IS THE SUPERCONDUCTING STATE
FOR THE CUPRATES REACHED THROUGH
A PERCOLATION TRANSITION?*

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(Received July 10, 2002)

Several recent experiments have revealed that the charge density $\rho$ in a given compound (mostly underdoped) is intrinsic inhomogeneous with large spatial variations. Therefore, it is appropriate to define a local charge density $\rho(r)$. These differences in the local charge concentration yield insulator and metallic regions, either in an intrinsic granular or in a stripe morphology. In the metallic region, the inhomogeneous charge density produces spatial or local distributions of superconducting critical temperatures $T_c(r)$ and zero temperature gap $\Delta_0(r)$. We propose that the superconducting phase in high $T_c$ oxides is reached when the temperature reach a value which superconduction regions with different critical temperatures percolates. We show also that this novel approach is able to reproduce the phase diagram for a family of cuprates and provides new insights on several experimental features of high $T_c$ oxides.

PACS numbers: 74.72.-h, 74.20.-z, 74.80.-g, 71.38.+i

The non usual properties of high $T_c$ superconductors have motivated several experiments and two features have been discovered which distinguish them from the overdoped compounds: firstly, the appearance of a pseudogap at a temperature $T^*$, that is, a discrete structure of the energy spectrum above $T_c$, identified by several different probes [1]. Second, there are increasing evidences that the electrical charges are highly inhomogeneous up to (and even further) the optimally doped region [2-4]. In fact, such intrinsic inhomogeneities are also consistent with the presence of charge domains either in a granular [3,4] or in a stripe form.

In a recently letter [5] we have proposed a new scenario in which a given HTSC compound with an average hole per Cu ion density $\langle \rho \rangle$ and with

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* Presented at the International Conference on Strongly Correlated Electron Systems, (SCES02), Cracow, Poland, July 10–13, 2002.
an inhomogeneous microscopic charge distribution $\rho(r)$ has a distribution of small clusters, each with a given $T_c(r)$. This percolating scenario can be understood by analyzing the scanning SQUID microscopy magnetic data which makes a map of the expelled magnetic flux (Meissner effect) domains on LSCO films [6]. This experiment shows the regions where the Meissner effect continuously develops from near $T^*$ to temperatures well below the percolating threshold $T_c$. Below we show that the percolating approach is capable to yield quantitative agreement with the HTSC phase diagrams and it provides also new physical insight on a number of phenomena detected in these materials.

Scanning Tunneling Microscopy/Spectroscopy (STM/S) [3] on optimally doped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$ measures nanoscale spatial variations in the local density of states and the superconducting gap at a very short length scale of $\approx 14\ \text{Å}$. These results suggest that instead of a single value, the zero temperature superconducting gap assumes different values at different spatial locations in the crystal and their statistics yield a Gaussian distribution [3]. New high resolution STM measurements [4] have revealed an interesting map of the superconducting gap spatial variation for underdoped Bi2212.

In order to model the above experimental observations we used a phenomenological combination of Poisson and Gaussian distributions for the charge distribution $\rho(r)$. Thus, for a given compound with an average charge density $\langle \rho \rangle$, the hole distribution function $P(\rho; \langle \rho \rangle)$ or simply $P(\rho)$ is a histogram of the probability of the local hole density $\rho$ inside the sample, separated in two branches or domains. The low density branch represents the hole-poor or isolating regions and the high density one represents the hole-rich or metallic regions. Such normalized charge probability distribution may be given by

$$P(\rho) = \frac{(\rho_c - \rho) \exp \left( \frac{-|\rho - \rho_c|^2}{2\sigma_-^2} \right)}{\sigma_-^2 \left( 2 - \exp \left( \frac{-|\rho|^2}{2\sigma_-^2} \right) \right)^2}, \quad \text{for } 0 < \rho < \rho_c, \quad (1)$$

$$P(\rho) = 0, \quad \text{for } \rho_c < \rho < \rho_m, \quad (2)$$

$$P(\rho) = \frac{(\rho - \rho_m) \exp \left( \frac{-|\rho - \rho_m|^2}{2\sigma_+^2} \right)}{\sigma_+^2 \left( 2 - \exp \left( \frac{-|\rho|^2}{2\sigma_+^2} \right) \right)^2}, \quad \text{for } \rho_m < \rho. \quad (3)$$

The values of $\sigma_-$ ($\sigma_+$) controls the width of the low (high) density branch. Here, $\rho_c$ is the end local density of the hole-poor branch. $\rho_m$ is the starting local density of the hole-rich or metallic branch. Both $\rho_c$ and $\rho_m$ are shown in Fig. 1(a) for the $\langle \rho \rangle = 0.16$ case. We can get a reliable estimation of the $\sigma_+$ values from the experimental STM/S Gaussian histogram distribution for the local gap [3] on an optimally doped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$ [5].
According to the percolation theory, the site percolation threshold occurs in a square lattice when 59% of the sites are filled [7]. Thus, we find the density where the hole-rich branch percolates integrating \( \int P(\rho)\,d\rho \) from \( \rho_m \) till the integral reaches the value of 0.59 where we define \( \rho_p \).

There are several different approaches which can be used to obtain \( T^*(\rho_p) \). Strictly speaking, due to the non-uniform charge distribution, we do not have translational symmetry and we should use a method which takes the disorder into account. Since our purpose here is to demonstrate the feasibility of the percolating scenario, we will take the simplest way, that is, we will follow a BCS-like approach with an extended Hubbard Hamiltonian to derive a curve for the temperature onset of vanishing gap as function of the local hole concentration \( \rho \). Using the experimental parameters for the dispersion relation and hopping integrals derived from experiments and band calculations appropriate to the LSCO family [8], we derived the onset of vanishing gap as function of the doping level. This curve is taken as the \( T^*(\rho) \) boundary as shown in Fig. 1 (b).

![Phase diagram for the LSCO family.](image)

**Fig. 1.** Phase diagram for the LSCO family. To explain how \( T^*(\rho) \) and \( T_c(\rho) \) are obtained, we plot in (a) the probability distribution for the optimal compound with \( \langle \rho \rangle = 0.16, P(\rho, 0.16) \). The arrows shows \( T^*(0.16) \) and the percolation threshold at \( \rho_p = 0.23 \) with \( T_c(\rho) = 0.16 = T_c(0.23) \). The experimental data are taken from Ref. [9] and \( T_{onset} \) is taken from the flux flow experiment (open squares) [10].

The theoretical curves in Fig. 1 are derived in the following way: For each local density \( \rho > \rho_m \), the maximum value of \( T^*(\rho) \) is equal \( T^*(\rho_m) \). This is the onset temperature of the superconducting gap and, therefore, \( T_c(\rho_m) = T^*(\rho) \). \( T_c(\rho) \) is estimated in the following manner: we calculate the
maximum temperature in which the superconducting region percolates in the metallic branch. The percolation occurs when all the clusters with local density between $\rho_m$ and $\rho_p$ are superconducting as shown for $\langle \rho \rangle = 0.16$. The value of $\rho_p$ can be seen in the panel following the arrow which shows that $T^\ast (\rho_p = 0.23)$ is equal the superconducting critical temperature of the compound, $T_c (\rho = 0.16)$.

There are several HTSC phenomena which are not well understood and can be explained with the percolating ideas derived from our model and calculations. Here we name just a few:

- (i) The steady decrease of the zero temperature gap $\Delta_0 (\langle \rho \rangle)$ with the doping $\langle \rho \rangle$.
- (ii) The resistivity deviation from the linear behavior at $T^\ast$.
- (iii) The existence of superconducting clusters between $T^\ast$ and $T_c$ easily explains the appearance of local diamagnetic or Meissner domains [6] and the Nernst flux flow effect above $T_c$ [10].
- (iv) The suppression in the specific heat $\gamma$ term found in underdoped compounds of different families.

In summary, we have proposed a quantitative novel and general approach to the phase diagram of the high $T_c$ cuprates superconductors in which the pseudo-gap is the largest superconducting gap among the superconducting regions in an inhomogeneous compound. The critical temperature $T_c$ is the maximum temperature for which these superconducting regions percolates. This method is also suitable to provide insights in the normal phase properties.

REFERENCES