TWO-PARTICLE PAIRING IN A TWO-DIMENSIONAL BOSE-GAS WITH TWO SORTS OF BOSONS*

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We consider a possibility of two-boson pairing in a dilute 2D Bose-gas on a lattice with strong hard-core repulsion U and a Van der Waals attractive tail V. The phase diagram of Bose gas consisting of one sort of structureless bosons contains only two regions: the usual one particle Bose–Einstein condensation (BEC) and the region of total phase separation on the Mott–Hubbard Bose solid and dilute Bose gas. But in the system with two sorts of structureless bosons the creation of the two-particle condensate ($\langle bb\rangle \neq 0$) is possible. We show that the full set of equations for stability of homogeneous two-particle condensate is satisfied.

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1. Introduction

The problem of two-particle pairing in Bose systems is interesting not only from the point of view of 2D Bose systems in magnetic traps but also for the theories of biexcitons in semiconductors, Schwinger bosons in magnetic systems and holons in HTSC. In the latter case a possible two-holon pairing in the slave–boson theories of superconductivity can restore a required charge $2e$ of a Cooper pair. The first attempt of the investigation of the possibility of two-particle pairing versus one particle Bose–Einstein condensation belongs to Valatin and Butler [1]. Later on Nozieres and Saint-James [2] showed that in 3D structureless Bose gas with attractive tail either standard one-particle BEC is more energetically beneficial or phase separation takes place earlier than the two-particle Bose pairing. As a next step Rice and Wang [3] conjectured that the two-particle Bose condensation is possible in 2D hard-core Bose gas with attractive tail. Later on the present authors showed

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that the Bose gas without internal structure with attractive interaction is unstable against phase separation [5]. Hence there is no region of two-particle Bose-condensate ($\langle bb\rangle \neq 0$ while $\langle b\rangle = 0$) in this type of models.

The aim of the present paper is to show that there is the set of models, in which the two-particle pairing can be realized. In particular we demonstrate that in the case of Hubbard model for a Bose gas consisting of two sorts of bosons with repulsion between the particles of the same sort and attraction between the particles of different sorts the stability conditions against the phase separation are satisfied. Hence in these systems the two-particle Bose condensate can exist.

2. Two-particle pairing of structureless bosons on a lattice

First let us remind the properties of two-particle condensate in a Bose gas. We will consider a Bose gas on a 2D square lattice with onsite repulsion U and nearest neighbors attraction V described by the Hamiltonian

$$H = -t\sum_{ij} b_i^\dagger b_j + \frac{U}{2}\sum_i n_i^2 - \frac{V}{2}\sum_{ij} n_i n_j\,, \tag{1}$$

where $n_i = b_i^\dagger b_i$ is a 2D boson density. The bound state energy is given by the pole in the T matrix problem [4, 5]. Due to hard-core repulsion the bound state exists only for the potential strength $V > V_\mathrm{c}$, where $V_\mathrm{c} = 4t$ is a threshold value (see for details [5]). The energy of the bound state is given by

$$|E_b| = 8W \exp\left\{\frac{-1}{\lambda}\right\}, \tag{2}$$

where W is a bandwidth and $\lambda = (V - V_\mathrm{c})/\pi V$.

The possibility of the pairing in medium is determined by the existence of the pole in the solution of the Bethe–Salpeter equation

$$1 + T(2\mu)\int\!\!\int \frac{dp_x dp_y}{(2\pi)^2}\frac{\coth\left(\frac{\varepsilon_{\boldsymbol{p}}-\mu}{2T}\right)}{2(\varepsilon_{\boldsymbol{p}}-\mu)}|\phi|^2 = 0\,, \tag{3}$$

where the $|\phi|$ are respectively eigenfunctions for s-, p- and d-wave pairing and $T(2\mu)$ is a corresponding T matrix. The conservation of particles gives an equation for the chemical potential

$$n_B = \int\!\!\int \frac{d^2p}{(2\pi)^2}\frac{1}{\exp\left\{\frac{E_p}{T}\right\} - 1}\,, \tag{4}$$

where $E_p = p^2/2m + |\tilde{\mu}|$ is the spectrum of almost an ideal Bose gas.

The system of the equations (3) and (4) has a solution for an s-wave channel at temperature $T_c \approx T_0/\ln\lambda$ where $T_0 = 2\pi n/m$ is degeneracy temperature. It corresponds to critical temperature of two-particle pairing.

The real collapse of the system is prohibited by strong onsite repulsion U, but the system can be unstable towards phase separation onto two clusters. First cluster has $n_1 \to 1$ and is localized due to Mott–Hubbard considerations. And second cluster has a density of particles $n_2 \to 0$. A simple analysis shows that the phase separation takes place for $V > V_{\rm ps} \sim 2t$.

As a result we obtain the following phase diagram for the system of the structureless bosons: for $V < 2t$ we have at low density the standard one-particle BEC. For $V > 2t$ the phase separation takes place. For large densities $n = n_c \leq 1$ ($n_c = 1$ for structureless bosons) the system undergoes a transition to the Mott–Hubbard Bose solid.

3. Possibility of two-particle pairing for the two-band Bose–Hubbard model

Now let us show that there is a class of models with two-particle condensation. We will consider the two-band Hubbard model for two sorts of structureless bosons. The Hamiltonian of the system has the form

$$H = -t_a \sum_{ij} a_i^\dagger a_j - t_b \sum_{ij} b_i^\dagger b_j + \frac{U_{aa}}{2} \sum_i n_{ia}^2 + \frac{U_{bb}}{2} \sum_i n_{ib}^2 - \frac{U_{ab}}{2} \sum_i n_{ia} n_{ib} \,,$$

where t_a and t_b, n_a and n_b are, respectively, the hopping matrix elements and densities for bosons of sorts a and b. For simplicity we will consider the case $t_a = t_b$ and assume that the bottoms of the bands coincide. In the Hamiltonian U_{aa} and U_{bb} are Hubbard onsite repulsions for bosons of sorts a and b. Finally U_{ab} is an onsite attraction between bosons of two different sorts. Note that the same Hamiltonian describes the two-layer situation with interlayer attraction and intra-layer repulsion.

Let us consider the low density limit with equal densities of both sorts of bosons, that is when both $n_a = n_b = n \ll 1$. In this limit we must replace the Hubbard interaction U_{ab} by the corresponding T matrix. The relevant expression for the T matrix T_{ab} is given by

$$T_{ab}(\tilde{E}) = \frac{U_{ab}}{1 - U_{ab} \int \frac{d^2p}{4\pi^2} \frac{1}{p^2/m + |\tilde{E}|}} \,, \tag{5}$$

where $\tilde{E} = E + W$.

The results of the previous section for the bound state (2) and critical temperature are still valid. But in this case we should substitute a coupling constant by $\lambda = mU_{ab}/(4\pi)$. Note that there is no threshold for pairing in this case. We also want to mention that in this case the coherence length can be larger than a mean distance between particles ($\xi = 1/\sqrt{2mE_b} \gg a = 1/\sqrt{n}$). Therefore, the pairs can be not only local but also extended.

Let us investigate the stability of the system against phase separation. The chemical potential in leading approximation can be written in the following form $\mu_a = T_{aa}n_a - |E_{\text{bound}}|/2$ and $\mu_b = T_{bb}n_b - |E_{\text{bound}}|/2$. By direct calculations it is easy to see that all required conditions for stability are satisfied: $\partial\mu_a/\partial n_a > 0$; $\partial\mu_b/\partial n_b > 0$ and $(\partial\mu_a/\partial n_a)(\partial\mu_b/\partial n_b) - (\partial\mu_a/\partial n_b)(\partial\mu_b/\partial n_a) > 0$ for $T_{aa} > 0$ and $T_{bb} > 0$. Hence the system is stable against phase separation.

Now we complete the phase diagram for the system with two sorts of bosons. The resulting phase diagram is quite different from one for the system consisting from one sort of structureless bosons. It has no region of phase separation. At low densities for repulsive U_{ab} we have usual BEC, but for attractive U_{ab} we have already the s-wave two-particle pairing. For large densities $n = n_c \sim 1$ the system undergoes a transition to the Mott–Hubbard Bose solid.

In conclusion we showed the possibility of realization of two-particle paring in the system of two sorts of bosons with repulsion between the same type of bosons and attraction between different sorts. We demonstrated that the necessary conditions for stability of homogeneous two particle condensation are satisfied. The more direct applications of the present results with two sorts of bosons are connected with SU(2) slave–boson theories of high T_c superconductivity and Schwinger-boson theories of 2D magnets.

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