GINZBURG–LANDAU FUNCTIONAL FOR METALS WITH SPIN–CHARGE SEPARATION: EFFECT OF THE MASS RENORMALIZATION*

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We evaluate the Ginzburg–Landau functional for the case of a superconductor with spin–charge separation. We have obtained analytical results for this functional when $T \leq T_{\rm c}$, in the limit of a spin–charge separation. For this case and, in the presence of the mass renormalization, we derived the form of the coherence length, $\varepsilon(T)$, the penetration depth, $\lambda(T)$, specific head jump, $\Delta C(T_{\rm c})/T_{\rm c}$, at the critical point, and the magnetic upper critical field, $H_{\rm c}2(T)$. The analytical results found here reduce to the BCS limit for a two–dimensional s-wave symmetry superconductor. We compare our results with recent works. In particular, we have performed a qualitative comparison with experimental results trying to fix the validity range of our spin–charge separation parameter, η. The d-wave order parameter symmetry does not change drastically the results presented here.

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1. Introduction

The discovery of high temperature superconductivity (HTSC) in 1986 by Bednorz and Müller [1] has caused a lot of enthusiasm among the physicists. Being a part of a largest family of strongly correlated electron systems, the cuprates exhibit anomalous properties in both the normal and the superconducting phases. As a consequence, standard theories, such as the Landau theory of the Fermi liquid and the BCS theory of the superconducting state, fail to describe correctly the physical properties of those materials. As an

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alternative, in the case of the normal state, several phenomenological models have been proposed in order to explain their nonmetallic behavior [2]. Despite the fact that the superconducting transition occurs at a relatively high temperatures, the characteristic of the ordered phase is the presence of electron pairs, leading to the idea that a modified BCS theory is appropriate for the description of their superconducting state.

For the description of the normal state we will follow Anderson [3, 4] proposal based on the hypothesis that a two dimensional (2D) system can be described by a Luttinger liquid type theory, similar to the one dimensional (1D) case. The generalization of the Luttinger liquid for the 2D case involves the use of the following Green's function (GF):

$$G(\boldsymbol{k}, i\omega_n) = \frac{g(\alpha)\mathrm{e}^{-i\pi\alpha/2}}{\omega_{\mathrm{c}}^{\alpha}(i\omega_n - \beta\eta\varepsilon_{\boldsymbol{k}})^{1/2}(i\omega_n - \beta\varepsilon_{\boldsymbol{k}})^{1/2-\alpha}}\ , \tag{1}$$

where ω_{c} is a cut-off frequency, $\eta = u_\sigma/u_\rho < 1$ is the ratio of the spin and charge velocities in the system, α is the non universal exponent related to the anomalous Fermi surface, $\beta = 2/(\eta + 1 - 2\alpha)$ is the mass renormalization factor, and $g(\alpha) = \pi\alpha/[2\sin(\pi\alpha/2)]$. Relations between the different parameters entering Eq. (1) can be obtained by studying different general properties of GF. $g(\alpha)$ was obtained by use of the first sum rule [5,6]. Based on this formalism [7], the necessity of the mass renormalization factor was predicted [6] using higher order sum rules. A similar NFL GF was proposed in Ref. [8] with normalization factor depending on two parameters, $g(\beta, \gamma)$. By using the time-reversal symmetry a charge–spin symmetric Green function comes out. Our $g(\alpha)$ depends on a single parameter. However, it also satisfies time-reversal symmetry. Here, we investigate metals with spin–charge separation in the superconducting state and we evaluate ξ.

2. General formalism of the Ginzburg–Landau functional

Let us first consider a pure s-wave superconductor. The difference between the superconducting and normal state free energy can be written as:

$$F_S(\boldsymbol{q}) - F_N(\boldsymbol{q}) = A|\Delta_{\boldsymbol{q}}|^2 + q^2 C|\Delta_{\boldsymbol{q}}|^2 + \frac{B}{2}|\Delta_{\boldsymbol{q}}|^4\ , \tag{2}$$

where the label S denotes the superconducting state, N the normal state, $\Delta_{\boldsymbol{q}}$ is the Fourier transform of the order parameter, and A, B, C are the temperature dependent Ginzburg–Landau coefficients [9].

2.1. The spin-charge separation liquid: $\alpha = 0$, $\eta \neq 1$

Here, we will focus our attention on the spin–charge separation liquid case, denoted by $\alpha = 0$ and $\eta \neq 1$.

The critical temperature, $T_c(\eta)$, which also includes the mass renormalization factor, leads to the following value:

$$T_c(\eta) = \frac{2\gamma_E}{\pi}\frac{2\omega_D}{1+\eta}\exp\left[-\frac{\pi}{(1+\eta)K(\sqrt{1-\eta^2})}\frac{1}{N_0 V}\right], \tag{3}$$

where γ_E is the Euler constant, ω_D is the Debye frequency, and $K(k)$ is the complete elliptic integral of the first kind.

A calculation of the G–L parameters leads to the following values:

$$\begin{aligned} A(\eta) &= N_0\frac{T - T_c(\eta)}{T_c(\eta)} f_A(\eta)\,, \\ B(\eta) &= \frac{7\zeta(3)N_0}{8\pi^2 T_c^2(\eta)} f_B(\eta)\,, \\ C(\eta) &= \frac{7\zeta(3)N_0 v_F^2}{32\pi^2 T_c^2(\eta)} f_C(\eta)\,, \end{aligned} \tag{4}$$

where we introduced the following notations

$$f_A(\eta) = \frac{(1+\eta)K(\sqrt{1-\eta^2})}{\pi} \qquad f_B(\eta) = \frac{1+\eta}{2}F\left(1,\frac{1}{2};2;1-\eta^2\right)\,, \tag{5}$$

$$f_C(\eta) = \frac{1}{2(1+\eta)}\left\{\frac{3}{2}\left[\frac{3}{2}F\left(\frac{1}{2},\frac{1}{2};3;1-\eta^2\right) + \eta F\left(\frac{3}{2},\frac{1}{2};3;1-\eta^2\right)\right.\right.$$
$$\left.\left. + \frac{3\eta^2}{2}F\left(\frac{5}{2},\frac{1}{2};3;1-\eta^2\right)\right] - F\left(\frac{1}{2},\frac{1}{2};2;1-\eta^2\right) - \eta^2 F\left(\frac{3}{2},\frac{1}{2};2;1-\eta^2\right)\right\}, \tag{6}$$

$F(\alpha,\beta;\gamma;z)$ being the hypergeometric function. Making the limit $\eta \to 1$ the standard BCS results are recovered. We present only ξ:

$$\frac{\xi(\eta,T)}{\xi_{BCS}(T)} = \frac{1}{f_T(\eta)}\sqrt{\frac{f_C(\eta)}{f_A(\eta)}}\sqrt{\frac{1 - T/T_{c0}}{1 - T/[f_T(\eta)T_{c0}]}}, \tag{7}$$

with

$$f_T(\eta) = \frac{T_c(\eta)}{T_{c0}} = \frac{2}{1+\eta}\exp\left[\left(1 - \frac{\pi}{(1+\eta)K(\sqrt{1-\eta^2})}\right)\frac{1}{N_0 V}\right]. \tag{8}$$

As we expect $f_T(\eta = 1) \to 1$. In Fig. 1 we plot the η-dependence of the ratio between the coherence length in the spin-charge separation liquid. We observe that, for $\eta \neq 1$, the value of the coherence length is lower than the one in the standard BCS case. The considered values of the T/T_{c0} are justified by the range of the critical region around the transition temperature.

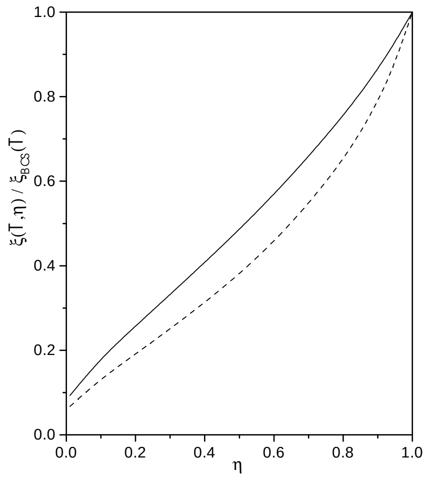

Fig. 1. The coherence length ratio as function of the spin-charge separation parameter η for different values of the T/T_{c0} ratio. The full line correspond to a value $T/T_{c0} = 0.8$, and the dashed line to $T/T_{c0} = 0.9$.

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