as λ_α appears in (D.4) three times. To check that

$$\frac{\partial (T^3)_\mu{}^\sigma}{\partial\lambda_1} = -R_1(T^2)_\nu{}^\sigma - T_\mu{}^\nu R'_1 T_\rho{}^\sigma - (T^2)_\mu{}^\rho R''_1 \tag{D.5}$$

vanishes let us write it out in extension

$$\begin{aligned} R_1(\lambda_1R'_1 + \lambda_2R'_2)(\lambda_1R''_1 + \lambda_2R''_2) &+ R'_1(\lambda_1R_1+\lambda_2R_2)(\lambda_1R''_1+\lambda_2R''_2)\\ &+ R''_1(\lambda_1R_1+\lambda_2R_2)(\lambda_1R'_1+\lambda_2R'_2)\\ &= R_1\lambda_1R'_1\lambda_2R''_2 + R_1\lambda_2R'_2\lambda_1R''_1 + R'_1\lambda_1R_1\lambda_2R''_2 + R'_1\lambda_2R_2\lambda_1R''_1\\ &+ R'_1\lambda_1R_1\lambda_2R'_2 + R'_1\lambda_2R_2\lambda_1R'_1 = 0\,. \end{aligned}$$

In particular

$$\frac{\partial \mathrm{Tr}\,(T^3)}{\partial\lambda_1} = -3\mathrm{Tr}\,(R_1T^2) = 0\,.$$

Further

$$\begin{aligned} \frac{\partial^2 (T^3)_\mu{}^\sigma}{\partial{\lambda_1}^2} = &- R_1R'_1\lambda_2R''_2 - R_1\lambda_2R'_2R''_1 - R'_1R_1\lambda_2R''_2\\ &- R'_1\lambda_2R_2R''_1 - R''_1R_1\lambda_2R'_2 - R'_1\lambda_2R_2R'_1 = 0\,. \end{aligned}$$

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