NUMERICAL RENORMALIZATION GROUP STUDY OF TWO-LEVEL KONDO EFFECT: DISCOVERY OF NEW FIXED POINT*

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The model of two-level Kondo effect is studied by the Wilson numerical renormalization group method. It is shown that there exist a new type of weak-coupling fixed point other than the strong-coupling fixed point found by Vladar and Zawadowski two decades ago by means of the one-loop and two-loop renormalization group methods.

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1. Introduction

The two-level Kondo (TLK) effect was first studied by Kondo more than two decades ago by means of a perturbation calculation [1]. In early 80's, Vladar–Zawadowski (VZ) showed on the basis of the poorman's scaling [2] and two-loop renormalization group [3] methods that the fixed point of the TLK model is given by that of the pseudo-spin-1/2 magnetic Kondo model. This problem attracted renewed interests in mid 80's in relation to the two-channel Kondo (TCK) effect which is characterized by the non-Fermi liquid fixed point [4–6]. It is because the Hamiltonian at strong-coupling fixed point of TLK system is inevitably mapped to the TCK Hamiltonian if the spin-degrees of freedom are read as the channel-degrees of freedom. The purpose of the present contribution is to investigate the nature of the fixed point of TLK system itself on a more solid calcualtion using the Wilson numerical renormalization group (NRG) method.

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2. Model

We use the same model Hamitonian as that adopted by VZ [2]:

$$H = \sum_{\vec{k}} \varepsilon_k a^\dagger_{\vec{k}} a_{\vec{k}} + \sum_{i=x,y,z} \Delta^i \tau^i + 2 \sum_{\vec{k}_1,\vec{k}_2} \sum_{i=x,y,z} (a^\dagger_{\vec{k}_2} V^i_{\hat{k}_2,\hat{k}_1} a_{\vec{k}_1}) \tau^i \,, \tag{1}$$

where $a_{\vec{k}}$ denotes annhilation operator of the conduction electron with the wavevector $\vec{k}$, τ^i the i-th component of the pseudo spin describing the two-level (TL) degrees of freedom, and the coupling V's are given as follows:

$$V^z_{\hat{k}_2,\hat{k}_1} = \frac{\mathrm{i}V_z}{4\pi}(a_0 + 3a_1 \cos\theta_{\hat{k}_2\hat{k}_1})(\hat{k}^z_1 - \hat{k}^z_2) \,, \tag{2}$$

$$V^x_{\hat{k}_2,\hat{k}_1} = \frac{V_x}{4\pi}(a_0 + 3a_1 \cos\theta_{\hat{k}_2\hat{k}_1})(\hat{k}^z_1 - \hat{k}^z_2)^2 \,, \tag{3}$$

and $V^y_{\hat{k}_2,\hat{k}_1} = 0$, with $V_z \equiv k_\mathrm{F} d/2$ and $V_x = (k_\mathrm{F} d)^2 \Delta_0 \lambda / V_\mathrm{B}$. Otherwise, the notations used in (2) and (3) are the same in Ref. [2]. We restrict the partial wave describing the conduction electrons within $\ell = 0,1,2,3$ and $m = 0$, m being the z-component of the angular momentum ℓ, around the TL system as in Ref. [2].

3. Results

Energy flow diagrams are classified into two types as shown in Figs. 1 and 2. Fig. 1 is for the parameter set, $V_z = 1$, $V_x = 0.01$, $a_0 = 0.6$, and $a_1 = 1.2$, and corresponds to the strong-coupling fixed point although it shows no even-odd alternation just as in the TCK effect. This is because the conduction electrons have 4-channel corresponding to partial wave $\ell = 0,1,2,3$, two of which are trapped by the TL system. The excitation energies and the degeneracy of each energy levels (shown in parentheses) can be explained completely by this picture. Small splitting of the energy levels is caused by the growth of the potential scattering term. Such a phenomenon occurs also in the sd-model if the potential scattering term is added to the model. The scaling unit Λ is set as $\Lambda = 3$ throughout this paper. The number of retained states at each step is 400. Fig. 2 is for the parameter set, $V_z = 1$, $V_x = 0.01$, $a_0 = 0.6$, and $a_1 = 0.4$, and represents the weak-coupling fixed point because it exhibits the even-odd alternation and can be understood as the coupling between conduction electrons and TL system is absent. Indeed, the degeneracy (shown in parentheses) can be completely explained by this weak-coupling assumption.

We have searched fixed points for a wide range of parameter sets, and determined the phase boundary between two regimes of weak- and strong-coupling fixed point as shown in Fig. 3. Although we have set $\Delta^i = 0$ here,

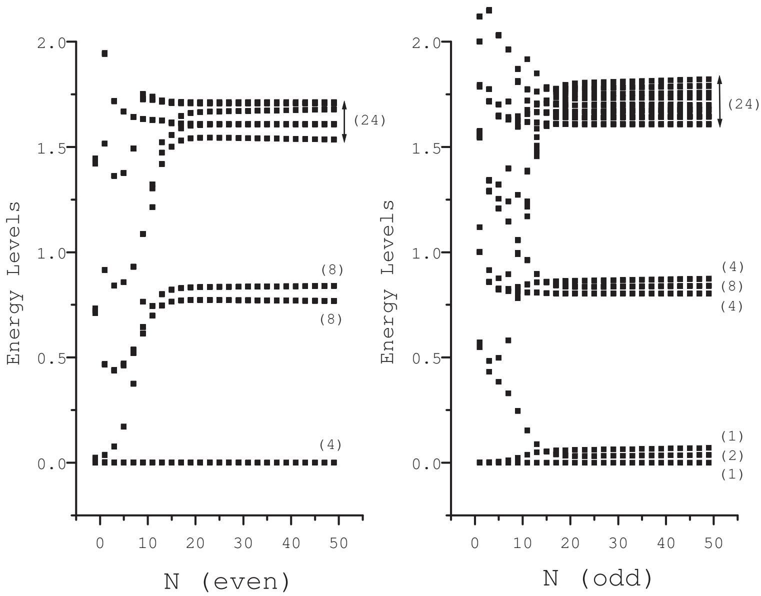

Fig. 1. Energy flow diagram for the parameter set, $V_z = 1$, $V_x = 0.01$, $a_0 = 0.6$, and $a_1 = 1.2$. The number in parentheses is the degeneracy.

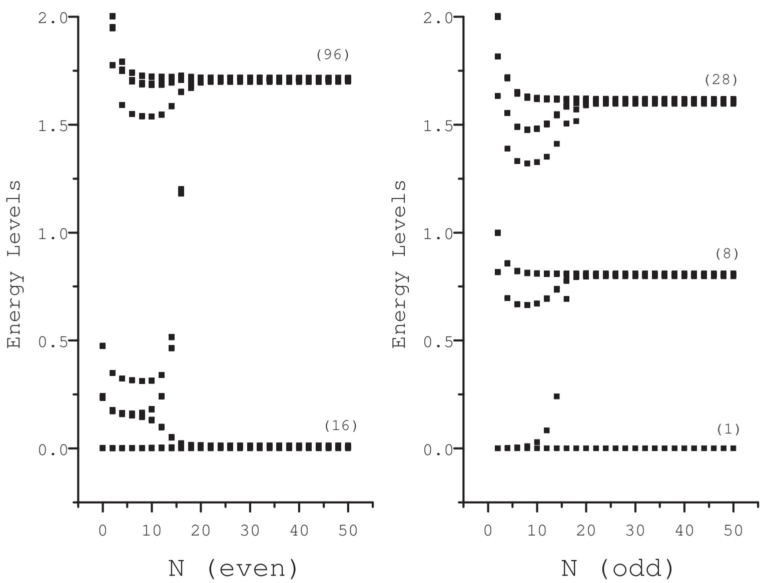

Fig. 2. Energy flow diagram for the parameter set, $V_z = 1$, $V_x = 0.01$, $a_0 = 0.6$, and $a_1 = 0.4$. The number in parentheses is the degeneracy.

the effect of finite field Δ^i does not change the phase diagram qualitatively unless $|\Delta^x| > |V_x|$. The weak-coupling region is located along the crossover regime of the two types of strong-coupling fixed point of the poorman's scaling equations given originally by VZ [2]. One type of the strong-coupling fixed points has already been found by VZ on the basis of a physical intuition, while another type of strong coupling fixed point is found for the frst time in this study by solving numerically the coupled scaling equations of 48 components:

$$\frac{\partial V^s_{\alpha\beta}}{\partial x} = -2\mathrm{i}\sum_{i,j}\epsilon^{ijs}\sum_{\gamma}V^i_{\alpha\gamma}(x)V^j_{\gamma\beta}(x), \tag{4}$$

where $V^i_{\alpha\beta}$'s are the matrix elements of $V^i_{\hat{k}_2\hat{k}_1}$, (2) and (3), in the space of $\ell = 0, 1, 2, 3$ and $m = 0$, and the scaling variable $x \equiv \ln(D_0/D)$ [2]. Both of the strong-coupling fixed points are characterized by the pseudo-spin-1/2 Kondo effect. It is noted that the region of weak-coupling fixed point is found only through the NRG analysis. Implication of new fixed point will be discussed elsewhere.

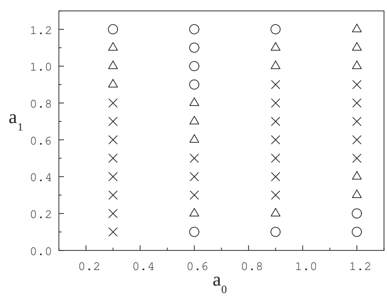

Fig. 3. Distribution of strong-coupling (circle) and weak-coupling (cross) fixed point. The points presented by triangle cannot be identified clearly with each fixed point due to numerical uncertainty.

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