DZYALOJINSKI-MORIYA INTERACTION IN $S = 1/2$ LADDER*

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Using a Majorana fermion representation, we discuss the influence of
Dzyaloshinski-Moriya interaction on the magnetic properties of a spin-1/2
ladder. We calculate the spin-echo decay rate and analyze the modifications
with respect to the isotropic ladder. Implications of our calculations for
experiments on ladder systems are discussed.

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1. Introduction

Recent years have seen a considerable effort in understanding the prop-
erties of low-dimensional quantum spin systems. The most notable example
of such systems are the spin ladders [1], whose properties are quite well
described by the isotropic Heisenberg model on a ladder. However, recent
electron spin resonance measurements (ESR) [2] on Sr$_4$Cu$_2$O$_9$ and experi-
ments on the compound CaCu$_2$O$_2$ [3] suggest the necessity of taking into
account spin anisotropies for an accurate description of the spin dynamics
in these systems. For spin 1/2 the leading anisotropy terms are of the form
Dzyaloshinski-Moriya (DM) [4]. In this paper we calculate the slowly varying
part of spin-spin correlation functions by means of a Majorana fermion
representation of the spin ladder model. In particular, we analyze the tem-
perature dependence of the Gaussian spin-echo decay rate $T_{2G}^{-1}$ and discuss
implications of the results for experiments.

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2. Model and spin–spin correlation function

The model of two weakly spin-coupled antiferromagnetic (AF) $S = 1/2$ Heisenberg chains with DM interaction along the rungs is:

$$H = H = J_{\parallel} \sum_{j=1,2,i} S_{j,i} \cdot S_{j,i+1} + J_{\perp} \sum_{i} S_{1,i} \cdot S_{2,i} + D \sum_{i} (S_{1,i} \times S_{2,i}), \quad (1)$$

where $J_{\parallel,\perp} > 0$ is the intra (inter)-chain AF interaction, $D$ is the DM vector and we use the quantization axis $\hat{z}$ for the spins such that $D = D\hat{z}$. This model is specially pertinent the experimental situation in CaCu$_2$O$_3$ [3]. Using the gauge transformation [6]:

$$S_{i,1}^+ = e^{-i\delta} \bar{S}_{i,1}^+; \quad S_{i,2}^+ = e^{i\delta} \bar{S}_{i,2}^+, \quad (2)$$

where $2\alpha = \text{arctan}(D/J_{\perp})$, the Hamiltonian $H$ is reduced to the simpler form:

$$H = \sum_{i} J_{\parallel} (\bar{S}_{1,i} \bar{S}_{1,i+1} + \bar{S}_{2,i} \bar{S}_{2,i+1}) + J_{\perp} (\bar{S}_{1,i}^\rho \bar{S}_{1,i}^\rho + \bar{S}_{1,i}^\sigma \bar{S}_{1,i}^\sigma) + J_{\perp} \bar{S}_{1,i}^\rho \bar{S}_{1,i}^\rho,$$

where $J_{\perp} = \sqrt{J_{\parallel}^2 + D^2 \text{sign}(J_{\perp})}$. For weak interchain $J_{\perp}$, we analyze the magnetic properties of the Hamiltonian (3) by using bosonization approach [5] and the Majorana fermion representation originally invoked by Shelton et al. [7] for the isotropic ladder. Based on this approach, we map the Hamiltonian (3) into that of four decoupled massive Majorana fermions $\xi_{\nu}^a (a = 1, \ldots, 4)$ where the index $\nu = L, R$ refers to right and left moving Majorana fermions respectively:

$$H = -\frac{im}{2} \sum_{a=1}^{4} \int dx \{ \xi_{\nu}^a \partial_x \xi_{\nu}^a - \xi_{\nu}^a \partial_x \xi_{\nu}^a \} - \frac{m_{\text{eff}}}{e} \xi_{\nu}^a \xi_{\nu}^a \xi_{\nu}^a \xi_{\nu}^a \}. \quad (3)$$

The spectrum is composed of a Majorana doublet $(\xi_1^L, \xi_2^L)$, $(\nu = L, R)$, with mass $m_{1,2} = m = J_{\perp}/2\pi$ and two singlets $\xi_0^3, \xi_0^4$ of masses $m_3 = (J_{\perp} - \frac{J_{\parallel}}{2\pi})$, $m_4 = -(\frac{J_{\parallel}}{2\pi} + \frac{J_{\perp}}{2\pi})$. In the isotropic ladder [7] the spectrum would instead be formed of a triplet $\xi_a^L, a = 1, 2, 3$ with mass $m$ and a singlet $\xi_0^4$ with a larger mass, $3|m|$ [7]. In the presence of a DM interaction, the loss of SU(2) symmetry thus manifests into the breaking of the triplet degeneracy.

To calculate the spin-spin correlation functions, we need to evaluate the spin density components along the three spatial directions $(1, 2, 3)$. These are expressed in terms of Majorana fermions operators via the relations:
\[ J_{\mu \nu}^1 = -i \cos \alpha (\xi^2 \xi_{\mu \nu}^3 - (-)^a \xi_{\mu \nu}^1 \xi^4) + i \sin \alpha (\xi^2 \xi_{\mu \nu}^4 + (-)^a \xi_{\mu \nu}^1 \xi^3), \]
\[ J_{\mu \nu}^2 = -i \cos \alpha (\xi^2 \xi_{\mu \nu}^3 - (-)^a \xi_{\mu \nu}^1 \xi^4) - i \sin \alpha (\xi^2 \xi_{\mu \nu}^4 + (-)^a \xi_{\mu \nu}^1 \xi^3), \]
\[ J_{\mu \nu}^3 = -i \xi_{\mu \nu}^1 \xi_{\mu \nu}^2 - i (-)^a \xi_{\mu \nu}^1 \xi_{\mu \nu}^3, \]  
where \( a = 1, 2 \) is the chain index. Differently from the isotropic ladder, the isotropic in-plane components are mixed by the DM interaction. The generalized spin susceptibility is a tensor:

\[ \chi^{AB}(q, i\omega_n) = \sum_{\mu, \nu = R, L} \sum_{a = 1, 2} \int d\tau dx e^{i(\omega_n - q^a \tau)} \langle T_{\tau} J_{\mu \nu}^A(x, \tau) J_{\nu \mu}^B(0, 0) \rangle. \]  

The Gaussian spin-echo decay rate is determined by indirect nuclear spin-spin interaction induced by spin fluctuations and can be calculated as [8]:

\[ T_{2G}^{-1} \propto \sum_q \chi(q, 0)^2, \]  
where a summation over the tensor components is understood. By using Eqs. (4), each tensor component can be expressed in terms of doublet and singlet and singlet-doublet correlation functions, that we denote by \( \Gamma^{\alpha\beta} \), as follows:

\[ \chi^{11}(q, i\omega_n) = [\cos^2 \alpha \Gamma^{23}(q, i\omega_n) - \sin^2 \alpha \Gamma^{24}(q, i\omega_n)] = \chi^{22}, \]
\[ \chi^{33}(q, i\omega_n) = \Gamma^{12}(q, i\omega_n), \]  
where

\[ \Gamma^{\alpha\beta}(q, i\omega_n) = -\beta^{-1} \sum_{\mu, \nu = R, L} \sum_k \sum_{i\omega_n'} G_{\mu \nu}^\alpha(k + q, i\omega_n + i\omega_n') G_{\nu \mu}^\beta(k, i\omega_n'). \]

Here \( G^{\alpha(\beta)} \) is the doublet or singlet Majorana fermion thermal Green's function. The explicit expression of (8) is obtained by the approach of Ref. [8]. In the limit \( T \to 0 \) we obtain a finite value of the spin susceptibility that can be ascribed to a Majorana fermion pair creation process with an explicit dependence on the DM interaction, whereas in the limit of large temperature, we have:

\[ \chi^{11}(q, 0) = \chi^{22}(q, 0) \simeq \cos(2\alpha)/T, \quad \chi^{33}(q, 0) \simeq 1/T, \]  
where only the third component is identical to the isotropic case.

In figure 1 we show the result of the full temperature dependence of \( T_{2G}^{-1} \) in presence of a DM interaction and without it. We observe a saturation
at low temperature that depends on the anisotropy. As an effect of DM interaction, the saturation value decreases. As evident from Eq. (9), the behavior of $T_2^{-1}$ could provide information on the DM interaction strength in experimental systems.

REFERENCES