DZYALOSHINSKI–MORIYA INTERACTION IN $S = 1/2$ LADDER*

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Using a Majorana fermion representation, we discuss the influence of Dzyaloshinski–Moriya interaction on the magnetic properties of a spin-1/2 ladder. We calculate the spin-echo decay rate and analyze the modifications with respect to the isotropic ladder. Implications of our calculations for experiments on ladder systems are discussed.

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1. Introduction

Recent years have seen a considerable effort in understanding the properties of low-dimensional quantum spin systems. The most notable example of such systems are the spin ladders [1], whose properties are quite well described by the *isotropic* Heisenberg model on a ladder. However, recent electron spin resonance measurements (ESR) [2] on $Sr_{14}Cu_{24}O_{41}$ and experiments on the compound $CaCu_2O_3$ [3] suggest the necessity of taking into account spin anisotropies for an accurate description of the spin dynamics in these systems. For spin 1/2 the leading anisotropy terms are of the form Dzyaloshinski–Moriya (DM) [4]. In this paper we calculate the slowly varying part of spin-spin correlation functions by means of a Majorana fermion representation of the spin ladder model. In particular, we analyze the temperature dependence of the Gaussian spin-echo decay rate T_{2G}^{-1} and discuss implications of the results for experiments.

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2. Model and spin–spin correlation function

The model of two weakly coupled antiferromagnetic (AF) $S = 1/2$ Heisenberg chains with DM interaction along the rungs is:

$$H = H = J_{||} \sum_{j=1,2,i} \boldsymbol{S}_{j,i} \cdot \boldsymbol{S}_{j,i+1} + J_{\perp} \sum_{i} \boldsymbol{S}_{1,i} \cdot \boldsymbol{S}_{2,i} + \boldsymbol{D} \cdot \sum_{i} (\boldsymbol{S}_{1,i} \times \boldsymbol{S}_{2,i}), \quad (1)$$

where $J_{||(\perp)} > 0$ is the intra(inter)-chain AF interaction, $\boldsymbol{D}$ is the DM vector and we use the quantization axis $\hat{z}$ for the spins such that $\boldsymbol{D} = D\hat{z}$. This model is specially pertinent the experimental situation in $\mathrm{CaCu_2O_3}$ [3]. Using the gauge transformation [6]:

$$S^{+}_{i,1} = \mathrm{e}^{-i\alpha} \tilde{S}^{+}_{i,1}; \quad S^{+}_{i,2} = \mathrm{e}^{i\alpha} \tilde{S}^{+}_{i,2}\,, \tag{2}$$

where $2\alpha = \arctan(D/J_{\perp})$, the Hamiltonian H is reduced to the simpler form:

$$H = \sum_{i} J_{||} (\tilde{\boldsymbol{S}}_{1,i} \tilde{\boldsymbol{S}}_{1,i+1} + \tilde{\boldsymbol{S}}_{2,i} \tilde{\boldsymbol{S}}_{2,i+1}) + \tilde{J}_{\perp} (\tilde{S}^{x}_{i,1} \tilde{S}^{x}_{i,2} + \tilde{S}^{y}_{i,1} \tilde{S}^{y}_{i,2}) + J_{\perp} \tilde{S}^{z}_{i,1} \tilde{S}^{z}_{i,2},$$

where $\tilde{J}_{\perp} = \sqrt{J_{\perp}^2 + D^2}\mathrm{sign}(J_{\perp})$. For weak interchain $J_{\perp}$, we analyze the magnetic properties of the Hamiltonian (3) by using bosonization approach [5] and the Majorana fermion representation originally invoked by Shelton *et al.* [7] for the isotropic ladder. Based on this approach, we map the Hamiltonian (3) into that of four decoupled massive Majorana fermions $\xi^{i}_{\nu} (i = 1, \ldots, 4)$ where the index $\nu =$ L, R refers to right and left moving Majorana fermions respectively:

$$H = -\frac{iu}{2} \sum_{a=1}^{4} \int dx \{ (\xi^a_R \partial_x \xi^a_R - \xi^a_L \partial_x \xi^a_L) - i m_a \xi^a_R \xi^a_L \}\,. \tag{3}$$

The spectrum is composed of a Majorana doublet $(\xi^1_{\nu}, \xi^2_{\nu})$, ($\nu =$ L,R), with mass $m_{1,2} = m = J'_{\perp}/2\pi$ and two singlets ξ^3_{ν}, ξ^4_{ν} of masses $m_3 = (\frac{\tilde{J}'_{\perp}}{\pi} - \frac{J'_{\perp}}{2\pi})$, $m_4 = -(\frac{\tilde{J}'_{\perp}}{\pi} + \frac{J'_{\perp}}{2\pi})$. In the isotropic ladder [7] the spectrum would instead be formed of a triplet ξ^a_{ν}, $a = 1, 2, 3$ with mass m and a singlet ξ^4_{ν} with a larger mass, $3|m|$ [7]. In the presence of a DM interaction, the loss of SU(2) symmetry thus manifests into the breaking of the triplet degeneracy.

To calculate the spin-spin correlation functions, we need to evaluate the spin density components along the three spatial directions $(1, 2, 3)$. These are expressed in terms of Majorana fermions operators via the relations:

$$
\begin{aligned}
J^1_{\nu a} &= -i\cos\alpha(\xi^2_\nu\xi^3_\nu - (-)^a\xi^1_\nu\xi^4_\nu) + i\sin\alpha(\xi^2_\nu\xi^4_\nu + (-)^a\xi^1_\nu\xi^3_\nu)\,, \\
J^2_{\nu a} &= -i\cos\alpha(\xi^3_\nu\xi^1_\nu - (-)^a\xi^2_\nu\xi^4_\nu) - i\sin\alpha(\xi^2_\nu\xi^4_\nu + (-)^a\xi^2_\nu\xi^3_\nu)\,, \\
J^3_{\nu a} &= -i\xi^1_\nu\xi^2_\nu - i(-)^a\xi^3_\nu\xi^4_\nu\,,
\end{aligned}
\tag{4}
$$

where $a = 1, 2$ is the chain index. Differently from the isotropic ladder, the isotropic in-plane components are mixed by the DM interaction. The generalized spin susceptibility is a tensor:

$$
\chi^{AB}(q, i\omega_n) = \sum_{\mu,\nu=R,L}\sum_{a,b=1,2}\int d\tau dx \mathrm{e}^{i(\omega_n\tau - qx)}\langle T_\tau J^A_{\mu a}(x,\tau)J^B_{\nu b}(0,0)\rangle\,. \tag{5}
$$

The Gaussian spin-echo decay rate is determined by indirect nuclear spin-spin interaction induced by spin fluctuations and can be calculated as [8]:

$$
T_{2G}^{-2} \propto \sum_q \chi(q,0)^2, \tag{6}
$$

where a summation over the tensor components is understood. By using Eqs. (4), each tensor component can be expressed in terms of doublet and singlet and singlet-doublet correlation functions, that we denote by $\Gamma^{\alpha\beta}$, as follows:

$$
\begin{aligned}
\chi^{11}(q, i\omega_n) &= [\cos^2\alpha\Gamma^{23}(q, i\omega_n) - \sin^2\alpha\Gamma^{24}(q, i\omega_n)] = \chi^{22}\,, \\
\chi^{33}(q, i\omega_n) &= \Gamma^{12}(q, i\omega_n)\,,
\end{aligned}
\tag{7}
$$

where

$$
\Gamma^{\alpha\beta}(q, i\omega_n) = -\beta^{-1}\sum_{\mu,\nu=R,L}\sum_k\sum_{i\omega'_n} G^\alpha_{\mu\nu}(k+q, i\omega'_n + i\omega_n)G^\beta_{\mu\nu}(k, i\omega'_n)\,. \tag{8}
$$

Here $G^{\alpha(\beta)}$ is the doublet or singlet Majorana fermion thermal Green's function. The explicit expression of (8) is obtained by the approach of Ref. [8]. In the limit $T \to 0$ we obtain a finite value of the spin susceptibility that can be ascribed to a Majorana fermion pair creation process with an explicit dependence on the DM interaction, whereas in the limit of large temperature, we have:

$$
\chi^{11}(q,0) = \chi^{22}(q,0) \simeq \cos(2\alpha)/T\,, \quad \chi^{33}(q,0) \simeq 1/T\,, \tag{9}
$$

where only the third component is identical to the isotropic case.

In figure 1 we show the result of the full temperature dependence of T_{2G}^{-1} in presence of a DM interaction and without it. We observe a saturation

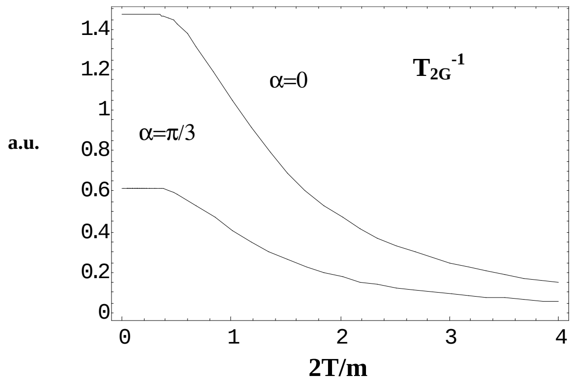

Fig. 1. Gaussian spin-echo decay rate versus temperature.

at low temperature that depends on the anisotropy. As an effect of DM interaction, the saturation value decreases. As evident from Eq. (9), the behavior of T_2^{-1} could provide information on the DM interaction strength in experimental systems.

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