OPTIMUM FINITE IMPULSE RESPONSE (FIR) LOW-PASS FILTERING OF MARKET DATA*

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The goal of this contribution is to compare 9 cases of FIR (Finite Impulse Response) type filters, by using approximation theory based norms for the following output parameters: delay and correlation between input and output, and "smoothness" of the output derivative. It was found that the most commonly used rectangular shape of impulse response is in general not an optimum solution. Indications concerning the optimum shape of impulse response subject to the assumed criteria are shortly presented and the triangular shape of impulse response is recommended.

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1. Introduction

One of the most common operations performed on market data is a smoothing, low-pass filtering, which enables us to obtain less noisy data. On the other hand, low-pass filtering results in delaying and distorting output data. Both, high noise-content in original data, delay and distortion in output data, have a negative impact on predictions of further market movements. Therefore, it is important to choose the appropriate filter impulse response to obtain a reasonable tradeoff between the delay and distortion on one hand, and the smoothness of output data on the other hand.

2. Fundamentals of linear filtering

Linear filtering is one of the most common operations performed on signals in various areas of science and technology. Basically, for the linear filter the superposition principle or its frequency domain equivalent *i.e.*,

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frequency preservation principle is to be met. The relation between input and output signal of a linear filter is given by convolution, *e.g.*, [1],

$$\boldsymbol{y} = \boldsymbol{h} * \boldsymbol{x}\,, \tag{1}$$

where $\boldsymbol{x}$ and $\boldsymbol{y}$ stand for filter input and output signal, respectively, $\boldsymbol{h}$ is the filter impulse response, and asterisk denotes convolution.

Filter impulse response in discrete time domain can be either Finite Impulse Response (FIR) or Infinite Impulse Response (IIR), *e.g.*, [2]. In practical applications, the unique representation of filter impulse response in frequency domain, called the transfer function or transmittance is often used. In terms of frequency domain terminology filters are divided into two basic categories, that is low-pass filter and high-pass filter. By using the superposition principle a number of sub-categories such as band-pass filter, band-stop filter and other can be composed. The aim of our contribution is to analyze the application of the FIR type low-pass filter in application to smoothing market data. For the sake of comparison of various cases of impulse response the approximation theory based criteria have been used. In Fig. 1 an example of market data filtering (Dow Jones Industrial Average daily quotations for 2004) by using triangular shape impulse response of length 15 is shown.

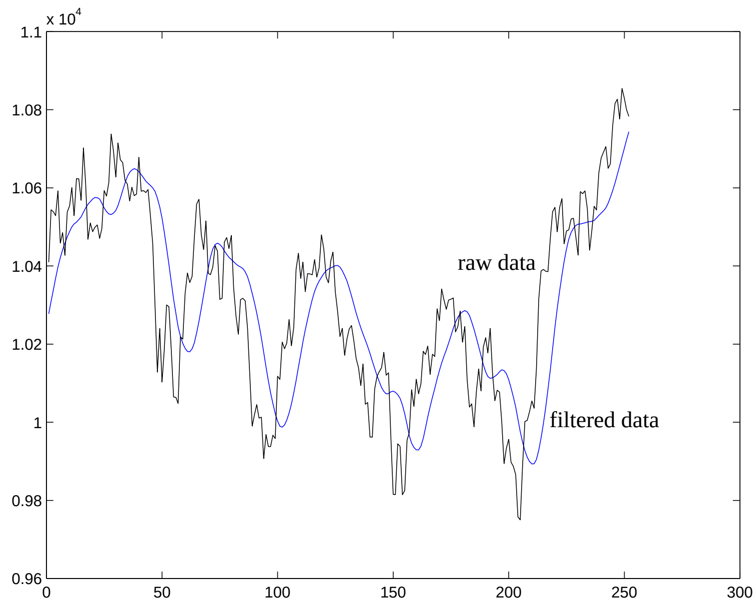

Fig. 1. Dow Jones daily quotations for 2004 (in color on line).

3. Definitions and norms

Denote here as $\boldsymbol{h}$ the filter impulse response and filter input signal $\boldsymbol{x}$, being a fragment of market quotations. Now, with reference to (1) let $\boldsymbol{y}$ be the convolution of $\boldsymbol{h}$ and $\boldsymbol{x}$ where

$$\boldsymbol{h} = (h_1, h_2, \ldots, h_k)\,, \tag{2}$$

is the filter impulse response of the length k, and k is an odd number

$$\boldsymbol{x} = (x_1, x_2, \ldots, x_q) \tag{3}$$

is the filter input signal of the length q ($q = 2 * k - 1$).

Now, we define the replica $\boldsymbol{r}$ as the steady-state part of the convolution $\boldsymbol{y}$, as follows

$$\boldsymbol{r} = (y_k, y_{k+1}, \ldots, y_{2k-1})\,. \tag{4}$$

A very specific and commonly used smoothing filter is a rectangular shape integrator of odd length k, whose impulse response is given as $\boldsymbol{h} = (1/k, \ldots, 1/k)$, [2]. For this case, the mean value of delay equals $(k-1)/2$. As a matter of fact for other impulse responses of the same length the delay would vary around this value. Therefore, the delay of $(k-1)/2$ is considered here as the predetermined, reference delay.

Consequently, it is assumed here, that the replica $\boldsymbol{r}$, defined by (4) corresponds to the fragment $\boldsymbol{s}$ of input signal $\boldsymbol{x}$, defined, as follows

$$\boldsymbol{s} = (x_{(k+1)/2}, \ldots, x_{(3k-1)/2})\,. \tag{5}$$

Signal $\boldsymbol{s}$, from now on, will be referred to as the original.

Now, consider the pair of the original $\boldsymbol{s}$ and replica $\boldsymbol{r}$. Define their varying part as follows

$$\boldsymbol{sv} = \boldsymbol{s} - \mathrm{mean}\,(\boldsymbol{s})\,, \tag{6}$$

and

$$\boldsymbol{rv} = \boldsymbol{r} - \mathrm{mean}\,(\boldsymbol{r})\,. \tag{7}$$

Consequently, define the five following criteria:

(a) *Delay* (del) *of replica* $\boldsymbol{r}$ *with respect to original* $\boldsymbol{s}$ defined as

$$\mathrm{del} = \frac{(tc_1 - tc_2)}{2}\,, \tag{8}$$

where tc_1 and tc_2 denote the coordinates for which cross-correlation functions $\mathrm{cor}(\boldsymbol{sv}, \boldsymbol{rv})$ and $\mathrm{cor}(\boldsymbol{rv}, \boldsymbol{sv})$ take the maximum value, respectively.

(b) *Maximum of cross-covariance* (crv) *of normalized in L_2 replica $\boldsymbol{r}$ and original $\boldsymbol{s}$* defined as

$$\mathrm{crv} = \max\left(\mathrm{cor}(\boldsymbol{sv}, \boldsymbol{rv})\right). \tag{9}$$

(c) *Product* (pro) *of normalized in L_2 original $\boldsymbol{s}$ and replica $\boldsymbol{r}$*

$$\mathrm{pro} = \boldsymbol{sv} \odot \boldsymbol{rv}\,, \tag{10}$$

where $\odot$ denotes calculation of product.

(d) *Standard deviation of the derivative* (sd) *of replica $\boldsymbol{r}$* defined as

$$\mathrm{sd} = \mathrm{std}\left[\mathrm{dif}\,(\boldsymbol{r})\right], \tag{11}$$

where std denotes standard deviation and dif is the first order difference, that is numerical derivative.

(e) *Maximum Chebyshev norm of the derivative* (mx) *of replica $\boldsymbol{r}$* defined as

$$\mathrm{mx} = \max\left(\mathrm{abs}\left[\mathrm{dif}\,(\boldsymbol{r})\right]\right). \tag{12}$$

Criteria (d) and (e) represent the measure of the "smoothness" of filter output signal.

4. Computations

For computations one minute quotations for the futures contracts on WIG 20 index of Warsaw Stock Exchange, were used. The database covers quotations from October 30, 2001 thru June 16, 2003, that is *ca.* 145 000 samples. Data has been divided into segments of $q = 2 * k + 1$, with the computational step equal q. This way for a given filter length k, a sequence of originals $\boldsymbol{s}$, and corresponding replicas $\boldsymbol{r}$, both of length k, were determined. Computations were performed for odd numbers of k ($9 \le k \le 299$) for the whole database, and the results were averaged. The following shapes of impulses response were considered

1. rectangular (boxcar), called **RECT**
2. triangular **TRIANG**
3. Hanning window **HAN**
4. Hamming window **HAM**
5. Blackman window **BLACK**
6. Chebyshev window with side-lobes ripple level −26 dB **CHEB26**
7. Chebyshev window with side-lobes ripple level −40 dB **CHEB40**
8. Chebyshev window with side-lobes ripple level −60 dB **CHEB60**
9. Chebyshev window with side-lobes ripple level −100 dB **CHEB100**.

The above impulse responses are by the definition even functions. In practical applications, in order to decrease the value of delay asymmetric shapes of impulse response are often considered. The rationale for that assumptions is to assign higher weights to more recent data. To cover the asymmetric case the right-hand halves of the specified symmetric shapes of impulse response were also taken into account.

5. Results

In Table I and II the results of computations for symmetric and asymmetric shape of impulse response are summarized, respectively. Denote, as follows

rdel — relative delay (del), that is defined by (8) divided by the respective,

del — for RECT, (in [%]),

crv — maximum of cross-covariance defined by (9), (in [%]),

pro — product defined by (10), (in [%]),

rsd — relative standard deviation (sd), that is defined in (11) divided by the respective (sd) for RECT, (in [%]),

rmx — relative maximum Chebyshev norm (mx), that is defined in (12) divided by the respective (mx) for RECT, (in [%]).

TABLE I

The results of computations for the symmetric shape of impulse response.

Shape	(rdel) [%]	(crv) [%]	(pro) [%]	(rsd) [%]	(rmx) [%]
RECT	0.00	67.95	55.59	100	100
TRIANG	−0.50	76.44	74.73	120.73	102.48
HAN	−0.36	78.09	76.89	144.29	114.43
HAM	−0.46	76.79	75.20	130.01	106.92
BLACK	−0.23	80.95	80.39	165.70	128.20
CHEB26	−0.01	71.90	67.00	116.87	124.07
CHEB40	−0.38	75.19	72.89	119.12	104.50
CHEB60	−0.27	78.77	77.78	143.86	114.94
CHEB100	−0.04	82.73	82.40	178.14	136.96

Notably, RECT impulse response, despite being an even (symmetric) function has been considered as the reference also for asymmetric impulse responses, due to its common usage.

TABLE II

The results of computations for the asymmetric shape of impulse response.

Shape	(rdel) [%]	(crv) [%]	(pro) [%]	(rsd) [%]	(rmx) [%]
TRIANG	−3.69	66.16	50.84	134.85	129.29
HAN	−3.63	65.47	48.22	151.48	138.03
HAM	−3.43	66.05	50.06	139.31	129.79
BLACK	−3.34	63.86	40.95	172.59	154.44
CHEB26	−2.71	67.56	54.52	138.85	155.03
CHEB40	−3.53	66.72	53.35	130.03	125.43
CHEB60	−3.41	65.05	46.17	153.72	140.54
CHEB100	−3.16	62.79	35.55	188.79	167.26

6. Conclusions

The following conclusions can be drawn from Table I and II:

1. Delay. Minimum value of delay is introduced by TRIANG, both for symmetric and asymmetric case.

2. "Smoothness". The highest degree of smoothing for the symmetric case, measured by means of standard deviation (sd), and Chebyshev maximum norm (mx), is attained in the case of RECT. However, for the case of maximum Chebyshev norm the following impulse responses: TRIANG, CHEB40 and HAM show only a very slightly inferior performance. In the asymmetric case CHEB40 and TRIANG give the best result.

3. Cross-covariance and product. For the symmetric case the highest value of (crv) and (pro) has been attained for CHEB100. In the asymmetric case CHEB26, CHEB40, and TRIANG are the best and similar in performance.

4. Cross-covariance and "smoothness", jointly. The filter for which the ratio (crv)/(sd) or (crv)/(mx) attains the maximum value is sought.

The best (crv)/(sd) show RECT, TRIANG and CHEB40 — for the symmetric case, and TRIANG and CHEB40 — for the asymmetric case.

5. Product and "smoothness", jointly. The filter for which the ratio (pro)/(sd) or (pro)/(mx) attains the maximum value is sought.

For both, (pro)/(sd) and (pro)/(mx) the best performance is attained, as follows: for TRIANG and CHEB40 — for the symmetric case, and CHEB40 and TRIANG — for the asymmetric case.

It is clear that the most commonly used filter RECT (boxcar) is in general not the best choice. For the case of impulse response symmetry a better choice is TRIANG or CHEB40 filter, or CHEB100 filter — the latter for the maximum of (crv) and (pro). For the asymmetric case where the main criterion is delay, the best choice is TRIANG.

In practical applications of market data filtering the delay is considered to be the most important figure of merit. From that viewpoint the optimum filter is the asymmetric TRIANG. The symmetric TRIANG also yields superior performance to the most of other filters. Therefore, the result of our analysis indicate that closest to the optimum solution is the TRIANG impulse response, both for the symmetric and asymmetric case.

REFERENCES

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