EFFECT OF ANISOTROPIC IMPURITY SCATTERING IN A *d*-WAVE SUPERCONDUCTOR*

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(Received July 10, 2002)

We study the impurity effect on the superconducting d-wave state accounting for the momentum-dependent impurity potential. We discuss the impurity-induced critical temperature suppression, local density of states in the vicinity of a single impurity, and the density of states in a superconductor with a spatial impurity distribution.

PACS numbers: 74.20.–z, 74.62.–c, 74.62.Dh, 74.72.–h

1. Introduction

Impurities provide a useful tool in probing the symmetry of unconventional superconducting states [1-5]. The most direct probe of a simple defect impact on superconductivity is provided by the scanning tunneling microscopy (STM) measurement of the position-dependent quasiparticle density of states. The STM images of Zn and Ni substitutions at the planar Cu sites in $Bi_2Sr_2CaCu_2O_{8+\delta}$ reveal a distinct four-fold symmetry of the local density of states (LDOS) [1, 2] predicted for the d-wave superconductor response to disorder [3, 4]. In addition to providing detailed information on the superconducting state, this kind of experiment may shed light on the nature of the quasiparticle scattering centers. Although the main feature of the four-fold symmetry of the tunneling currents is captured by the model of isotropic impurity scattering, the observed spatial dependence of the quasiparticle density of states is far more complex, and may originate from a non trivial structure of the impurity potential. The issue of anisotropy of the impurity potential has been evoked in the discussion of the disorder-induced suppression of the critical temperature in the cuprates [6-8]. It has been shown that the low impurity pair-breaking effect in these compounds can be

* Presented at the International Conference on Strongly Correlated Electron Systems, (SCES 02), Cracow, Poland, July 10–13, 2002.

understood within a scenario of a momentum-dependent (anisotropic) impurity scattering in the d-wave superconductor [11-14]. In the present paper we compare the effect of the anisotropic impurity potential on the critical temperature, quasiparticle density of states and local density of states in the vicinity of a single impurity, and conclude to what extend the momentum-dependence of the scattering potential is responsible for the disorder phenomena observed in the cuprates. We consider the momentum-dependent impurity potential [11-15] $v\left(\boldsymbol{k},\boldsymbol{k}'\right) = v_i + v_a f\left(\boldsymbol{k}\right) f\left(\boldsymbol{k}'\right)$, where v_i, v_a are isotropic and anisotropic scattering amplitudes, respectively, $f\left(\boldsymbol{k}\right)$ is the anisotropy function that vanishes after integration over the Fermi surface. We study the effect of anisotropy by referring to isotropic impurity potential v_0 that determines the scattering strength in both isotropic and anisotropic scattering channels: $v_i = \alpha v_0$, $v_a = (1-\alpha)\, v_0$; where $0 \leq \alpha \leq 1$ is a partition parameter. Scattering is isotropic for $\alpha = 1$ and purely anisotropic for $\alpha = 0$. For the analysis of different scattering limits we introduce a convenient parameter $c = 1/\left(\pi N_0 v_0\right)$, where N_0 is the density of states per spin at the Fermi level in the normal state. In the calculations we assume a two-dimensional superconductivity. We discuss the impurity effect on the superconducting state determined by the order parameter $\Delta\left(\boldsymbol{k}\right) = \Delta_0 e\left(\boldsymbol{k}\right)$, where $e\left(\boldsymbol{k}\right) = \sqrt{2}\cos 2\phi$ and the amplitude of $\Delta\left(\boldsymbol{k}\right)$ is defined as $\Delta = \sqrt{2}\Delta_0$.

2. Critical temperature

Potential determined by $f\left(\boldsymbol{k}\right) = \sqrt{2}\left(k_x^2 - k_y^2\right) = \sqrt{2}\cos 2\phi$ which is in phase with the d-wave superconducting order parameter leads to a particularly moderate suppression of the critical temperature [10]. A significantly lowered pair-breaking for the in phase scattering is apparent for a model potential given by $f\left(\boldsymbol{k}\right) = \pm 1$. In the Born limit the critical temperature is determined by [11,14,15]

$$\begin{aligned} \ln\frac{T_{\mathrm{c}}}{T_{\mathrm{c}_0}} &= \left(\langle e\rangle^2 + \langle ef\rangle^2 - 1\right)\left[\psi\left(\frac{1}{2} + \frac{\pi N_0\left(v_i^2+v_a^2\right)}{2\pi T_{\mathrm{c}}}\right) - \psi\left(\frac{1}{2}\right)\right] \\ &\quad + \langle ef\rangle^2\left[\psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{\pi N_0\left(v_i^2+v_a^2\right)}{2\pi T_{\mathrm{c}}}\left(1 - \frac{2v_iv_a}{v_i^2+v_a^2}\right)\right)\right] \end{aligned} \tag{1}$$

and for the resonant impurity scattering we have [10]

$$\ln\frac{T_{\mathrm{c}}}{T_{\mathrm{c}_0}} = \left(\langle e\rangle^2 + \langle ef\rangle^2 - 1\right)\left[\psi\left(\frac{1}{2} + \frac{2\mathit{\Gamma}}{2\pi T_{\mathrm{c}}}\right) - \psi\left(\frac{1}{2}\right)\right], \tag{2}$$

where T_{c_0} is the critical temperature of a pure system, $\mathit{\Gamma} = n/\pi N_0$, and n is the impurity concentration. A straightforward analysis of Eqs. (1) and

(2) shows that T_c is maximal for the in phase scattering. It has been shown that scattering being close to the in phase scattering reproduces the observed initial T_c suppression in the cuprates [13, 14].

3. Local density of states

The position-dependent change of the quasiparticle density of states around a single impurity is determined by the real space transform of the retarded Green's function $\hat{G}$ and reads $\delta N(\boldsymbol{r},\omega) = -\frac{1}{\pi}\mathrm{Im}\{\delta G_{11}(\boldsymbol{r},\omega)\}$. The single impurity approximation means that the impurity-induced self-energy is not determined self-consistently, *i.e.*, the t-matrix equation is solved with the Green's function of a pure system. We have evaluated the

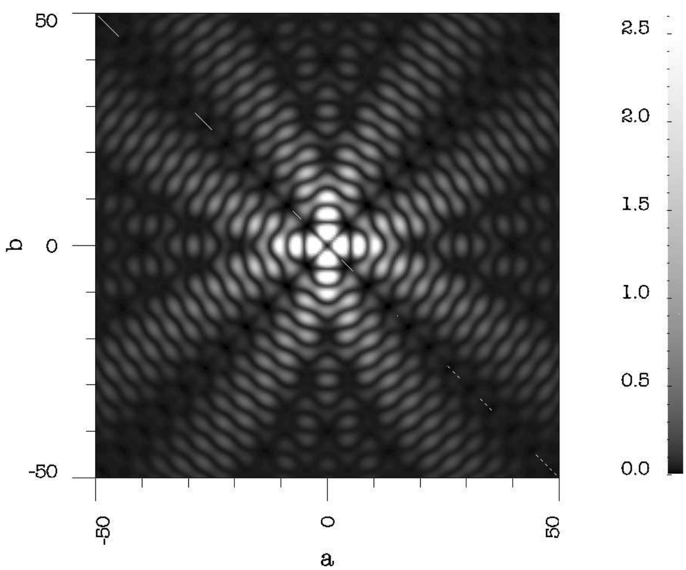

Fig. 1. LDOS-map at the resonance frequency $\omega = 0.032\Delta$ around the impurity located at (0,0) in the ab plane of the d-wave superconductor for isotropic impurity potential. The density of states is given in the units of the FS two-spin density of states $N(0)$ and varies from the lowest (black) to the highest (white) value according to the scale next to the map. The distance is measured in k_F^{-1} units.

LDOS for equal impurity scattering strength in isotropic and anisotropic channel $\alpha = 0.5$ and for $c = 0.05$ that simulates resonant scattering. The coherence length has been set to $\xi_0 = 12\pi k_F^{-1}$ and the parabolic electron energy band has been assumed. The out of phase impurity potential defined by $f(\boldsymbol{k}) = \sqrt{2}\sin 2\phi$ leads to exactly the same LDOS on the impurity

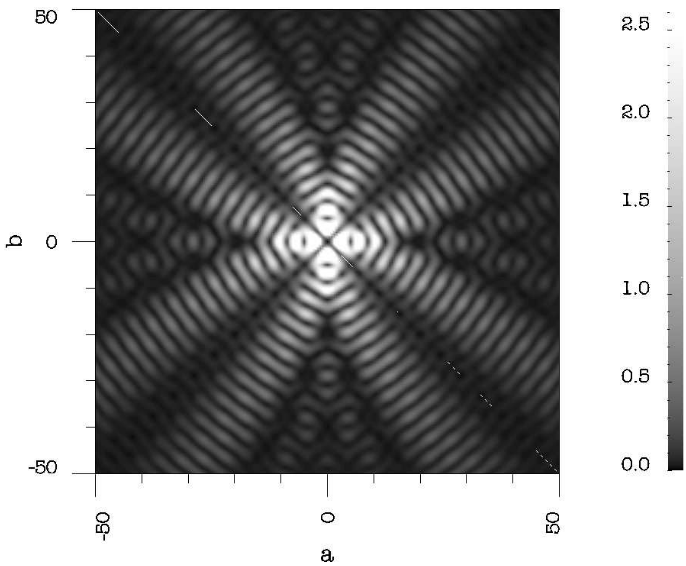

Fig. 2. LDOS-map at the resonance frequency $\omega = 0.032\Delta$ around the impurity located at (0,0) in the ab plane of the d-wave superconductor for the out of phase impurity potential ($f(\boldsymbol{k}) = \sqrt{2}\sin 2\phi$). Units as in Fig. 1.

as the isotropic scattering. It gives rise to a bound state at the energy $\omega = -0.032\Delta$. The spatial distribution of the quasiparticle states differs, however, from the one around the isotropic impurity (Figs. 1 and 2). We have also checked that the in phase impurity potential does not generate the impurity-bound state except for a low anisotropy content, that is, $\alpha \sim 0.9$ or for a high anisotropy, *i.e.*, $\alpha \sim 0.1$.

4. Density of states

We perform a fully self-consistent t-matrix evaluation of the self-energy for the d-wave superconductor with a uniform impurity distribution assuming a constant particle-hole symmetric density of states in the normal state. The quasiparticle density of states in the superconducting state DOS $= -\frac{1}{\pi}\sum_{\boldsymbol{k}} \mathrm{Im} G_{11}\left(\boldsymbol{k},\omega\right)$ is shown in Fig. 3 for several scattering regimes: Born ($c = 10/\pi$), intermediate ($c = 1/\pi$) and unitary ($c = 0.001/\pi$). We note, that compared to isotropic scattering the in phase ($f \sim \cos 2\phi$) impurity potential reduces the density of states, for the intermediate scattering it can change the low-energy dependence of the density of states and for the unitary scattering does not generate the impurity band. The out of phase potential ($f \sim \sin 2\phi$) effect on the d-wave superconducting state resembles the one of the isotropic impurity but has a slightly stronger pair-breaking effect seen in a larger density of states.

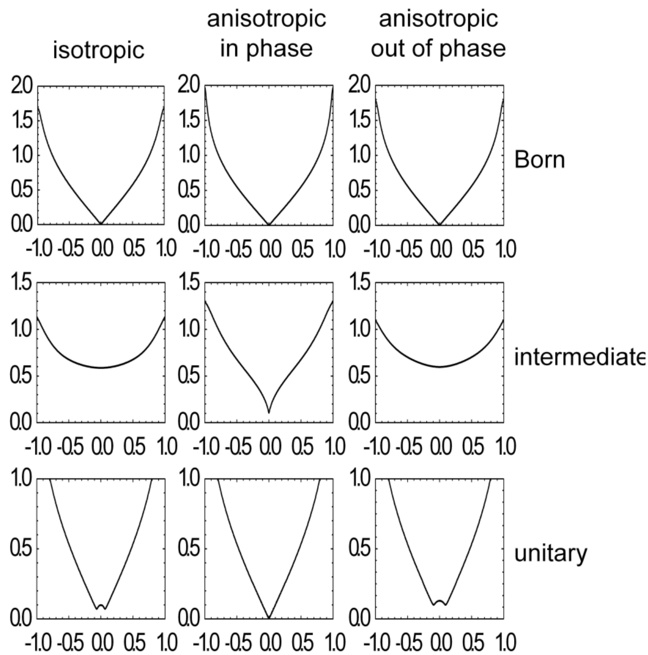

Fig. 3. The density of states for weak (Born) impurity scattering $c = 10/\pi$ and impurity concentration $n = 1$ a.u.: (a) isotropic potential, (b) anisotropic potential $f(\boldsymbol{k}) = \sqrt{2}\cos 2\phi$, (c) anisotropic potential $f(\boldsymbol{k}) = \sqrt{2}\sin 2\phi$; intermediate impurity scattering $c = 1/\pi$ and impurity concentration $n = 1$ a.u.: (d) isotropic potential, (e) anisotropic potential $f(\boldsymbol{k}) = \sqrt{2}\cos 2\phi$, (f) anisotropic potential $f(\boldsymbol{k}) = \sqrt{2}\sin 2\phi$; resonant impurity scattering $c = 0.001/\pi$ and impurity concentration $n = 0.01$ a.u.: (g) isotropic potential, (h) anisotropic potential $f(\boldsymbol{k}) = \sqrt{2}\cos 2\phi$, (i) anisotropic potential $f(\boldsymbol{k}) = \sqrt{2}\sin 2\phi$. The potential partition parameter $\alpha = 0.8$.

5. Conclusion

The scenario of anisotropic impurity scattering reproduces experimentally determined suppression of the critical temperature in cuprates for the impurity potential in phase with the order parameter, that is, $f(\boldsymbol{k}) \sim \cos 2\phi$. Such an impurity potential, however, leads to a low density of states in the d-wave superconductor and does not generate the impurity-bound state observed in the STM images of the cuprates except for a low ($\alpha \sim 0.9$) and a high anisotropy ($\alpha \sim 0.1$) of the scattering potential.

The work was supported in part by Polish State Committee for Scientific Research (KBN) grant No. 5 P03B 058 20.

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