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MULTIPHOTON EXCHANGES IN PERIPHERAL HEAVY ION COLLISIONS*

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We have analyzed generic features of lepton pair production via multiphoton processes in peripheral heavy ion scattering. This process serves as an archetype reaction for perturbative QCD multigluon hard processes in collisions of ultrarelativistic nuclei. Explicit results for lepton pair production by annihilation of two photons from one nucleus and two photons from another one were obtained.

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1. Introduction

Investigation of lepton pair photoproduction in ultrarelativistic ion collisions has rising interest connected mainly with the long expected running of LHC. Main arguments for deeper examination of this production are the following [1–3]:

- Huge Born cross section $\sigma_{\rm Born}^{\gamma^*\gamma^*\to e^+e^-}$ of lepton pair photoproduction leads to high experiment background, effects the beam lifetime and overall luminosity;
- The produced lepton pair interacts with the Coulomb fields of the ions and the corresponding unitary and Coulomb corrections have a noticeable impact on the total cross section σ of the process under consideration;

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- There is no clear extension of results achieved in QED (*e.g.*, summation of the perturbation series corresponding to multiple interaction) to perturbative QCD.

Theoretical investigation of Coulomb corrections for the spectrum of lepton pairs photoproduced in the Coulomb field of ultrarelativistic nucleus starts with papers of Bethe and Maximon [4]. In their work the exact theory of Coulomb corrections based on the description of leptons by exact solutions of the Dirac equation in the Coulomb field (see, *e.g.*, [5]) has been developed. Despite the high activity in this area [6–11] the issue of correct allowance for final state interaction of produced leptons with the colliding ion Coulomb fields is lacking yet.

In our series of papers [12–15] we have performed extended studies of production and interaction of lepton pairs in Coulomb fields of ultrarelativistic heavy ions. Unlike the approach of Bethe and Maximon, we have chosen Feynman diagram technique, in which one has to sum multiphoton exchanges between produced electrons and positrons and the target nucleus. For ultrarelativistic leptons this reduces to the eikonal factors in the impact parameter representation. In the momentum space the same eikonal form leads to simple recurrence relations between the $(n+1)$ and n-photon exchange amplitudes [16], the incoming photon can be either real or virtual.

2. The aim of work

In the paper [12] we mentioned the amplitude $M_{(2)}^{(2)}$ for 4 photon exchange which is irrelevant in leading and next-to-leading logarithmic approximations in QED. Nevertheless, the knowledge of such a kind contributions becomes important for similar processes in QCD with multigluon exchanges between the color constituents of each of the colliding hadrons and the created quark–antiquark pair. Therefore the aim of presented work was to investigate final state interaction of produced lepton pairs with the Coulomb fields of colliding ions A, B with charge numbers Z_1, Z_2 through four photon exchange (Fig. 1).

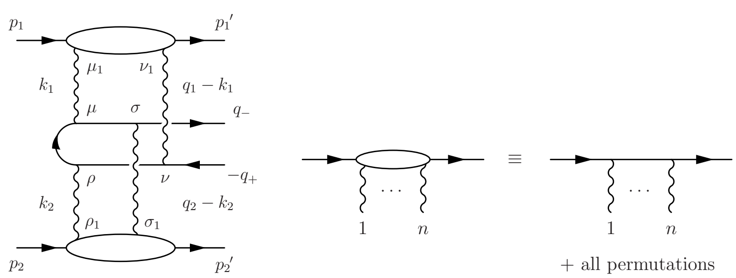

Fig. 1. Typical Feynman diagram for the amplitude $M_{(2)}^{(2)}$ and the notation for the permutations of n virtual photons emitted by the heavy ion.

The kinematical invariants of the process are $s=(p_1+p_2)^2$ — c.m.s. total energy of colliding ions, $q_1^2=(p_1-p_1')^2$, $q_2^2=(p_2-p_2')^2$, $s_1=(q_+ + q_-)^2$ — invariant mass of the pair, $p_1^2=p_1'^2=M_1^2$, $p_2^2=p_2'^2=M_2^2$ — ion masses, $q_\pm^2=m^2$ — lepton mass. We have worked in peripheral kinematics which corresponds to small scattering angles of ions A and B or in other words $s \gg s_1$, M_1^2, M_2^2, $|q_1^2|$, $|q_2^2| \gg m^2$. Whenever it was possible, we have neglected the lepton masses and the terms of the order M^2/s. Sudakov parametrization for 4–momenta of all exchanged photons k_i was used, *i.e.* $k_i=\alpha_i\tilde{p}_2+\beta_i\tilde{p}_1+k_{i\perp}$, $d^4k_i=(s/2)\,d\alpha_i\,d\beta_i\,d^2k_{i\perp}$, $k_{i\perp}=(0,0,\boldsymbol{k}_i)$, $k_{i\perp}^2=-\boldsymbol{k}_i^2$, with lightcone 4-momenta $\tilde{p}_1^2=\tilde{p}_2^2=0$, $\tilde{p}_{1,2}.q_\perp=0$, $2\tilde{p}_1\cdot\tilde{p}_2=s$.

After summation of all contributions of Feynman diagrams from Fig. 1 one can get following compact expression for the amplitude $M_{(2)}^{(2)}$

$$
\begin{aligned}
M_{(2)}^{(2)} &= \frac{\mathrm{i}s}{(2!)^2}(16\pi\alpha^2 Z_1 Z_2)^2 N_1 N_2 \\
&\quad\times \int \frac{d^2\boldsymbol{k}_1}{\pi}\,\frac{d^2\boldsymbol{k}_2}{\pi}\,\frac{\bar{u}(q_-)\,R_{(2)}^{(2)}\,\frac{\hat{p}_2}{s}\,v(q_+)}{\boldsymbol{k}_1^2\boldsymbol{k}_2^2(\boldsymbol{q}_1-\boldsymbol{k}_1)^2(\boldsymbol{q}_2-\boldsymbol{k}_2)^2}\,, \qquad (1)\\
R_{(2)}^{(2)} &= \beta_-\boldsymbol{b}_n^2+\beta_+\boldsymbol{a}_n^2 \\
&\quad-\sum_{n=3}^{10}\frac{[\hat{a}_n\hat{b}_n\hat{c}_n\hat{d}_n]_\perp}{2[\beta_-\boldsymbol{b}_n^2\boldsymbol{d}_n^2+\beta_+\boldsymbol{a}_n^2\boldsymbol{c}_n^2]}\left(1+\mathrm{i}\,\frac{(-1)^{n+1}}{\pi}\ln\frac{\beta_-\boldsymbol{b}_n^2\boldsymbol{d}_n^2}{\beta_+\boldsymbol{a}_n^2\boldsymbol{c}_n^2}\right) \\
&\quad+\sum_{n=11}^{12}\mathrm{i}\,\frac{(-1)^{n+1}}{\pi}\,\frac{[\hat{a}_n\hat{b}_n]_\perp}{\beta_-\boldsymbol{b}_n^2+\beta_+\boldsymbol{a}_n^2}\ln\frac{\beta_-\boldsymbol{b}_n^2}{\beta_+\boldsymbol{a}_n^2}\,, \qquad (2)
\end{aligned}
$$

with factor $N_1=\bar{u}^\eta(p_1')\hat{p}_2u^\eta(p_1)/s$, $\sum_\eta|N_1|^2=2$ for the ion A and a similar factor N_2 for the ion B; coefficients a_n, b_n, c_n, d_n are in Table I. It was confirmed that real part, as well as the whole expression (2) is equal zero if one of the following conditions is true: $\boldsymbol{k}_1=0$; $\boldsymbol{k}_2=0$; $\boldsymbol{k}_1=\boldsymbol{q}_1$; $\boldsymbol{k}_2=\boldsymbol{q}_2$, what is crucial for the gauge invariance and infrared convergence of integrations over $d^2\boldsymbol{k}_i$.

We have considered the behavior of expression (1) in the case where the transverse component of lepton momenta is large compared to the momenta transferred to the ions (wide angle limit), the main contribution to the matrix element is then given by the region $|\boldsymbol{q}_i| \ll |\boldsymbol{k}_i| \ll |\boldsymbol{q}|$. The quantity (2) plays a role of cut-off in the region $|\boldsymbol{k}_i|>|\boldsymbol{q}|$. From general arguments it could be cast to the form

$$
\mathrm{Re}\,R_{(2)}^{(2)} \approx \frac{[k_1^\mu(q_1-k_1)^\nu k_2^\alpha(q_2-k_2)^\beta]_\perp}{(\boldsymbol{q}^2)^2}\,R_{\mu\nu\alpha\beta}\,, \qquad (3)
$$

with some dimensionless tensor matrix $R_{\mu\nu\alpha\beta}$ independent of $\boldsymbol{k}_i, \boldsymbol{q}_i$. Then for $M_{(2)}^{(2)}$ one gets

$$\int \frac{d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2}{\pi^2} \frac{\mathrm{Re}\, R_{(2)}^{(2)}}{\boldsymbol{k}_1^2 \boldsymbol{k}_2^2 (\boldsymbol{q}_1 - \boldsymbol{k}_1)^2 (\boldsymbol{q}_2 - \boldsymbol{k}_2)^2} \approx \frac{I}{(\boldsymbol{q}^2)^2} \frac{4(\beta_+ - \beta_-)}{(\beta_- + \beta_+)^2} \ln \frac{\boldsymbol{q}_{\max}^2}{\boldsymbol{q}_1^2} \ln \frac{\boldsymbol{q}_{\max}^2}{\boldsymbol{q}_2^2}\,, \tag{4}$$

where I is the unit matrix and $q_{\max}$ is the upper integration limit $q_{\max} \simeq 1/R$, R is the nucleus radius. Such enhancement is absent if the number of exchanged photons from every ion exceeds two. In fact, the amplitudes $M_{(n)}^{(2)}$, $M_{(2)}^{(n)}$, $n > 2$ contain only the first power of large logarithm, whereas $M_{(n)}^{(m)}$, $m, n > 2$ do not contain such a factor at all, because the corresponding loop momenta integrals are convergent in both infrared and ultraviolet regions and one can safely put $|\boldsymbol{q}_{1(2)}| = 0$ over loop integrations.

TABLE I

The coefficients for formula (2). The brackets denote index permutation, *e.g.*, $(12) \equiv 12 + 21$.

n	R_{ijkl}	a_n	b_n	c_n	d_n
1	$R_{(12)(34)}$	q_-	$q_- - q_1$	—	—
2	$R_{(34)(12)}$	$q_1 - q_+$	q_+	—	—
3	R_{1324}	q_-	$q_- - k_1$	$q_- - k_1 - k_2$	$q_- - q_1 - k_2$
4	R_{1423}	q_-	$q_- - k_1$	$q_- - q_2 + k_2 - k_1$	$-q_+ + k_2$
5	R_{2314}	q_-	$q_- - q_1 + k_1$	$q_- - q_1 + k_1 - k_2$	$-q_+ + q_2 - k_2$
6	R_{2413}	q_-	$q_- - q_1 + k_1$	$-q_+ + k_1 + k_2$	$-q_+ + k_2$
7	R_{4231}	$q_- - q_2 + k_2$	$-q_+ + k_1 + k_2$	$-q_+ + k_1$	q_+
8	R_{3241}	$q_- - k_2$	$q_- - q_1 + k_1 - k_2$	$-q_+ + k_1$	q_+
9	R_{4132}	$q_- - q_2 + k_2$	$q_- - q_2 + k_2 - k_1$	$-q_+ + q_1 - k_1$	q_+
10	R_{3142}	$q_- - k_2$	$q_- - k_1 - k_2$	$-q_+ + q_1 - k_1$	q_+
11	$R_{3(12)4}$	$q_- - k_2$	$-q_+ + q_2 - k_2$	—	—
12	$R_{4(12)3}$	$q_- - q_2 + k_2$	$-q_+ + k_2$	—	—

Finally, from very straightforward generalization of (3) it can be shown that the set of amplitudes with an odd number of exchanges with one or both nuclei is suppressed in the limit of wide angle production

$$M^{(2m)}_{(2n+1)} \sim O\left(\frac{|\,\boldsymbol{q}_1\,|}{|\,\boldsymbol{q}\,|}\right), \qquad M^{(2m+1)}_{(2n)} \sim O\left(\frac{|\,\boldsymbol{q}_2\,|}{|\,\boldsymbol{q}\,|}\right),$$
$$M^{(2m+1)}_{(2n+1)} \sim O\left(\frac{|\,\boldsymbol{q}_1\,||\,\boldsymbol{q}_2\,|}{|\,\boldsymbol{q}^2\,|}\right). \tag{5}$$

3. Conclusions

We have obtained the complex expression for the amplitude of the process $2\gamma+2\gamma \to \mathrm{l}^+\mathrm{l}^-$ and found its wide angle limit. We have shown the existence of logarithmic enhancement of the mentioned contribution to production of lepton pairs for the case of large transverse momentum. In the case of multiphoton collisions it was shown that such logarithmic enhancement is missing.

The result reveals that the familiar eikonalization of Coulomb distortion breaks down for oppositely moving heavy ion Coulombic fields. In such a way the braking of eikonalization in QED suggests the complete failure of linear $k_\perp$-factorization in multigluon perturbative QCD processes.

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