Vol. 39 (2008) *ACTA PHYSICA POLONICA B* No 11

GRAVITY THEORY BASED ON MASS–ENERGY EQUIVALENCE

Stephen A. Lipinski, Hubert M. Lipinski

Unified Gravity Corporation
P.O. Box 60940, Palo Alto, CA 94306, USA
feedback@unifiedgravity.com

(Received May 9, 2008; revised version received July 8, 2008)

In this theory, an object is a mass density field in the fabric of space (FS) that satisfies mass–energy equivalence. In contrast with General Relativity (GR), the theory posits a preferred reference frame — namely the reference frame in which the FS is at rest. Also in contrast with GR, gravity between two objects results from the interaction of their mass density fields integrated over the entire FS. This interaction results in two types of gravity: Type I gravity which includes classical gravity, and under certain conditions, Type II gravity which includes a very strong wave gravity. Gravity exerted by large on small objects reduces to classical gravity. Gravity exerted by small on large objects is 3 times the classical value at small kinetic energies. When the small object becomes relativistic, then gravity becomes much larger. Every object has a gravity wavelength, and for the object being acted upon, classical type gravity occurs at distances less than its gravity wavelength while wave gravity occurs at distances greater than its gravity wavelength. The theory yields a set of 8 logarithmic singularities in the gravity force as well as a first-order singularity in the gravity potential. If the FS is quantized into discrete units, these singularities act on the FS to effect changes and interactions in mass density fields instantaneously. As a result, gravity acts instantaneously. We suggest that the 3 degree K cosmic background radiation results from kinetic energy released by the FS units. The theory then predicts that the rest mass of each FS unit is 2 proton masses and its characteristic length is approximately 2 mm. We extend the gravity theory to photons and predict the same results as GR for the classical experimental tests as well as for the change in period of binary pulsars. Finally, we show that the gravity theory makes possible a derivation of the Coulomb force.

PACS numbers: 04.50.–h, 04.80.Cc, 04.90.+e, 95.30.Sf

1. Introduction

The observation of the cosmic background radiation suggests the existence of a preferred reference frame and calls into question the relativistic invariance foundation of General Relativity (GR). Even with GR's widespread acceptance and numerous precise validations (Will 1993), we can ask whether there is another gravity theory that might explain the nature of the cosmic background radiation and its preferred reference frame.

The derivation of such a gravity theory began as an attempt to answer where kinetic energy is stored and how the storage of kinetic energy affects gravity. Since we could find no answers to these questions, we started with the only equation that seemed relevant, namely that of mass–energy equivalence (Einstein 1905).

We believed that both rest mass and kinetic energy distort the fabric of space (FS) — not space-time as in GR. Accordingly, we looked for a rest mass and kinetic energy density function that when integrated over all space would give the answer predicted by mass–energy equivalence. We found only one function and that was in the table of Fourier cosine transforms of Bessel functions (Erdelyi, Magnus, Tricomi 1954). The transform was originally derived by Weber and is also called a Weber discontinuous integral.

Thus we hypothesize that an object is the following mass density field $D_{\rm G}(r)$ in the FS:

$$D_{\rm G}(r) = M/4\pi\lambda_{\rm G}\, J_0(r/\lambda_{\rm G})\cos(vr/c\lambda_{\rm G})/r^2\,, \tag{1}$$

where M is the rest mass of the object, $\lambda_{\rm G}$ is its gravity wavelength, J_0 is the 0th order Bessel function of the first kind, r is the distance from the object, v is the speed of the object, and c is the speed of light. The J_0 Bessel function (also called a cylindrical harmonic) corresponds to the space distortion due to rest mass, while the cosine function corresponds to the space distortion due to kinetic energy.

The integral over all space in spherical coordinates of the mass density field $D_{\rm G}(r)$ reduces to the Fourier cosine transform of the J_0 Bessel function (Erdelyi, Magnus, Tricomi 1954):

$$\begin{aligned}\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \int_0^{\infty} dr r^2 D_{\rm G}(r) &= M/\sqrt{(1-v^2/c^2)}\,, \qquad v/c<1\,,\\ &= 0\,, \qquad v/c>1\,. \end{aligned} \tag{2}$$

The integral over all space of the mass density field $D_{\rm G}(r)$ now predicts mass–energy equivalence and also that the speed of an object is limited by the speed of light. The mass density field may have $v = 0$ since in that case the integral reduces to the Bessel function normalization integral (Wolfram 1998):

$$M/4\pi\lambda_{\rm G}\int_0^{2\pi}d\varphi\int_0^{\pi}d\theta\sin\theta\int_0^{\infty}dr\,J_0(r/\lambda_{\rm G})=M\,. \tag{3}$$

The mass density field differs from those in current theories of gravity since it includes negative values and must be integrated to infinity. We interpreted the negative values as resulting from rest mass waves and kinetic energy waves in the FS. The speed of an object is defined relative to the FS since the mass density field of the object exists in the FS. Thus the reference frame in which the FS is at rest is the preferred reference frame. This conclusion is consistent with the cosmic background radiation, since we suggest that this radiation results from kinetic energy being released from the FS.

In contrast with GR, gravity between two objects results from the interaction of the two individual mass density fields integrated over the entire FS. For the object being acted upon, the theory predicts either classical type gravity or wave gravity, depending upon the distance between the two objects and the object's gravity wavelength. The derivation of the gravity force is exact and the values of the constants in the theory are determined from observational data.

If gravity acts at the speed of light, the integration of the mass density fields to infinity poses a serious problem. This problem is resolved as the theory includes a set of 8 logarithmic singularities (*i.e.* proportional to $-\log(\varepsilon)|_{\varepsilon=0}$) and a first-order singularity (*i.e.* proportional to $1/\varepsilon|_{\varepsilon=0}$) in the gravity force. As we develop later, if the FS is quantized into discrete units, the singularity equations show that both types of singularities act on the FS to effect any changes and interactions in the mass density fields. Since the singularities are infinite forces, the changes and interactions occur instantaneously and hence gravity acts instantaneously.

We also describe the gravitational interaction of photons and compare the predictions of the gravity theory with the experimental tests of GR. Our theory predicts the same results for the classical tests as well as for the change in period of binary pulsars.

It is noteworthy that the singularities of Type I and Type II gravity display many of the same characteristics as phenomena such as Cooper pairs, Brownian motion, and Aurora Borealis whose forces are not well explained by conventional theory.

We should add what this theory is not. It is not a quantum theory of gravity but rather a classical theory, even though the FS is quantized into discrete units. This theory is not Lorentz invariant since gravity acts instantaneously. Gravity is also not symmetric in that gravity exerted by an object A on an object B is not the same as gravity exerted by object B on object A. As we show later, Newton's third law is preserved in a two-body system, but the gravity theory does violate the equivalence principle in a novel way. Also, the gravity force is not applied directly by the object exerting the gravity, but by the FS. As a result, energy and momentum are conserved if the FS is included. This is possible since the theory requires that the FS has mass and kinetic energy and we suggest, based on observational data, that each FS unit has a rest mass of 2 proton masses, a characteristic length of approximately 2mm, and the capability to store and transfer kinetic energy as vibration energy.

2. Gravity constants

We hypothesize that the gravity wavelength $\lambda_{\rm G}$ of an object is linearly proportional to its rest mass and is referenced to the gravity wavelength $\lambda_{\rm FS}$ of a FS unit as follows:

$$\lambda_{\rm G} = \lambda_{\rm FS}\, M/m_{\rm FS}\,, \tag{4}$$

where M is the object rest mass and $m_{\rm FS}$ is the rest mass of a FS unit. We use the atomic mass formula to replace the rest mass of a FS unit:

$$a_{\rm FS} = N_{\rm A} m_{\rm FS} \quad \left[\text{kg mole}^{-1}\right]\,, \tag{5}$$

where $a_{\rm FS}$ is the atomic mass of a FS unit in kilograms per mole and $N_{\rm A}$ is Avogadro's number. We define a constant κ and rewrite the gravity wavelength of an object as:

$$\kappa = a_{\rm FS}/\lambda_{\rm FS} \quad \left[\text{kg mole}^{-1}\ \text{m}^{-1}\right]\,, \tag{6}$$

$$\lambda_{\rm G} = N_{\rm A}/\kappa\, M\,. \tag{7}$$

The constant κ is the FS atomic mass linear density since the gravity wavelength $\lambda_{\rm FS}$ of a FS unit is its characteristic length.

To determine the constant $N_{\rm A}/\kappa$ from observational data, we suggest that the yellow–green glow occurring in nuclear reactions (Rohringer 1968) corresponds to the electron gravity wavelength. As we show later, wave gravity exerted by fusion or fission byproducts in motion strongly vibrates electrons at the electron gravity wavelength. As a result, the electrons release the vibration kinetic energy as radiation at the electron gravity wavelength. We divide the electron gravity wavelength $\lambda_e \approx 0.55\times10^{-6}$ m (yellow–green light) by its rest mass m_e and obtain:

$$N_{\rm A}/\kappa = \lambda_e/m_e \approx 6.0\times10^{23} \quad \left[\text{m kg}^{-1}\right]\,. \tag{8}$$

This result provides observational support that the value of $N_{\rm A}/\kappa$ is Avogadro's number and that the value of the FS atomic mass linear density κ is one.

Wave gravity can transfer kinetic energy not only to electrons, but also to FS units by strongly vibrating the units at the FS gravity wavelength. As with electrons, the units release the vibration kinetic energy as radiation which we should observe at the FS gravity wavelength. One type of radiation connected with the FS is the 3 degree K cosmic background radiation. If we assume that this radiation results from cosmic kinetic energy stored in the FS at the instant of the Big Bang and released since that time, then the FS gravity wavelength λ_{FS} is the wavelength of the cosmic background radiation (Penzias, Wilson 1965):

$$\lambda_{\mathrm{FS}} \approx 2.0 \times 10^{-3}\ \mathrm{m}\,. \tag{9}$$

We now use Eq. (7) for the gravity wavelength to obtain the rest mass of a FS unit:

$$m_{\mathrm{FS}} = \lambda_{\mathrm{FS}}/(N_{\mathrm{A}}/\kappa) = 2m_p\,. \tag{10}$$

The result is so close to 2 proton masses that we hypothesize that the rest mass m_{FS} of a FS unit is indeed 2 proton masses ($2m_p$). We would further hypothesize that a FS unit is, in fact, a vibrating proton–antiproton pair.

3. Gravity fields

We show later that the gravity force exerted by an object A on an object B that reduces to classical gravity is:

$$F_{\mathrm{G}}(r_{\mathrm{B}}) = A_{\mathrm{G}} G m_{\mathrm{A}} m_{\mathrm{B}} J_0(r_{\mathrm{B}}/\lambda_{\mathrm{B}})/r_{\mathrm{B}}^2\,, \tag{11}$$

where A_{G} is an amplification factor, G is the gravitational constant, m_{A} is the mass of object A, m_{B} is the mass of object B, r_{B} is the distance of object A from object B, and λ_{B} the gravity wavelength of object B. For example, aside from relativistic rest mass corrections, $A_{\mathrm{G}} = 1$ for gravity exerted by large on small objects. The deviation of gravity from an inverse square force arises from the J_0 Bessel function.

Classical gravity corresponds to gravity in the near-zero region of the J_0 Bessel function. If $r_{\mathrm{B}}/\lambda_{\mathrm{B}} \ll 1$, the near-zero expansion of the Bessel function $J_0(x) = 1 - x^2/4 + \ldots$ shows that $J_0(r_{\mathrm{B}}/\lambda_{\mathrm{B}}) \approx 1$ and we have the classical gravity force times the amplification factor. Since an object's gravity wavelength in meters is 6.0×10^{23} times its rest mass in kilograms, an object's gravity wavelength is extremely large except for elementary particles and nuclei. As a result, the near-zero region for larger objects is very large and gravity reduces to classical gravity for most objects and distances.

Wave gravity occurs in the region $r_{\mathrm{B}}/\lambda_{\mathrm{B}} > 1$ as the Bessel function $J_0(r_{\mathrm{B}}/\lambda_{\mathrm{B}})$ becomes harmonic. For example, in the asymptotic region $r_{\mathrm{B}}/\lambda_{\mathrm{B}} \gg 1$ we have:

$$\begin{aligned} J_0(r_{\rm B}/\lambda_{\rm B}) &\approx \sqrt{(2\lambda_{\rm B}/\pi r_{\rm B})}\cos(r_{\rm B}/\lambda_{\rm B}-\pi/4)\,, &(12)\\ F_{\rm G}(r_{\rm B}) &\approx A_{\rm G}Gm_{\rm A}m_{\rm B}\sqrt{(2\lambda_{\rm B}/\pi)}\cos(r_{\rm B}/\lambda_{\rm B}-\pi/4)/r_{\rm B}^{5/2}\,. &(13)\end{aligned}$$

However, the wave gravity region for planetary masses like the Earth or Sun is very far away since $\lambda_{\rm Earth} = 3.8\times10^{32}$ light-years and $\lambda_{\rm Sun} = 1.3\times10^{38}$ light-years. These gravity wavelengths should be compared to the size of the observable universe — about 4.2×10^{10} light-years. Thus gravity for planetary masses as for most objects is a classical $1/r^2$ force. But, as we show later, gravity is on the average larger than classical gravity since $A_{\rm G} = 3$ in the case of gravity exerted by small on large objects and $A_{\rm G}$ has a logarithmic singularity as the masses become equal. In the former case, when the small object is relativistic, $A_{\rm G} \gg 3$ and gravity is also very much larger than classical gravity.

If we examine the mass density field in the asymptotic region $r/\lambda_{\rm G} \gg 1$, we obtain:

$$\begin{aligned} D_{\rm G}(r) \approx{}& M/4\pi\lambda_{\rm G}\,\sqrt{(\lambda_{\rm G}/2\pi)}\\ &\times\{\cos((1+v/c)r/\lambda_{\rm G}-\pi/4)+\cos((1-v/c)r/\lambda_{\rm G}-\pi/4)\}/r^{5/2}\,. \quad (14)\end{aligned}$$

This asymptotic behavior suggests that gravity can be best understood using wave theory. In this view, the kinetic energy creates a mass density wave that has no carrier but only two sidebands. This is an extremely efficient method to transfer information or energy. Wave theory also suggests that the mass density field of the receiving object acts as a receiving antenna and demodulator. Accordingly, we hypothesize that the gravity force occurs at the mass density field level.

The calculation of the gravity force experienced by an object follows the standard calculation of classical gravity exerted by an object with a spherically symmetric mass density. However, we use the mass density fields of both objects and a coupling constant between the two mass density fields. In order that gravity experienced by a very large object reduces to classical gravity, the coupling constant must be $G4\pi\lambda_{\rm G}$ where G is the gravitational constant and $\lambda_{\rm G}$ is its gravity wavelength. Thus the gravity force exerted by an object A with mass $m_{\rm A}$, gravity wavelength $\lambda_{\rm A}$, and speed $v_{\rm A}$ on an object B with mass $m_{\rm B}$, gravity wavelength $\lambda_{\rm B}$, and speed $v_{\rm B}$ is:

$$\begin{aligned} F_{\rm G}(r_{\rm B}) ={}& Gm_{\rm A}m_{\rm B}/4\pi\lambda_{\rm A}\int_0^\infty dr_{\rm A}r_{\rm A}^2\int_0^\pi d\theta\sin\theta\int_0^{2\pi}d\varphi J_0(r_{\rm A}/\lambda_{\rm A})\\ &\times\cos\left(v_{\rm A}r_{\rm A}/c\lambda_{\rm A}\right)/r_{\rm A}^2\;J_0\left(\left(r_{\rm B}^2+r_{\rm A}^2-2r_{\rm B}r_{\rm A}\cos\theta\right)^{1/2}/\lambda_{\rm B}\right)\end{aligned}$$

$$\times \cos\left(v_{\rm B}\left(r_{\rm B}^2+r_{\rm A}^2-2r_{\rm B}r_{\rm A}\cos\theta\right)^{1/2}/c\lambda_{\rm B}\right)$$
$$\times(r_{\rm B}-r_{\rm A}\cos\theta)/\left(r_{\rm B}^2+r_{\rm A}^2-2r_{\rm B}r_{\rm A}\cos\theta\right)^{3/2}\,. \tag{15}$$

We integrate over φ and twice by parts over θ, collect terms, and then make the substitution $x=(r_{\rm B}^2+r_{\rm A}^2-2r_{\rm B}r_{\rm A}\cos\theta)^{1/2}$ with the result:

$$F_{\rm G}(r_{\rm B})=Gm_{\rm A}m_{\rm B}/2\lambda_{\rm A}r_{\rm B}^2\int_0^\infty dr_{\rm A}\,J_0(r_{\rm A}/\lambda_{\rm A})\cos(v_{\rm A}r_{\rm A}/c\lambda_{\rm A})$$
$$\times\{[J_0((r_{\rm B}+r_{\rm A})/\lambda_{\rm B})\cos(v_{\rm B}(r_{\rm B}+r_{\rm A})/c\lambda_{\rm B})+J_0((r_{\rm B}-r_{\rm A})/\lambda_{\rm B})$$
$$\times\cos(v_{\rm B}(r_{\rm B}-r_{\rm A})/c\lambda_{\rm B})]-1/2\lambda_{\rm B}\int_{r_{\rm B}-r_{\rm A}}^{r_{\rm B}+r_{\rm A}}dx/x\left(r_{\rm A}^2-r_{\rm B}^2+x^2\right)/r_{\rm A}\;J_0'(x/\lambda_{\rm B})$$
$$\times\cos(v_{\rm B}x/c\lambda_{\rm B})+v_{\rm B}/c2\lambda_{\rm B}\int_{r_{\rm B}-r_{\rm A}}^{r_{\rm B}+r_{\rm A}}dx/x\left(r_{\rm A}^2-r_{\rm B}^2+x^2\right)/r_{\rm A}\;J_0(x/\lambda_{\rm B})$$
$$\times\sin(v_{\rm B}x/c\lambda_{\rm B})\}\,. \tag{16}$$

There are really three integrals here. The first integral which includes the two J_0 terms is gravity that arises from the density of space and is evaluated in Appendix A. The second is gravity that arises from the change in the density of space due to the rest mass and is evaluated in Appendix B. The third is gravity that arises from the change in the density of space due to kinetic energy and is evaluated in Appendix C. The integration shows that there are two types of gravity which we call Type I and Type II gravity.

4. Type I gravity

We now calculate what we call Type I gravity which reduces to classical gravity in the classical limit. We bring the integrals back together, grouping them by whether they contain a "$\cos(r_{\rm A}\ldots)$" or "$\sin(r_{\rm A}\ldots)r_{\rm A}$" term. We evaluate the "$\cos(r_{\rm A}\ldots)$" integrals first. If we define $s=r_{\rm B}/\lambda_{\rm B}$ and the functions $A(s)$, $B(s)$, and $C(s)$, we then have:

$$F_{\rm G1}(r_{\rm B})\;=\;Gm_{\rm A}m_{\rm B}\lambda_{\rm B}/4\lambda_{\rm A}r_{\rm B}^2\,A(s)\,, \tag{17}$$
$$A(s)\;=\;1/\lambda_{\rm B}\int_0^\infty dr_{\rm A}\,J_0(r_{\rm A}/\lambda_{\rm A})\;1/\pi\int_0^\pi d\theta\{\exp(iys)$$
$$\times[\cos(r_{\rm A}(y/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))+\cos(r_{\rm A}(y/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))]+\exp(izs)$$
$$\times[\cos(r_{\rm A}(z/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))+\cos(r_{\rm A}(z/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))]\}\,, \tag{18}$$

where

$$\begin{aligned} y &= \cos\theta + v_{\rm B}/c\,, \\ z &= \cos\theta - v_{\rm B}/c\,. \end{aligned}$$

$$F_{\rm G22}(r_{\rm B}) = G m_{\rm A} m_{\rm B} \lambda_{\rm B}/4\lambda_{\rm A} r_{\rm B}^2\, B(s)\,, \tag{19}$$

$$\begin{aligned} B(s) = {} & 1/\lambda_{\rm B} \int_0^\infty dr_{\rm A} J_0(r_{\rm A}/\lambda_{\rm A})\; 1/\pi \int_0^\pi d\theta \left(1-\cos^2\theta\right) \\ & \times\Big\{ \exp(iys)\left(s/iy - 1/(iy)^2\right) [\cos(r_{\rm A}(y/\lambda_{\rm B} + v_{\rm A}/c\lambda_{\rm A})) \\ & + \cos(r_{\rm A}(y/\lambda_{\rm B} - v_{\rm A}/c\lambda_{\rm A}))] + \exp(izs)\left(s/iz - 1/(iz)^2\right) \\ & \times[\cos(r_{\rm A}(z/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))+\cos(r_{\rm A}(z/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))]\Big\}, \end{aligned} \tag{20}$$

$$F_{\rm G31}(r_{\rm B}) = G m_{\rm A} m_{\rm B} \lambda_{\rm B}/4\lambda_{\rm A} r_{\rm B}^2\, C(s)\,, \tag{21}$$

$$\begin{aligned} C(s) = {} & -iv_{\rm B}/c\; 1/\lambda_{\rm B} \int_0^\infty dr_{\rm A} J_0(r_{\rm A}/\lambda_{\rm A})\; 1/\pi \int_0^\pi d\theta \\ & \times\{\exp(iys)[\cos(r_{\rm A}(y/\lambda_{\rm B} + v_{\rm A}/c\lambda_{\rm A})) \\ & + \cos(r_{\rm A}(y/\lambda_{\rm B} - v_{\rm A}/c\lambda_{\rm A}))]/iy \\ & - \exp(izs)[\cos(r_{\rm A}(z/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A})) \\ & + \cos(r_{\rm A}(z/\lambda_{\rm B} - v_{\rm A}/c\lambda_{\rm A}))]/iz\}\,. \end{aligned} \tag{22}$$

We now reverse the order of integration, noting that the integrals over $r_{\rm A}$ are the same Weber discontinuous integrals as for the mass density integral. However, as we show in Appendix D, the "$\sin(r_{\rm A}\ldots)r_{\rm A}$" terms cancel the integrals when the resulting inverse square root terms are imaginary. The integration over $r_{\rm A}$ gives:

$$\begin{aligned} A(s) = {} & 1/\pi \int_0^\pi d\theta \exp(iys)\Big[\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2 - (y + v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2} \\ & + \left(\lambda_{\rm B}^2/\lambda_{\rm A}^2 - (y - v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2} \Big] \\ & + 1/\pi \int_0^\pi d\theta \exp(izs)\Big[\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2 - (z + v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2} \\ & + \left(\lambda_{\rm B}^2/\lambda_{\rm A}^2 - (z - v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2} \Big]\,, \end{aligned} \tag{23}$$

$$\begin{aligned}
B(s) \;=\; & 1/\pi \int_0^\pi d\theta \left(1-\cos^2\theta\right) \exp(iys) \left(s/iy - 1/(iy)^2\right) \\
& \times \Big[\left(\lambda_B^2/\lambda_A^2 - (y + v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \\
& + \left(\lambda_B^2/\lambda_A^2 - (y - v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \Big] \\
& + 1/\pi \int_0^\pi d\theta \left(1-\cos^2\theta\right) \exp(izs) \left(s/iz - 1/(iz)^2\right) \\
& \times \Big[\left(\lambda_B^2/\lambda_A^2 - (z + v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \\
& + \left(\lambda_B^2/\lambda_A^2 - (z - v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \Big] ,
\end{aligned} \tag{24}$$

$$\begin{aligned}
C(s) \;=\; & -iv_B/c\; 1/\pi \int_0^\pi d\theta \exp(iys)/iy \\
& \times \Big[\left(\lambda_B^2/\lambda_A^2 - (y + v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \\
& + \left(\lambda_B^2/\lambda_A^2 - (y - v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \Big] \\
& + iv_B/c\; 1/\pi \int_0^\pi d\theta \exp(izs)/iz \\
& \times \Big[\left(\lambda_B^2/\lambda_A^2 - (z + v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \\
& + \left(\lambda_B^2/\lambda_A^2 - (z - v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \Big] .
\end{aligned} \tag{25}$$

The integrals are non-zero for all values of λ_B/λ_A, but the integration limits may be reduced so that the inverse square root terms are real. For the most part, when $\lambda_B/\lambda_A > 1$ (*i.e.* gravity exerted by object A on object B that has larger mass), the integral limits are 0 to π as the zeros of the Weber terms lie outside the integration interval. As λ_B/λ_A approaches one, the zeros of the Weber terms approach the integration interval and the limits must be carefully specified. When $\lambda_B/\lambda_A < 1$ (*i.e.* gravity exerted by object A on an object B that has smaller mass), the integration limits are the zeros of the Weber inverse square root terms even though we may display the limits as 0 to π.

We first show, however, that the Type I gravity force experienced by object B is an amplification factor multiplied by $J_0(r_{\rm B}/\lambda_{\rm B})/r_{\rm B}^2$. We do this by showing that the Type I gravity force satisfies Bessel's equation of order zero. If we add the function $A(s)$ and its second derivative, we obtain:

$$\begin{aligned}
& d^2/ds^2 A(s)+A(s)=1/\pi\int_0^\pi d\theta\exp(iys)\left[\left(1-\cos^2\theta\right)-yv_{\rm B}/c-\cos\theta v_{\rm B}/c\right]\\
& \times\left[\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(y+v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}+\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(y-v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}\right]\\
& +1/\pi\int_0^\pi d\theta\exp(izs)\left[\left(1-\cos^2\theta\right)+zv_{\rm B}/c+\cos\theta v_{\rm B}/c\right]\\
& \times\left[\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(z+v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}+\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(z-v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}\right].
\end{aligned}\tag{26}$$

Taking the derivative of $B(s)$ and dividing by s gives:

$$\begin{aligned}
& 1/s\ d/ds\ B(s)=1/\pi\int_0^\pi d\theta\exp(iys)(1-\cos^2\theta)\\
& \times\left[\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(y+v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}+\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(y-v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}\right]\\
& +1/\pi\int_0^\pi d\theta\exp(izs)\left(1-\cos^2\theta\right)\left[\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(z+v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}\right.\\
& \left.+\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(z-v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}\right].
\end{aligned}\tag{27}$$

The function $C(s)$ is the kinetic energy correction term that contributes to the second derivative of $A(s)$ to remove the first of its kinetic energy terms. Thus we take the second derivative of $C(s)$ to obtain:

$$\begin{aligned}
& d^2/ds^2\ C(s)=1/\pi\int_0^\pi d\theta\exp(iys)yv_{\rm B}/c\\
& \times\left[\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(y+v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}+\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(y-v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}\right]\\
& -1/\pi\int_0^\pi d\theta\exp(izs)zv_{\rm B}/c\\
& \times\left[\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(z+v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}+\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(z-v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}\right].
\end{aligned}\tag{28}$$

We show in Appendix E that the remaining kinetic energy term in the second derivative of $A(s)$ integrates to zero and so the Type I gravity force satisfies an equation that we compare to Bessel's equation:

$$d^2/ds^2\{A(s)+C(s)\} - 1/s\; d/ds\; B(s) + A(s) \;=\; 0\,, \qquad (29)$$

$$d^2/ds^2 J_n(s) + 1/s\; d/ds\; J_n(s) + \left(1-n^2/s^2\right) J_n(s) \;=\; 0\,. \qquad (30)$$

Thus the Type I gravity force is a Bessel function of order zero and the functions $A(s)$ and $C(s)$ are proportional to $J_0(s)$ and $B(s)$ to $-J_0(s)$. By grouping together the terms in s and noting that $\exp(is\cos\theta)$ is a Bessel generating function, we can replace the terms in s by $J_0(s)$. We begin with the $A(s)$ term, changing the integration variable to $t = \cos\theta$:

$$\exp(is\cos\theta) = J_0(s) + 2\sum_{n=1}^{\infty} i^n J_n(s)\cos(n\theta)\,, \qquad (31)$$

$$A(s) = J_0(s)\; 1/\pi \int_{-1}^{1} dt\left(1-t^2\right)^{-1/2}$$
$$\times\Big\{((t{+}v_{\rm B}/c{+}\alpha)(\beta{-}t{-}v_{\rm B}/c))^{-1/2}{+}((t+v_{\rm B}/c{+}\beta)(\alpha{-}t{-}v_{\rm B}/c))^{-1/2}$$
$$+((t{-}v_{\rm B}/c{+}\alpha)(\beta{-}t{+}v_{\rm B}/c))^{-1/2}{+}((t{-}v_{\rm B}/c{+}\beta)(\alpha{-}t{+}v_{\rm B}/c))^{-1/2}\Big\}, \qquad (32)$$

where

$$\alpha \;=\; \lambda_{\rm B}/\lambda_{\rm A}(1+v_{\rm A}/c)\,,$$
$$\beta \;=\; \lambda_{\rm B}/\lambda_{\rm A}(1-v_{\rm A}/c)\,.$$

When the integral limits are -1 to 1 or the Weber zeros, each integral has the following form where $K(m)$ is the complete elliptic integral K of the first kind (Wolfram 1996):

$$\int_{-1}^{1} dt\left(1-t^2\right)^{-1/2}(a+t)^{-1/2}(b-t)^{-1/2} = 2((1+a)(1+b))^{-1/2}K(m)\,, \qquad (33)$$

where $m = 2(a+b)/((1+a)(1+b))$.

When object A is much smaller than object B (*i.e.* $\lambda_{\rm B}/\lambda_{\rm A} \gg 1$), then $a \gg 1$ and $b \gg 1$, and the argument of the complete elliptic integral $K(m)$ is very small. We can then use the identity $K(0) = \pi/2$ to approximate each of the 4 integrals:

$$A(s) \;\approx\; 4J_0(s)(\alpha\beta)^{-1/2}\,, \qquad (34)$$

$$F_{\rm G1}(r_{\rm B}) \;\approx\; Gm_{\rm A}m_{\rm B}\left(1-v_{\rm A}^2/c^2\right)^{-1/2} J_0(r_{\rm B}/\lambda_{\rm B})/r_{\rm B}^2\,, \qquad (35)$$

where $\lambda_{\rm B}/\lambda_{\rm A} \gg 1$.

Now we take the classical limit in which the mass of object B is large (*i.e.* $r_{\rm B}/\lambda_{\rm B} \ll 1$) and recover classical gravity with a rest mass increase for object A relativistic:

$$F_{\rm G\ Classical}(r_{\rm B}) = Gm_{\rm A}m_{\rm B}\left(1 - v_{\rm A}^2/c^2\right)^{-1/2}/r_{\rm B}^2\,, \tag{36}$$

where $\lambda_{\rm B}/\lambda_{\rm A} \gg 1$, $r_{\rm B}/\lambda_{\rm B} \ll 1$.

When object A is much larger than object B (*i.e.* $\lambda_{\rm B}/\lambda_{\rm A} \ll 1$), then $\alpha \ll 1$ and $\beta \ll 1$. If $\lambda_{\rm B}/\lambda_{\rm A} \ll (1 - v_{\rm B}^2/c^2)$, then $m \ll 1$, $K(m) \approx \pi/2$, and $A(s) \approx 4J_0(s)(1 - v_{\rm B}^2/c^2)^{-1/2}$. Therefore the contribution of $A(s)$ to the gravity of a larger object on a smaller object is negligible.

We now examine the 8 logarithmic singularities in the functions $A(s)$ and $C(s)$ which occur whenever $\lambda_{\rm A}$, $\lambda_{\rm B}$, $v_{\rm A}$, and $v_{\rm B}$ satisfy one of the 4 following conditions:

$$\lambda_{\rm B}/\lambda_{\rm A} = (1 \pm v_{\rm B}/c)/(1 \pm v_{\rm A}/c)\,. \tag{37}$$

The singularities occur when the zeros of the Weber terms are coincident with the zeros at the edge of the Bessel integration region and result in a factor $1/(1-t)$ or $1/(1+t)$. Since the singularities are at the integration limits, this results in Type I gravity having two logarithmic singularities from each of its four terms.

For example, we evaluate the first $A(s)$ term in Eq. (32) when the Weber zeros in Eq. (33) occur at $a = 2\lambda_{\rm B}/\lambda_{\rm A} - 1$ and $b = 1$, corresponding to $\lambda_{\rm B}/\lambda_{\rm A} = (1 + v_{\rm B}/c)/(1 - v_{\rm A}/c)$. The integration region is from -1 to 1 since $\lambda_{\rm B}/\lambda_{\rm A} \geq 1$ and we have:

$$A_1(s) = J_0(s)\ 2/\pi\ ((1+a)(1+b))^{-1/2}K(m)\,, \tag{38}$$

where

$$\begin{aligned} m &= 2(a+b)/((1+a)(1+b))\,, \\ a &= \lambda_{\rm B}/\lambda_{\rm A}(1 + v_{\rm A}/c) + v_{\rm B}/c\,, \\ b &= \lambda_{\rm B}/\lambda_{\rm A}(1 - v_{\rm A}/c) - v_{\rm B}/c\,. \end{aligned}$$

The argument of the complete elliptic integral $K(m)$ is close to 1 so $K(m) \approx -1/2\log(1-m) + \log(4)$ and we obtain at the singularity:

$$\begin{aligned} F_{\rm G11}(r_{\rm B}) &= Gm_{\rm A}m_{\rm B}J_0(r_{\rm B}/\lambda_{\rm B})/r_{\rm B}^2\ 1/8\pi\ (\lambda_{\rm B}/\lambda_{\rm A})^{1/2} \\ &\quad \times\{\log(32\lambda_{\rm B}/\lambda_{\rm A}/(\lambda_{\rm B}/\lambda_{\rm A} - 1)) - \log(b-1)|_{\rm b=1}\}\,, \end{aligned} \tag{39}$$

where $\lambda_{\rm B}/\lambda_{\rm A} = (1 + v_{\rm B}/c)/(1 - v_{\rm A}/c)$.

The other singularity in the first term occurs at $a=1$ and $b=2\lambda_B/\lambda_A-1$, corresponding to $\lambda_B/\lambda_A = (1-v_B/c)/(1+v_A/c)$. The integration region is from -1 to b since $\lambda_B/\lambda_A \le 1$. Such integrals with mixed limits have the following form (Wolfram 1996):

$$\int_{-1}^{b} dt\left(1-t^2\right)^{-1/2}(a+t)^{-1/2}(b-t)^{-1/2} = 2(2(a+b))^{-1/2}K(1/m)\,. \quad (40)$$

The argument of the complete elliptic integral $K(1/m)$ is again close to 1 and we obtain:

$$\begin{aligned} F_{G11}(r_B) &= Gm_A m_B J_0(r_B/\lambda_B)/r_B^2\; 1/8\pi\,(\lambda_B/\lambda_A)^{1/2} \\ &\quad \times\{\log(32\lambda_B/\lambda_A/(1-\lambda_B/\lambda_A)) - \log(a-1)|_{a=1}\}\,, \end{aligned} \quad (41)$$

where $\lambda_B/\lambda_A = (1-v_B/c)/(1+v_A/c)$.

The singularities in the $B(s)$ terms give rise both to gravity exerted by a larger object A on a smaller object B and to gravity exerted by a smaller object A on a larger object B. We also show that this gravity reduces to classical gravity in the classical limit. We first note that the singularities in $B(s)$ are removed by differentiation so that its $J_0(s)$ nature arises from the combination of both the $1/iy$ and $1/(iy)^2$ terms. As a result, we replace the common terms in s and the Bessel generating functions (*i.e.* "$(s-1/iy)\exp(iys)$" and "$(s-1/iz)\exp(izs)$") by $-J_0(s)$. In addition, we change the integration variable to $t=\cos\theta$:

$$\begin{aligned} B(s) &= -J_0(s)\,1/i\pi\int_{-1}^{1} dt\left(1-t^2\right)^{1/2} \\ &\quad \times\Big\{\Big[(t+v_B/c+\alpha)^{-1/2}(\beta-t-v_B/c)^{-1/2} \\ &\quad +(t+v_B/c+\beta)^{-1/2}(\alpha-t-v_B/c)^{-1/2}\Big]/(t+v_B/c) \\ &\quad +\Big[(t-v_B/c+\alpha)^{-1/2}(\beta-t+v_B/c)^{-1/2} \\ &\quad +(t-v_B/c+\beta)^{-1/2}(\alpha-t+v_B/c)^{-1/2}\Big]/(t-v_B/c)\Big\}\,. \end{aligned} \quad (42)$$

For gravity exerted by a larger object A on a smaller object B, the zeros of the Weber terms lie inside the integration interval 0 to π and so the integration interval is really between the zeros of each Weber term. We shift the integration variable so that each integral has the following form where $\Pi(n|m)$ is the complete elliptic integral Π of the third kind (Wolfram 1996):

$$\int_{-a}^{b} dt(e+t)^{1/2}(d-t)^{1/2}(a+t)^{-1/2}(b-t)^{-1/2}/t$$
$$= 2(d-b)((a+d)(b+e))^{-1/2}\{\Pi((a+b)/(a+d)|m)$$
$$+e/b\,\Pi(d(a+b)/(b(a+d))|m)\}\,, \qquad (43)$$

where $m = 2(a+b)/((a+d)(b+e))$.

If $\lambda_{\rm B}/\lambda_{\rm A} \ll 1$, then $a \ll 1$ and $b \ll 1$. If $(a+b) \ll ed$, then $m \ll 1$ and we can use the identity $\Pi(n|0) = (1-n)^{-1/2}\pi/2$ to approximate the terms. The integral evaluates to $\{\pi(d-b)^{1/2}(b+e)^{-1/2}(1-ie(ab)^{-1/2})\}$. Thus gravity exerted by a much larger object A on a smaller object B is:

$$B(s) \approx 4J_0(s)(\alpha\beta)^{-1/2}(ed)^{1/2}\,, \qquad (44)$$
$$F_{G22}(r_{\rm B}) \approx Gm_{\rm A}m_{\rm B}(1-v_{\rm A}^2/c^2)^{-1/2}(1-v_{\rm B}^2/c^2)^{1/2}J_0(r_{\rm B}/\lambda_{\rm B})/r_{\rm B}^2\,, \qquad (45)$$

where $\lambda_{\rm B}/\lambda_{\rm A} \ll \left(1-v_{\rm B}^2/c^2\right)$.

We note that the gravity exhibits a rest mass increase for object A relativistic, but a rest mass decrease for object B relativistic. We now take the classical limit in which the mass of object B is large (*i.e.* $r_{\rm B}/\lambda_{\rm B} \ll 1$) and we obtain classical gravity with a rest mass increase for object A relativistic, but with a rest mass decrease for object B relativistic:

$$F_{\rm G\ Classical}(r_{\rm B}) = Gm_{\rm A}m_{\rm B}\left(1-v_{\rm A}^2/c^2\right)^{-1/2}\left(1-v_{\rm B}^2/c^2\right)^{1/2}/r_{\rm B}^2\,, \qquad (46)$$

where

$$\lambda_{\rm B}/\lambda_{\rm A} \ll \left(1-v_B^2/c^2\right)\,, \qquad r_{\rm B}/\lambda_{\rm B} \ll 1\,.$$

When object A is smaller than object B, the integration limits are the Bessel limits. We shift the integration variable so that each integral has the following form (Wolfram 1996):

$$\int_{-e}^{d} dt(e+t)^{1/2}(d-t)^{1/2}(a+t)^{-1/2}(b-t)^{-1/2}/t$$
$$= 2(b-d)((a+d)(b+e))^{-1/2}\{-(b+e)/b\,K(m)$$
$$+\Pi(2/(b+e)|m) + e/b\,\Pi(2b/((b+e)d))|m)\}\,, \qquad (47)$$

where $m = 2(a+b)/((a+d)(b+e))$.

If $\lambda_B/\lambda_A \gg 1$, then $a \gg 1$, $b \gg 1$, $m \ll 1$, and we can use the identities $K(0) = \pi/2$ and $\Pi(n|0) = (1-n)^{-1/2}\pi/2$ to approximate the terms. Consequently the $K(m)$ term cancels the first $\Pi(n|m)$ term and we have:

$$B(s) \;\approx\; 4J_0(s)(\alpha\beta)^{-1/2}(ed)^{1/2}\,, \tag{48}$$

$$F_{G22}(r_B) \;\approx\; Gm_Am_B(1-v_A^2/c^2)^{-1/2}(1-v_B^2/c^2)^{1/2}J_0(r_B/\lambda_B)/r_B^2\,, \tag{49}$$

where $\lambda_B/\lambda_A \gg 1$.

While this result implies that gravity exerted by a smaller object A on a much larger object B is now twice the classical value, it is in fact three times the classical value since we show later that Type II gravity also gives the same result. However, in the case of Type II gravity, the gravity is only the same for $v_A/c \ll 1$. As object A becomes relativistic, Type II gravity becomes much larger than just a relativistic increase in the rest mass of object A.

We conclude Type I gravity by evaluating the kinetic energy correction term $C(s)$. We show that $C(s)$ is negligible in the classical limit when object A is larger than object B and integrates to zero when object A is smaller than object B. As with $A(s)$, we replace the terms in s by $J_0(s)$ and change the integration variable to $t = \cos\theta$ to obtain:

$$\begin{aligned} C(s) \;=\; & -v_B/c\; J_0(s)\; 1/\pi \int_{-1}^{1} dt\left(1-t^2\right)^{-1/2} \\ & \times\Big\{\Big[(t+v_B/c+\alpha)^{-1/2}(\beta-t-v_B/c)^{-1/2} \\ & +(t+v_B/c+\beta)^{-1/2}(\alpha-t-v_B/c)^{-1/2}\Big]/(t+v_B/c) \\ & -\Big[(t-v_B/c+\alpha)^{-1/2}(\beta-t+v_B/c)^{-1/2} \\ & +(t-v_B/c+\beta)^{-1/2}(\alpha-t+v_B/c)^{-1/2}\Big]/(t-v_B/c)\Big\}\,. \end{aligned} \tag{50}$$

For gravity exerted by a larger object A on a smaller object B, the zeros of the Weber terms lie inside the integration interval 0 to π and so the integration interval is between the zeros of each Weber term. We shift the integration variable so that each integral has the following form (Wolfram 1996):

$$\begin{aligned} & \int_{-a}^{b} dt(e+t)^{-1/2}(d-t)^{-1/2}(a+t)^{-1/2}(b-t)^{-1/2}/t \\ & = 2((a+d)(b+e))^{-1/2}\{1/d\;K(m) + (d-b)/bd\;\Pi(d(a+b)/(b(a+d))|m)\}, \end{aligned} \tag{51}$$

where $m = 2(a+b)/((a+d)(b+e))$.

If $\lambda_B/\lambda_A \ll 1$, then $a \ll 1$ and $b \ll 1$. If $(a+b) \ll de$ then $m \ll 1$, and we can use the identities $K(0) = \pi/2$ and $\Pi(n|0) = (1-n)^{-1/2}\pi/2$ to approximate the terms. The integral evaluates to $\{\pi(de)^{-1/2}(1/d - i(ab)^{-1/2})\}$ with the result that the imaginary part of the 4 terms in $C(s)$ cancel and the real part is negligible.

When object A is smaller than object B, the integration limits are the Bessel limits. The first and fourth terms and the second and third terms cancel as they are mirror images with respect to the integration interval and occur with opposite sign. Thus the kinetic energy correction term $C(s)$ integrates to zero.

5. Type II gravity

We can follow a similar procedure for Type II gravity and find that this gravity is far stronger than classical gravity. For Type II gravity, we evaluate the "$\sin(r_A \ldots)/r_A$" integrals. We define $s = r_B/\lambda_B$ and the functions $E(s)$ and $F(s)$ as follows:

$$\begin{aligned} F_{G23}(r_B) &= G m_A m_B \lambda_B / 4\lambda_A r_B^2 \, E(s)\,, \qquad (52)\\ E(s) &= \int_0^\infty dr_A/r_A \; J_0(r_A/\lambda_A)\; i/\pi \int_0^\pi d\theta \left(1-\cos^2\theta\right) \\ &\times \Big\{ \exp(iys)\left(1/(iy)^3 - s/(iy)^2\right) \Big[\sin(r_A(y/\lambda_B + v_A/c\lambda_A)) \\ &+ \sin(r_A(y/\lambda_B - v_A/c\lambda_A))\Big] \\ &+ \exp(izs)\left(1/(iz)^3 - s/(iz)^2\right) \Big[\sin(r_A(z/\lambda_B + v_A/c\lambda_A)) \\ &+ \sin(r_A(z/\lambda_B - v_A/c\lambda_A))\Big]\Big\}\,, \qquad (53) \end{aligned}$$

where

$$\begin{aligned} y &= \cos\theta + v_B/c\,, \\ z &= \cos\theta - v_B/c\,. \end{aligned}$$

$$\begin{aligned} F_{G32}(r_B) &= G m_A m_B \lambda_B / 4\lambda_A r_B^2 \, F(s)\,, \qquad (54)\\ F(s) &= i v_B^2/c^2 \int_0^\infty dr_A/r_A \; J_0(r_A/\lambda_A)\; 1/\pi \int_0^\pi d\theta \int_0^1 dt \\ &\times \Big\{ \exp(iy_t s)\left(1/(iy_t)^3 - s/(iy_t)^2\right) \end{aligned}$$

$$\times\Big[\sin(r_{\mathrm{A}}(y_t/\lambda_{\mathrm{B}}+v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))+\sin(r_{\mathrm{A}}(y_t/\lambda_{\mathrm{B}}-v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\Big]$$
$$+\exp(iz_t s)\left(1/(iz_t)^3-s/(iz_t)^2\right)$$
$$\times\Big[\sin(r_{\mathrm{A}}(z_t/\lambda_{\mathrm{B}}+v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))+\sin(r_{\mathrm{A}}(z_t/\lambda_{\mathrm{B}}-v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\Big]\Big\}\,, \quad (55)$$

where

$$y_t = \cos\theta + v_{\mathrm{B}}t/c\,,$$
$$z_t = \cos\theta - v_{\mathrm{B}}t/c\,.$$

The $F(s)$ term is the kinetic energy correction term and has the same functional form as $E(s)$. When object A is the same size or larger than object B (*i.e.* $\lambda_{\mathrm{B}}/\lambda_{\mathrm{A}} \le 1$), $E(s)$ has a first-order singularity while $F(s)$ does not, so we neglect the $F(s)$ term. When object A is smaller than object B (*i.e.* $\lambda_{\mathrm{B}}/\lambda_{\mathrm{A}} > 1$), we show later that $F(s)$ integrates to zero.

To determine the functional form of $E(s)$, we first define the functions $P(s)$ and $Q(s)$ as:

$$E(s) = P(s) - sQ(s)\,, \quad (56)$$

$$P(s) = \int_0^\infty dr_{\mathrm{A}}/r_{\mathrm{A}}\; J_0(r_{\mathrm{A}}/\lambda_{\mathrm{A}})\; i/\pi \int_0^\pi d\theta\left(1-\cos^2\theta\right)$$
$$\times\Big\{\exp(iys)\;1/(iy)^3\Big[\sin(r_{\mathrm{A}}(y/\lambda_{\mathrm{B}}+v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))$$
$$+\sin(r_{\mathrm{A}}(y/\lambda_{\mathrm{B}}-v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\Big]$$
$$+\exp(izs)\;1/(iz)^3\Big[\sin(r_{\mathrm{A}}(z/\lambda_{\mathrm{B}}+v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))$$
$$+\sin(r_{\mathrm{A}}(z/\lambda_{\mathrm{B}}-v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\Big]\Big\}\,, \quad (57)$$

$$Q(s) = \int_0^\infty dr_{\mathrm{A}}/r_{\mathrm{A}}\; J_0(r_{\mathrm{A}}/\lambda_{\mathrm{A}})\; i/\pi \int_0^\pi d\theta\left(1-\cos^2\theta\right)$$
$$\times\Big\{\exp(iys)\;1/(iy)^2\Big[\sin(r_{\mathrm{A}}(y/\lambda_{\mathrm{B}}+v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))$$
$$+\sin(r_{\mathrm{A}}(y/\lambda_{\mathrm{B}}-v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\Big]$$
$$+\exp(izs)\;1/(iz)^2\Big[\sin(r_{\mathrm{A}}(z/\lambda_{\mathrm{B}}+v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))$$
$$+\sin(r_{\mathrm{A}}(z/\lambda_{\mathrm{B}}-v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\Big]\Big\}\,. \quad (58)$$

We can then write the following equation which removes the singularities in $E(s)$, add and subtract $Q(s)$, and use that $Q(s) = d/dsP(s)$ to obtain:

$$d/ds(1/s\; d/ds(P(s) - sQ(s))) = -d/ds(P''(s))\,, \tag{59}$$

$$d/ds(P''(s)+1/s\; P'(s)+P(s))-(Q''(s)+1/s\; Q'(s)+\left(1-1/s^2\right) Q(s))=0\,. \tag{60}$$

Since $Q(s) = d/dsP(s)$, this equation is zero only if both the equations in $P(s)$ and $Q(s)$ are zero. Thus $P(s)$ is the $J_0(s)$ Bessel function and $Q(s)$ is the $J_1(s)$ Bessel function. To try to simplify $E(s)$, we use that $P(s)$ satisfies Bessel's equation of order zero, $Q(s) = P'(s)$, and $Q(s)$ satisfies Bessel's equation of order one to obtain:

$$E(s) = -2/s\; Q(s) + sQ''(s)\,. \tag{61}$$

Rather than evaluate the Type II gravity force at this time, we calculate the Type II gravity potential as seen by object B. We integrate the second term in $E(s)$ twice by parts and find that the resulting integral cancels the first term in $E(s)$:

$$V_{\mathrm{G23}}(r_{\mathrm{B}}) \;=\; Gm_{\mathrm{A}}m_{\mathrm{B}}/4\lambda_{\mathrm{A}}\; V_E(s)\,, \tag{62}$$

$$V_E(s) \;=\; \int dsE(s)/s^2 = -P(s)/s\,. \tag{63}$$

Thus the Type II gravity potential is proportional to $-J_0(r_{\mathrm{B}}/\lambda_{\mathrm{B}})/r_{\mathrm{B}}$. We replace the Bessel generating functions in the $P(s)$ integrals by $J_0(s)$ as follows:

$$\begin{aligned} P(s) \;=\; & J_0(s) \int_0^\infty dr_{\mathrm{A}}/r_{\mathrm{A}}\; J_0(r_{\mathrm{A}}/\lambda_{\mathrm{A}})\; i/\pi \int_0^\pi d\theta \left(1-\cos^2\theta\right) \\ & \times\Big\{1/(iy)^3\Big[\sin(r_{\mathrm{A}}(y/\lambda_{\mathrm{B}}+v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))+\sin(r_{\mathrm{A}}(y/\lambda_{\mathrm{B}}-v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\Big] \\ & +1/(iz)^3\Big[\sin(r_{\mathrm{A}}(z/\lambda_{\mathrm{B}}+v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))+\sin(r_{\mathrm{A}}(z/\lambda_{\mathrm{B}}-v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\Big]\Big\}. \end{aligned} \tag{64}$$

We reverse the order of integration and note that the integrals over r_{A} are the Fourier sine transform $(a > 0)$:

$$\begin{aligned} \int_0^\infty dt/t\; J_0(at)\sin(xt) \;&=\; -\pi/2\,, && -\infty < x < -a\,, \\ &=\; \sin^{-1}(x/a)\,, && -a < x < a\,, \\ &=\; \pi/2\,, && a < x < \infty\,. \end{aligned} \tag{65}$$

We now examine the $P(s)$ integrals. When $\lambda_B/\lambda_A \le 1$, the Fourier transform provides three regions of integration. There is the region of the inverse sine with the limits of the inverse sine, and $-\pi/2$ and $\pi/2$ regions with the limits from the inverse sine to the Bessel limits. When $\lambda_B/\lambda_A > 1$, the integration limits are 0 to π from the Bessel limits and the inverse sine is incomplete. We examine this case later.

When object A is the same size or larger than object B (*i.e.* $\lambda_B/\lambda_A \le 1$), all the $P(s)$ integrals have a first-order singularity in the region of the inverse sine so we can neglect the other two regions:

$$\begin{aligned} P(s) &= -J_0(s)\,1/\pi \int_0^\pi d\theta \left(1-\cos^2\theta\right) \\ &\quad \times\Big\{1/y^3\Big[\sin^{-1}(y\lambda_A/\lambda_B + v_A/c) + \sin^{-1}(y\lambda_A/\lambda_B - v_A/c)\Big] \\ &\quad +1/z^3\Big[\sin^{-1}(z\lambda_A/\lambda_B + v_A/c) + \sin^{-1}(z\lambda_A/\lambda_B - v_A/c)\Big]\Big\}. \end{aligned} \tag{66}$$

If we change the integration variable to argument of the inverse sine, all the $P(s)$ integrals have the following form:

$$\int_{-1}^{1} dt(a+t)^{1/2}(b-t)^{1/2}\sin^{-1}(t)/(t-d)^3\,. \tag{67}$$

We then set the square root terms to their value at the singularity, evaluate the resulting integrals (Wolfram 1996), and keep only the term with the singularity which we express as $(1/\varepsilon|_{\varepsilon=0})$. As with $B(s)$ in Type I gravity, $P(s)$ exhibits a rest mass increase for object A relativistic and a rest mass decrease for object B relativistic:

$$P(s) = J_0(s)\lambda_A^2/\lambda_B^2\left(1-v_A^2/c^2\right)^{-1/2}\left(1-v_B^2/c^2\right)^{1/2}4/\pi\;(1/\varepsilon|_{\varepsilon=0}), \tag{68}$$

$$\begin{aligned} V_{G23}(r_B) &= -Gm_Am_B\lambda_A/\lambda_B\;J_0(r_B/\lambda_B)/r_B\left(1-v_A^2/c^2\right)^{-1/2} \\ &\quad \times\left(1-v_B^2/c^2\right)^{1/2}1/\pi\;(1/\varepsilon|_{\varepsilon=0})\,, \end{aligned} \tag{69}$$

where $\lambda_B/\lambda_A \le 1$.

Since the Type II gravity potential has a first-order singularity, the Type II gravity force experienced by object B is zero for distances less than its gravity wavelength. For distances greater than its gravity wavelength, a very large gravity force occurs whenever $J_0(r_B/\lambda_B)$ changes sign:

$$\begin{aligned} F_{G23}(r_B) &= Gm_Am_B\lambda_A/\lambda_B^2\;J_1(r_B/\lambda_B)/r_B\left(1-v_A^2/c^2\right)^{-1/2} \\ &\quad \times\left(1-v_B^2/c^2\right)^{1/2}1/\pi\,(1/\varepsilon|_{\varepsilon=0})\;, \end{aligned} \tag{70}$$

where J_1 is the 1st order Bessel function of the first kind and $r_{\rm B}/\lambda_{\rm B}$ is a zero of the J_0 Bessel function. For example, the first zero of the J_0 Bessel function occurs at $r_{\rm B}/\lambda_{\rm B} \approx 2.4$.

Since a force results in a change in momentum, we hypothesize that the Type II gravity force imparts a momentum addition to object B in the direction of the Type II gravity force as object B moves through the zeros of the J_0 Bessel function. For example, an alpha particle emitted from a fusion reaction transfers kinetic energy into vibrating electrons in the surrounding environment at the electron gravity wavelength.

As shown earlier, the electron gravity wavelength $\approx 0.55 \times 10^{-6}$ m. Thus at atomic distances which are about 10^{-10} m, gravity experienced by an electron is the small classical force and does not appear to affect atomic quantum mechanical phenomena.

We now examine Type II gravity when object A is smaller than object B (*i.e.* $\lambda_{\rm B}/\lambda_{\rm A} > 1$). In this region, the integration limits are 0 to π from the Bessel limits and the inverse sine is incomplete. We change the integration variable to $t = \cos\theta$ and obtain:

$$\begin{aligned} P(s) \; = \; & -J_0(s)\, 1/\pi \int_{-1}^{1} dt \left(1-t^2\right)^{1/2} \\ & \times\Big\{ \Big[\sin^{-1}(\lambda_{\rm A}/\lambda_{\rm B}(t+v_{\rm B}/c)+v_{\rm A}/c) \\ & + \sin^{-1}(\lambda_{\rm A}/\lambda_{\rm B}(t+v_{\rm B}/c)-v_{\rm A}/c)\Big]/(t+v_{\rm B}/c)^3 \\ & + \Big[\sin^{-1}(\lambda_{\rm A}/\lambda_{\rm B}(t-v_{\rm B}/c)+v_{\rm A}/c) \\ & + \sin^{-1}(\lambda_{\rm A}/\lambda_{\rm B}(t-v_{\rm B}/c)-v_{\rm A}/c)\Big]/(t-v_{\rm B}/c)^3 \Big\} . \end{aligned} \tag{71}$$

Each integral has the following form (Wolfram 1996):

$$\begin{aligned} & \int_{-1}^{1} dt \left(1-t^2\right)^{1/2} \sin^{-1}\{a(t+d)+b\}/(t+d)^3 \\ & = 1/2 \, \sin^{-1} \Big\{ -a\pi + \left(b - 2ad\left(1-d^2\right)\right) \\ & \times \log\Big[-4\left(1-d^2\right)^{1/2} / \left(b-2ad\left(1-d^2\right)\right)\Big] \Big/ \left(1-d^2\right)^{3/2} \Big\} \\ & -1/2 \, \sin^{-1} \Big\{ a\pi + \left(b - 2ad\left(1-d^2\right)\right) \\ & \times \log\Big[4\left(1-d^2\right)^{1/2} / \left(b-2ad\left(1-d^2\right)\right)\Big] \Big/ \left(1-d^2\right)^{3/2} \Big\} . \end{aligned} \tag{72}$$

If $v_A/c \ll 1$ and $v_B/c \ll 1$, then $P(s)$ is nearly independent of v_A/c and v_B/c and we have:

$$P(s) \approx 4/\pi \, J_0(s) \sin^{-1}(\pi\lambda_A/\lambda_B)\,, \tag{73}$$
$$V_{G23}(r_B) \approx -Gm_A m_B \lambda_B/\lambda_A \, 1/\pi \, \sin^{-1}(\pi\lambda_A/\lambda_B) J_0(r_B/\lambda_B)/r_B\,, \tag{74}$$

where

$$\lambda_B/\lambda_A > 1\,, \qquad v_A/c \ll 1\,, \qquad v_B/c \ll 1\,.$$

In the classical limit for object A much smaller than object B (*i.e.* $\lambda_B/\lambda_A \gg 1$, $r_B/\lambda_B \ll 1$), the Type II gravity potential reduces to the classical gravity potential:

$$V_{G23}(r_B) \approx -Gm_A m_B/r_B\,, \tag{75}$$

where

$$\lambda_B/\lambda_A \gg 1\,, \qquad r_B/\lambda_B \ll 1\,, \qquad v_A/c \ll 1\,, \qquad v_B/c \ll 1\,.$$

For object A much smaller than object B (*i.e.* $\lambda_B/\lambda_A \gg 1$), object A highly relativistic (*i.e.* $v_A/c \approx 1$), and $v_B/c \ll 1$, each integral contributes the same real part while the imaginary parts cancel, and the factor λ_B/λ_A no longer cancels in the Type II gravity potential. As a result, the Type II gravity potential and the gravity force in the classical limit are very large since the mass of object A is effectively replaced by the mass of object B:

$$P(s) \approx 2/\pi \, J_0(s) \, \mathrm{Re}\left\{\sin^{-1}(\log 4) - \sin^{-1}(\log 4 + i\pi)\right\}\,, \tag{76}$$
$$\begin{aligned} V_{G23}(r_B) \approx\; & -Gm_A m_B/r_B \, \lambda_B/\lambda_A \, 1/2\pi \\ & \times \mathrm{Re}\left\{\sin^{-1}(\log 4) - \sin^{-1}(\log 4 + i\pi)\right\}\,, \end{aligned} \tag{77}$$

where

$$\lambda_B/\lambda_A \gg 1\,, \qquad r_B/\lambda_B \ll 1\,, \qquad v_A/c \approx 1\,, \qquad v_B/c \ll 1\,.$$

Thus a relativistic small object or FS unit is able to exert a very large classical type force on a large object.

We conclude Type II gravity by showing that the kinetic energy correction term $F(s)$ integrates to zero when object A is smaller than object B (*i.e.* $\lambda_B/\lambda_A > 1$). As with $E(s)$, we define $F(s) = P_{KE}(s) - sQ_{KE}(s)$, show that the gravity potential $V_{G32}(r_B)$ is proportional to $-P_{KE}(s)/s$, evaluate the integral over r_A, and replace the Bessel generating functions in $P_{KE}(s)$ by $J_0(s)$ to obtain:

$$V_{G32}(r_B) = -Gm_A m_B \lambda_B/4\lambda_A \, P_{KE}(s)/r_B\,, \tag{78}$$

$$P_{\mathrm{KE}}(s) = -v_{\mathrm{B}}^2/c^2 \; J_0(s)\; 1/\pi \int_0^\pi d\theta \int_0^1 dt$$
$$\times\Big\{ \left[\sin^{-1}(y_t\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c) + \sin^{-1}(y_t\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c)\right]/y_t^3$$
$$+ \left[\sin^{-1}(z_t\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c) + \sin^{-1}(z_t\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c)\right]/z_t^3\Big\}. \quad (79)$$

In evaluating the integrals over t, the values at the lower limits cancel, and each integral has the following form which we substitute in the $P_{\mathrm{KE}}(s)$ integrals (Wolfram 1996):

$$\int_0^1 dt \sin^{-1}(at+b)/(et+d)^3$$
$$= -1/2e \left\{ \sin^{-1}(b+a)/(d+e)^2 + \lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} \left(1 - v_{\mathrm{A}}^2/c^2\right)^{-1}\right.$$
$$\times \left(1-(b+a)^2\right)^{1/2}/(d+e) + (\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}})^2 (b - ad/e)\left(1 - v_{\mathrm{A}}^2/c^2\right)^{-3/2}$$
$$\left.\times \log\left[\left(1-(b-ad/e)(b+a) + \left(1-v_{\mathrm{A}}^2/c^2\right)^{1/2}\left(1-(b+a)^2\right)^{1/2}\right)\Big/(d+e)\right]\right\}, (80)$$

$$P_{\mathrm{KE}}(s) = 1/2\; v_{\mathrm{B}}/c\; J_0(s)\; 1/\pi \int_0^\pi d\theta$$
$$\times\Big\{\Big\{ \left[\sin^{-1}(y\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c) + \sin^{-1}(y\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c)\right]/y^2$$
$$- \left[\sin^{-1}(z\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c) + \sin^{-1}(z\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c)\right]/z^2\Big\}$$
$$+\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}}\left(1 - v_{\mathrm{A}}^2/c^2\right)^{-1}\Big\{\Big[(1-(y\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c)^2)^{1/2}$$
$$+(1-(y\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c)^2)^{1/2}\Big]/y - \Big[\left(1-(z\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c)^2\right)^{1/2}$$
$$+\left(1-(z\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c)^2\right)^{1/2}\Big]/z\Big\} + (\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}})^2\; v_{\mathrm{A}}/c\; \left(1 - v_{\mathrm{A}}^2/c^2\right)^{-3/2}$$
$$\times\Big\{\log\left[\left(1-v_{\mathrm{A}}/c\;(y\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}}+v_{\mathrm{A}}/c)+\left(1-v_{\mathrm{A}}^2/c^2\right)^{1/2}\left(1-(y\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}}+v_{\mathrm{A}}/c)^2\right)^{1/2}\right)/y\right]$$
$$-\log\left[\left(1+v_{\mathrm{A}}/c\;(y\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}}-v_{\mathrm{A}}/c)+\left(1-v_{\mathrm{A}}^2/c^2\right)^{1/2}\left(1-(y\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}}-v_{\mathrm{A}}/c)^2\right)^{1/2}\right)/y\right]$$
$$-\log\left[\left(1-v_{\mathrm{A}}/c\;(z\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}}+v_{\mathrm{A}}/c)+\left(1-v_{\mathrm{A}}^2/c^2\right)^{1/2}\left(1-(z\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}}+v_{\mathrm{A}}/c)^2\right)^{1/2}\right)/z\right]$$
$$+\log\left[\left(1+v_{\mathrm{A}}/c\;(z\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}}-v_{\mathrm{A}}/c)+\left(1-v_{\mathrm{A}}^2/c^2\right)^{1/2}\left(1-(z\lambda_{\mathrm{A}}/\lambda_{\mathrm{B}}-v_{\mathrm{A}}/c)^2\right)^{1/2}\right)/z\right]\Big\}\Big\}. (81)$$

In the first two sets of integrals, the first and fourth terms and the second and third terms cancel as they are mirror images with respect to the integration interval and occur with opposite sign. In the third set of integrals, the $\log(y)$ and $\log(z)$ factors cancel. Then the first and fourth terms and the second and third terms are negative mirror images and cancel as well.

6. Gravity acts instantaneously

In this section we show that if the FS is quantized into discrete units, the mass density fields in the FS are created or changed instantaneously by both the logarithmic and first-order singularities in the gravity force since the singularities are infinite forces. We suggest that each FS unit has a rest mass equal to 2 proton masses, a characteristic length equal to its gravity wavelength (2 mm), and a speed parameter that corresponds to its kinetic energy. The mass density field of an arbitrary object is then defined by its density value at each FS unit.

We first consider the gravity force exerted by a FS unit denoted as unit A on a FS unit denoted as unit B. The rest mass of each FS unit is twice the proton mass and the kinetic energy of each FS unit is the same. As a result, the Type I gravity force has 4 logarithmic singularities according to Eq. (37) since the gravity wavelengths λ_{FS} of the two FS units are identical and the speed parameters are also identical. Thus the Type I gravity force exerted by unit A on unit B is:

$$F_{\mathrm{GI}}(r_{\mathrm{B}}) = A_{\mathrm{GIFS}} G(2m_p)^2 J_0(r_{\mathrm{B}}/\lambda_{\mathrm{FS}})/r_{\mathrm{B}}^2 \,, \tag{82}$$

where the FS amplification factor A_{GIFS} contains the four logarithmic singularities of the Type I gravity force exerted on unit B.

The net Type I gravity force exerted by all units A in a radial line on unit B is:

$$\begin{aligned} \int_0^\infty dr_{\mathrm{B}} r_{\mathrm{B}}^2 F_{\mathrm{GI}}(r_{\mathrm{B}}) &= A_{\mathrm{GIFS}} G(2m_p)^2 \int_0^\infty dr_{\mathrm{B}} J_0(r_{\mathrm{B}}/\lambda_{\mathrm{FS}}) \\ &= A_{\mathrm{GIFS}} G(2m_p)^2 \lambda_{\mathrm{FS}} \,. \end{aligned} \tag{83}$$

What is important is that the value of the integral is positive and thus the net force from all units A along the radial line is proportional to the amplification factor A_{GIFS} and hence is infinite. This net force, however, is exactly balanced by the net Type I gravity force exerted by the units A in the opposite radial direction. The same is true for every radial direction for every unit B of the arbitrary object mass density field.

The same calculation can be done for Type II gravity. Because the gravity wavelengths for both unit A and unit B are identical, the Type II gravity force has a first-order singularity according to Eq. (70) at the zeros of the J_0 Bessel function:

$$F_{\mathrm{GII}}(r_{\mathrm{B}}) = A_{\mathrm{GIIFS}} G(2m_p)^2/\lambda_{\mathrm{FS}}\ J_1(r_{\mathrm{B}}/\lambda_{\mathrm{FS}})/r_{\mathrm{B}}\,, \tag{84}$$

where the FS amplification factor A_{GIIFS} contains the first-order singularity of Type II gravity, J_1 is the 1st order Bessel function of the first kind, and $r_{\mathrm{B}}/\lambda_{\mathrm{FS}}$ is a zero of the J_0 Bessel function.

The net Type II gravity force exerted by all units A in a radial line on unit B is the sum of the Type II gravity force at the zeros of the J_0 Bessel function:

$$\sum_{\text{Zeros of } J_0} F_{\mathrm{GII}}(r_{\mathrm{B}}) = A_{\mathrm{GIIFS}} G(2m_p)^2/\lambda_{\mathrm{FS}} \sum_{\text{Zeros of } J_0} J_1(r_{\mathrm{B}}/\lambda_{\mathrm{FS}})/r_{\mathrm{B}}\,. \tag{85}$$

The value of the sum is positive and thus the net force from all units A along the radial line is proportional to the amplification factor A_{GIIFS} and hence is infinite. This net force, however, is exactly balanced by the net Type II gravity force exerted by the units A in the opposite radial direction. The same is true for every radial direction for every unit B of the arbitrary object mass density field. We note that Type II singularities occur at the points in the FS where the Type I singularities are zero, *i.e.* at the zeros of the J_0 Bessel function. We also note that the Type I and Type II forces on the FS are symmetric in that the gravity force exerted by unit A on unit B is the same as the gravity force exerted by unit B on unit A.

Thus when an arbitrary object mass density field is created or changed, every part of the mass field is acted on by the singularities in all directions and at all distances to bring the mass density field to its new state. Since the singularities are infinite forces that act at all distances, any changes or interactions in the mass density fields occur instantaneously and gravity acts instantaneously.

7. Contraction and expansion of the fabric of space

We show that if kinetic energy is transferred into or released from the FS, then the FS contracts or expands. Again we assume that the FS is quantized into discrete units with each FS unit having a rest mass of 2 proton masses, a characteristic length equal to its gravity wavelength (2 mm), and a speed parameter v that corresponds to its kinetic energy KE according to mass–energy equivalence:

$$\mathrm{KE} = 2m_p c^2 \left(\left(1 - v^2/c^2\right)^{-1/2} - 1\right)\,. \tag{86}$$

Since the gravity wavelengths for any two units are identical, the Type II gravity force has a first-order singularity according to Eq. (70) at the zeros of the J_0 Bessel function. Consider the following units in which the central unit and all units to the left have a speed parameter v_1 and all units to the right have a speed parameter v_2:

$$\begin{array}{cccccccccc} \ldots & \square & \square & \square & \square & \square & \square & \square & \ldots \\ & v_1 & v_1 & v_1 & v_1 & v_2 & v_2 & v_2 & \\ & & \leftarrow & r_1 & | & r_1, r_2 & \rightarrow & & \end{array} \tag{87}$$

The Type II gravity forces exerted on the central unit by the units on the left and right of the central unit are as follows where the FS amplification factor A_{GIIFS} includes the first-order singularity, J_1 is the 1st order Bessel function of the first kind, r_1 is the unit of radial distance, λ_{FS} is the FS gravity wavelength, and r_1/λ_{FS} is a zero of the J_0 Bessel function:

$$F_{\text{GII LEFT}}(r_1) = A_{\text{GIIFS}} G(2m_p)^2/\lambda_{\text{FS}}\, J_1(r_1/\lambda_{\text{FS}})/r_1\,, \tag{88}$$

$$\begin{aligned} F_{\text{GII RIGHT}}(r_1) = {} & A_{\text{GIIFS}} G(2m_p)^2/\lambda_{\text{FS}} \left(1 - v_1^2/c^2\right)^{1/2} \\ & \times (1 - v_2^2/c^2)^{-1/2}\, J_1(r_1/\lambda_{\text{FS}})/r_1\,, \end{aligned} \tag{89}$$

and the forces are zero if r_1/λ_{FS} is not a zero of the J_0 Bessel function. The only way that these forces balance exactly for all radial distances is if the FS on the right is contracted or expanded and the new unit of distance r_2 in the FS on the right is related to the old unit of distance r_1 as follows:

$$r_2 = r_1 \left(1 - v_2^2/c^2\right)^{1/2} \left(1 - v_1^2/c^2\right)^{-1/2}\,. \tag{90}$$

Thus as kinetic energy is transferred into the FS, its speed parameter increases and the FS contracts. The opposite is also true, namely as kinetic energy is released from the FS, its speed parameter decreases and the FS expands. This contraction and expansion of the FS are unrelated to the mass or distribution of mass in the FS.

We may then hypothesize that the universe was created with such a large amount of kinetic energy stored in the FS that space was immensely contracted with all mass concentrated in a small volume. When time began, the cosmic kinetic energy began to be released from the FS and the FS and mass began to expand and are still expanding today. The cosmic kinetic energy released from the FS units is in fact the 3 degree K cosmic background radiation. This gravity theory suggests that the universe expands once.

8. Gravity for photons and other zero mass particles

The gravity theory can also describe the gravitational interactions of photons, other zero mass particles, or any type of gravitating energy. We hypothesize that an object without mass is a mass density field $D_\mathrm{G}(r)$ in the FS without the kinetic energy cosine term:

$$D_\mathrm{G}(r) = E/c^2 \, 1/4\pi\lambda_\mathrm{G} \, J_0(r/\lambda_\mathrm{G})/r^2 \,, \tag{91}$$

where E is its energy, E/c^2 is its effective mass, λ_G is its gravity wavelength, J_0 is the 0th order Bessel function of the first kind, and r is the distance from the object. We also hypothesize that the gravity wavelength λ_G of a gravitating energy or other zero mass particle is given by Eq. (7) as for objects with mass, but proportional to its effective mass.

For a photon, however, we hypothesize that its gravity wavelength is equivalent to that of a bound particle–antiparticle pair with the same energy. For example, particle A has mass m_A, gravity wavelength λ_A, and speed v_A, and its antiparticle B has mass m_B, gravity wavelength λ_B, and speed v_B. The non-singular gravity force exerted by particle A on its antiparticle B is given by Eq. (19) and the function $B(r_\mathrm{B}/\lambda_\mathrm{B})$ by Eq. (42).

We further hypothesize that the gravity wavelength λ_G of a photon is its energy E divided by the gravity force experienced by the particle or antiparticle at a distance equal to its gravity wavelength. Since this force does not include Type II gravity, photons experience only Type I gravity and not Type II gravity. Using that $m_\mathrm{A} = m_\mathrm{B}$, $\lambda_\mathrm{A} = \lambda_\mathrm{B}$, $\lambda_\mathrm{B} = N_\mathrm{A}/\kappa \, m_\mathrm{B}$, $v_\mathrm{A} = v_\mathrm{B}$, and $B(r_\mathrm{B}/\lambda_\mathrm{B}) = 4J_0(r_\mathrm{B}/\lambda_\mathrm{B})$ when $\lambda_\mathrm{A} = \lambda_\mathrm{B}$ and $v_\mathrm{A} = v_\mathrm{B}$, we obtain:

$$\lambda_\mathrm{G} = E/F_\mathrm{G}(\lambda_\mathrm{B}) = E/\left[Gm_\mathrm{B}^2/\lambda_\mathrm{B}^2 J_0(1)\right] = E(N_\mathrm{A}/\kappa)^2/[GJ_0(1)] \,, \tag{92}$$

where N_A is Avogadro's number, κ is the FS atomic mass linear density, and $J_0(1)$ is the value of the J_0 Bessel function at unit argument.

If a photon is isolated, its gravity wavelength is proportional to its own energy. However, if the photon is a constituent of an electromagnetic wave, then its gravity wavelength is proportional to the energy of the electromagnetic wave. This behavior of the gravity wavelength for photons is comparable to an isolated particle with mass and the same particle bound within a larger object or wave.

9. Comparison with experimental tests

The gravity theory requires the mass density fields to be integrated to infinity to obtain mass–energy equivalence and the gravity forces, and as a result requires the FS to extend to infinity. Since the FS and the mass in the universe are not related, the size and age of the visible universe as measured by the Hubble radius are unaffected, except that the FS units expand according to Eq. (90) as the FS units release cosmic kinetic energy. As we show elsewhere, by comparing the Hubble radius to an object's gravity wavelength, we can determine if and when gravity changes from classical to wave gravity.

As a classical gravity test, we consider two experimental situations — the measurement of gravity on a beam of neutrons and on a single neutron. If a neutron is isolated, its gravity wavelength $\lambda_{\mathrm{G}} \approx 1$ mm and the neutron experiences wave gravity exerted by objects at distances greater than 1 mm. However, if a neutron is part of a beam of neutrons, it experiences the first-order singularities of Type II gravity exerted by the other neutrons in the beam. Since the neutron is now a constituent of a wave, its gravity wavelength is proportional to the mass of all the neutrons in the beam. As a result, each neutron in the beam experiences classical gravity, as exerted by the Earth for example.

In an experimental situation that measures gravity on a single neutron, gravity is measured quantum mechanically. Hence the single neutron is not isolated, but is part of the quantum mechanical system that includes the measurement apparatus. Thus the neutron's gravity wavelength includes the mass of the measurement apparatus and the neutron again experiences classical gravity.

Also, if gravity exerted by small on large objects is $3\times$ classical gravity, the Earth's tides (which result primarily from gravity exerted by the Moon) should be much larger than what is actually observed. Why then does classical gravity accurately predict the Earth's tides? The answer lies in the fact that tidal water particles, unlike particles that comprise the solid Earth, have very different speeds. Thus, whereas the mass density fields of solid particles that comprise the Earth are collectively combined by the fabric of space units that exert gravity, the mass density fields of tidal water particles combine separately. As a result, the gravity wavelength of tidal water particles is proportional only to the mass of tidal water particles with similar speeds, and the gravity force exerted by the Moon on tidal waters is the large on small gravity force, which is equal to classical gravity.

We now compare the predictions of the gravity theory based on mass–energy equivalence with the experimental tests of General Relativity (GR). With the mass density field given by Eq. (91), a photon experiences gravity according to Eq. (11) in which object B is now a photon, m_{B} is its effective mass, and λ_{B} is its gravity wavelength. Using that a photon energy $E = hf$ where h is Planck's constant and f is its frequency, we can rewrite the photon gravity wavelength λ_{G} in Eq. (92) as:

$$\lambda_{\mathrm{G}} = f(N_A/\kappa)^2/[G/h\ J_0(1)] = 4.7 \times 10^{24} f \text{ [meters]} . \tag{93}$$

For example, an isolated photon with frequency $f = 5.5 \times 10^{14}$ cps (yellow-green light) has a gravity wavelength $\lambda_{\rm G} = 2.6 \times 10^{39}$ m or 2.7×10^{23} light-years. Since the size of the observable universe is about 4.2×10^{10} light-years, we have $r_{\rm B}/\lambda_{\rm B} \ll 1$ and the Bessel function $J_0(r_{\rm B}/\lambda_{\rm B}) \approx 1$ in Eq. (11). As a result, for this example as well as for most photon frequencies, the photon experiences classical type gravity.

The first comparison test with GR is gravitational redshifting. For example, we consider a star such as our Sun whose gravity wavelength $\lambda_{\rm Sun} = 1.3 \times 10^{38}$ light-years and a yellow-green photon. Since $\lambda_{\rm G}/\lambda_{\rm Sun} \ll 1$, we have large object on small object gravity and $A_{\rm G} = 1$ in Eq. (11). Thus the gravity exerted by a star on an emitted photon is classical gravity and the photon experiences gravitational redshifting in agreement with GR.

The second comparison test with GR is the bending of light by a massive object. The degree of bending of light predicted by the gravity theory depends on the ratio of the gravity wavelengths of the massive object and the photon gravity wavelength. If the photon or parent wave gravity wavelength is much less than the gravity wavelength of the massive object (*i.e.* $\lambda_{\rm B}/\lambda_{\rm A} \ll 1$), then we have large on small gravity from $B(s)$ in Eq. (46). As a result, $A_{\rm G} = 1$ in Eq. (11) and the bending of light is the classical Newtonian value.

However, if the photon is a constituent of an electromagnetic wave whose gravity wavelength is very much larger than that of the massive object (*i.e.* $\lambda_{\rm B}/\lambda_{\rm A} \gg 1$), the small on large gravity receives equal Type I gravity contributions from $A(s)$ in Eq. (36) and $B(s)$ in Eq. (49). As a result, $A_{\rm G} = 2$ in Eq. (11) and the bending of light is twice the classical Newtonian value.

For example, assuming an average sunlight frequency of 5.5×10^{14} cps and intensity of 1400 W/m^2 at the Earth's surface, the number of photons impacting a square meter per second is 3.8×10^{21}. Thus the gravity wavelength of a one light-second long, one square meter cylinder of photons is $\lambda_{\rm G} = 1.1 \times 10^{45}$ light-years. Even this gravity wavelength for such a small volume of the parent light wave is much larger than the gravity wavelength of the Sun which is $\lambda_{\rm Sun} = 1.3 \times 10^{38}$ light-years. Since $\lambda_{\rm G}/\lambda_{\rm Sun} \gg 1$, the bending of light is twice the classical Newtonian value in agreement with GR.

The third comparison test with GR is the related Shapiro time delay which results from the additional distance traveled due to bending of a parent wave by a massive object such as the Sun. For example (MIT 2008), with a radar pulse energy $E = 15$ J (150 KW pulse power, 100 μs pulse width), the gravity wavelength of photons in the pulse according to Eq. (92) is $\lambda_{\rm G} = 1.07 \times 10^{59}$ m or 1.13×10^{43} light-years. Since the photon gravity wavelength is very much larger than that of the Sun (*i.e.* $\lambda_{\rm G}/\lambda_{\rm Sun} \gg 1$), the small on large gravity exerted by the Sun on the photons in the radar pulse receives equal Type I gravity contributions from $A(s)$ in Eq. (36) and

$B(s)$ in Eq. (49). As a result, $A_G = 2$ in Eq. (11) and the time delay of the radar signal is twice the classical Newtonian value, again in agreement with GR.

The fourth comparison test with GR is the precession of the perihelion of Mercury. We consider the Sun and Mercury as a two-body gravitational system, denoting the larger object (Sun) as object 1 with inertial mass m_1 and the smaller object (Mercury) as object 2 with inertial mass m_2. When two non-relativistic objects have inertial masses that are very unequal, the small on large gravity force is 3 times the large on small gravity force. If we assume that the gravitational mass (g-mass) of the smaller object is equal to 3 times its inertial mass, then Newton's third law is preserved as we show in Eqs. (94)–(99).

Accordingly, we define the distances r_1 and r_2 of each object from the center of g-mass. We then include the relativistic increases of the inertial masses in the acceleration terms and the relativistic corrections to the gravity force from Eqs. (36), (46), (49), and (75) in the equations of motion about the center of g-mass:

$$r_1 = 3m_2 r/M\,, \qquad r_2 = m_1 r/M\,, \qquad r = r_1 + r_2\,, \qquad M = m_1 + 3m_2\,, \quad (94)$$

$$\begin{aligned} m_1\left(1 - v_1^2/c^2\right)^{-1/2} d^2\boldsymbol{r}_1/dt^2 &= -Gm_1m_2/r^2\, \underline{\boldsymbol{r}}_1 \Big[\left(1 - v_2^2/c^2\right)^{-1/2} \\ &\quad + \left(1 - v_1^2/c^2\right)^{1/2}\left(1 - v_2^2/c^2\right)^{-1/2} + 1\Big], \quad (95) \\ m_2\left(1 - v_2^2/c^2\right)^{-1/2} d^2\boldsymbol{r}_2/dt^2 &= -Gm_1m_2/r^2\, \underline{\boldsymbol{r}}_2 \left(1 - v_2^2/c^2\right)^{1/2} \\ &\quad \times \left(1 - v_1^2/c^2\right)^{-1/2}, \quad (96) \end{aligned}$$

where v_1 and v_2 are the speeds of objects 1 and 2 relative to the FS (or CMBR) at rest and c is the speed of light. We expand the relativistic factors and obtain to order $1/c^2$:

$$\begin{aligned} m_1 d^2\boldsymbol{r}_1/dt^2 &= -Gm_1m_2/r^2\, \underline{\boldsymbol{r}}_1 \left(3 - 2v_1^2/c^2 + v_2/c^2\right), \quad (97) \\ m_2 d^2\boldsymbol{r}_2/dt^2 &= -Gm_1m_2/r^2\, \underline{\boldsymbol{r}}_2 \left(1 + 1/2\; v_1^2/c^2 - v_2^2/c^2\right). \quad (98) \end{aligned}$$

The motion of the center of g-mass relative to the FS at rest is then specified by only relativistic terms:

$$m_1 d^2\boldsymbol{r}_1/dt^2 + 3m_2 d^2\boldsymbol{r}_2/dt^2 = \boldsymbol{F}_{\mathrm{CM}} = -Gm_1m_2/r^2\, \underline{\boldsymbol{r}}_2 \left(7/2\; v_1^2/c^2 - 4v_2^2/c^2\right). \quad (99)$$

These equations result in the conservation of energy, linear momentum, and angular momentum that follow from Newton's third law. While the ratio between g-mass and inertial mass of the smaller object is 3 when the inertial masses are very unequal and non-relativistic, the ratio decreases to 1 as the inertial masses become equal as we show in Eq. (114). The observance of Newton's third law by the gravity theory does imply, however, that the gravity theory violates the equivalence principle in a novel way.

Since $\boldsymbol{r}_2$ is defined relative to the center of g-mass, the force on the center of g-mass is the negative of the same force on object 2. Thus corrected for the motion of the center of g-mass, the equation of motion for $\boldsymbol{r}_2$ in the reference frame that is the FS at rest is:

$$m_2 d^2\boldsymbol{r}_2/dt^2 = -Gm_1m_2/r^2\, \underline{\boldsymbol{r}}_2 \left(1 - 3v_1^2/c^2 + 3v_2^2/c^2\right) . \tag{100}$$

The speeds v_1 and v_2 are each vector combinations of the velocity of the center of g-mass relative to the FS at rest and the velocity relative to the center of g-mass. The terms proportional to v_{CM}^2/c^2 in v_1^2/c^2 and v_2^2/c^2 cancel, where v_{CM} is the speed of the center of g-mass relative to the FS at rest. In addition, we neglect the remaining speed terms in v_1^2/c^2 since they are much smaller for the Sun than the corresponding terms in v_2^2/c^2 for Mercury. We then write the equations of motion for Mercury in polar coordinates about the center of g-mass in the reference frame that is the FS at rest:

$$\begin{aligned} d^2r_2/dt^2 - r_2(d\varphi/dt)^2 = -Gm_1(m_1/M)^2/r_2^2 \Big\{ 1 + 3/c^2\left[(dr_2/dt)^2 + r_2^2(d\varphi/dt)^2\right] \\ +6v_{\mathrm{CM}}/c^2\left(\cos\theta\, dr_2/dt - \sin\theta\, r_2 d\varphi/dt\right)\Big\} , \end{aligned} \tag{101}$$

$$r_2^2\, d\varphi/dt = h_2 , \tag{102}$$

where θ is the angle between position vector of Mercury relative to the center of g-mass and the velocity vector of the center of g-mass relative to the FS at rest and h_2 is the angular momentum of Mercury per unit mass around the center of g-mass. We make the usual $u_2 \equiv 1/r_2$ substitution where $\alpha = Gm_1(m_1/M)^2/h_2^2$ and $\lambda = 3h_2^2/c^2$, and obtain:

$$\begin{aligned} d^2u_2/d\varphi^2 + u_2 = \alpha\Big\{1 + \lambda\left[(du_2/d\varphi)^2 + u_2^2\right] \\ -6v_{\mathrm{CM}}h_2/c^2\left(\cos\theta\, du_2/d\varphi + \sin\theta\, u_2\right)\Big\} . \end{aligned} \tag{103}$$

We then expand the periodic solutions of Eq. (103) in a Fourier cosine series (see for example (Bergmann 1942)) and add a term proportional to $\sin\theta$:

$$u_2 = \alpha + \lambda\beta_0 + \alpha\varepsilon\cos\rho\varphi + \alpha k\sin\theta + \lambda\sum_{n=2}^{\infty}\beta_n\cos n\rho\varphi , \tag{104}$$

where ε is Mercury's eccentricity and $\theta = \rho_{\rm CM}\varphi + \Delta$. We substitute Eq. (104) into Eq. (103), use that $\rho \approx 1$ and $\rho_{\rm CM} \approx 1$, and obtain to first order:

$$\begin{aligned}
&\alpha\varepsilon\left(1-\rho^2\right)\cos\rho\varphi + \alpha k\left(1-\rho_{\rm CM}^2\right)\sin\theta + \alpha + \lambda\beta_0 \\
&= \alpha\Big\{1 + \lambda\alpha^2\left(1+\varepsilon^2+2\varepsilon\cos\rho\varphi + k^2 + 2\varepsilon k\sin\Delta + 2k\sin\theta\right) \\
&-6\alpha v_{\rm CM}h_2/c^2\left(k+\varepsilon\sin\Delta+\sin\theta\right)\Big\}\,. \qquad (105)
\end{aligned}$$

In Eq. (105), we compare the terms proportional to $\cos\rho\varphi$, $\sin\theta$, and a constant, and obtain the following equations:

$$\begin{aligned}
\left(1-\rho^2\right) &= 2\lambda\alpha^2\,, \qquad \rho \approx 1-\lambda\alpha^2\,, \qquad (106)\\
\left(1-\rho_{\rm CM}^2\right)k &= -6\alpha v_{\rm CM}h_2/c^2 + 2\lambda\alpha^2 k\,, \qquad (107)\\
\beta_0 &= \alpha^3\left(1+\varepsilon^2+k^2+2\varepsilon k\sin\Delta\right) - 2\alpha^2 v_{\rm CM}/h_2(k+\varepsilon\sin\Delta)\,. \qquad (108)
\end{aligned}$$

Eq. (106) is the identical equation to that derived in GR. Thus the gravity theory based on mass–energy equivalence contributes the same 43 seconds of arc per century for the precession of the perihelion of Mercury as does GR.

In Eq. (107), the constant k is dimensionless and thus the parameters of the $\sin\theta$ term are:

$$\begin{aligned}
k &= v_{\rm CM}/c\,,\\
\rho_{\rm CM} &\approx 1 + 3\alpha h_2/c - \lambda\alpha^2\,. \qquad (109)
\end{aligned}$$

As a result, the gravity theory based on mass–energy equivalence, like GR, is consistent with lunar laser ranging measurements of the Moon's orbit. In these measurements there is no preferred reference frame effect, since they are undertaken in the reference frame that is the center of g-mass at rest, where the $\sin\theta$ term in Eq. (104) is zero.

The fifth comparison test with GR is the geodetic precession of a gyroscope orbiting the Earth as measured by Gravity Probe B. As derived for gravity exerted by a large on small object, the increased gravity force in Eq. (100) results in a smaller circumference for the gyroscope orbit than would occur in Newtonian gravity. The velocities of object 1 (Earth) and object 2 (gyroscope) are $\boldsymbol{v}_1 = \boldsymbol{v}_{\rm E}$ and $\boldsymbol{v}_2 = \boldsymbol{v}_{\rm E} + \boldsymbol{v}$, where $\boldsymbol{v}_{\rm E}$ is the velocity of the Earth relative to the FS at rest and $\boldsymbol{v}$ is the velocity of the gyroscope relative to the Earth. Then in the reference frame that is the Earth at rest, the contracted radial distance R specified by Eq. (100) is $R = r(1 - 3/2\ v^2/c^2)$. Thus in this gravity theory, geodetic precession is entirely due to a novel gravito-electric field which is larger than in GR (*i.e.* 3/2 rather than 1/2), and results in the same geodetic precession as GR.

The sixth comparison test with GR is the change in period of a binary pulsar. We show that this change in period can be explained by energy loss due to the motion of the center of g-mass. In a binary pulsar such as PSR $1913+16$, the two stars are approximately the same size. Thus in the equations of motion, we use the gravity force from Eqs. (32), (42), and (74) for approximately equal masses:

$$m_1 \left(1-v^2/c^2\right)^{-1/2} d^2\boldsymbol{r}_1/dt^2 = -Gm_1m_2/r^2\ \underline{\boldsymbol{r}}_1 \times \left[\left(1-v^2/c^2\right)^{-1/2} K\left(z\left(1-v^2/c^2\right)^{-1}\right)/2\pi + K(z)/2\pi + 3/2\right], \quad (110)$$

$$m_2 \left(1-v^2/c^2\right)^{-1/2} d^2\boldsymbol{r}_2/dt^2 = -Gm_1m_2/r^2\ \underline{\boldsymbol{r}}_2 \times \left[\left(1-v^2/c^2\right)^{-1/2} K\left(z\left(1-v^2/c^2\right)^{-1}\right)/2\pi + K(z)/2\pi + 1\right], \quad (111)$$

where $m_1 \approx m_2$, $m_1 > m_2$, $v_1 \approx v_2 = v$, $K(z)$ is the complete elliptic integral K of the first kind, and its argument $z = (4m_2/m_1)/(1+m_2/m_1)^2 = (4m_1/m_2)/(1+m_1/m_2)^2$. We expand the relativistic factors and obtain to order $1/c^2$:

$$\begin{aligned} m_1 d^2\boldsymbol{r}_1/dt^2 &= -Gm_1m_2/r^2\ \underline{\boldsymbol{r}}_1 \Big[(K(z)/\pi + 3/2) \\ &\quad \times \left(1 - 1/2\ v^2/c^2\right) + 1/2\ v^2/c^2\ (1-z)^{-1}\Big], \quad (112) \\ m_2 d^2\boldsymbol{r}_2/dt^2 &= -Gm_1m_2/r^2\ \underline{\boldsymbol{r}}_2 \Big[(K(z)/\pi + 1) \\ &\quad \times \left(1 - 1/2\ v^2/c^2\right) + 1/2\ v^2/c^2\ (1-z)^{-1}\Big]. \quad (113) \end{aligned}$$

As a result, the motion of the center of g-mass is given by:

$$m_1 d^2\boldsymbol{r}_1/dt^2 + Xm_2 d^2\boldsymbol{r}_2/dt^2 = \boldsymbol{F}_{\rm CM} = -Gm_1m_2/r^2\ \underline{\boldsymbol{r}}_2\ v^2/c^2 \times \left[1/2\ (1-z)^{-1}(X-1)\right], \quad (114)$$

where $X = (K(z)/\pi + 3/2)/(K(z)/\pi + 1)$.

We define the total mass $M = m_1 + m_2$, reduced mass $\mu = m_1m_2/M$, angular momentum per unit mass $h = r^2 d\varphi/dt$, and effective gravitational constant $G_{\rm E} = (K(z)/\pi + 1)G$. We next make the substitutions $r = a(1-\varepsilon^2)/(1+\varepsilon\cos\varphi)$, $h^2 = G_{\rm E}Ma(1-\varepsilon^2)$, and $v^2 = 1/4\ G_{\rm E}M/(a(1-\varepsilon^2))[(1+\varepsilon\cos\varphi)^2 + \varepsilon^2\sin^2\varphi]$, where a is the semi-major axis and ε is the orbital eccentricity. We then write the relativistic force $\boldsymbol{F}_{\rm CM}$ on the center of g-mass as:

$$\begin{aligned} \boldsymbol{F}_{\rm CM} &= -\underline{\boldsymbol{r}}_2 G^2\mu M^2 / \left(a^3\left(1-\varepsilon^2\right)^3 c^2\right)(16(1-z))^{-1} \\ &\quad \times (1+\varepsilon\cos\varphi)^2\left[(1+\varepsilon\cos\varphi)^2 + \varepsilon^2\sin^2\varphi\right]. \quad (115) \end{aligned}$$

We change to polar coordinates in the equation of motion and find that the motion of the center of g-mass is almost circular. The equation of motion, the radius r_{CM}, and the period P_{CM} of the center of g-mass are:

$$Mh^2/r^2 \left[d/d\varphi(1/r^2\, dr_{\mathrm{CM}}/d\varphi) - r_{\mathrm{CM}}/r^2\right] = F_{\mathrm{CM}}\,, \tag{116}$$

$$r_{\mathrm{CM}} \approx G\mu/c^2(K(z)/\pi+1)^{-1}(16(1-z))^{-1}\,, \tag{117}$$

$$P_{\mathrm{CM}} \approx 2\pi G\mu/c^3(K(z)/\pi+1)^{-2}(16(1-z))^{-3/2}\,. \tag{118}$$

If we consider the center of g-mass as revolving around the objects, then the distance D_{CM} traversed by the center of g-mass relative to the objects per unit time of the center of g-mass is:

$$D_{\mathrm{CM}} = r\, d\varphi/dt\; P_{\mathrm{CM}}/2\pi = h/r\; P_{\mathrm{CM}}/2\pi\,. \tag{119}$$

The distance traversed by the center of g-mass is always parallel to the force on the center of g-mass. As a result, the energy E transferred per unit time by the force F_{CM} in moving the center of g-mass a distance D_{CM} is given by:

$$\begin{aligned} dE/dt \;&=\; -(K(z)/\pi+1)^{-3/2}(16(1-z))^{-5/2}G^{7/2}\mu^2M^{5/2}c^{-5}a^{-7/2} \\ &\quad\times\left(1-\varepsilon^2\right)^{-7/2}(1+\varepsilon\cos\varphi)^3\left[(1+\varepsilon\cos\varphi)^2+\varepsilon^2\sin^2\varphi\right]\,. \end{aligned} \tag{120}$$

We average the change in energy $E = -1/2G_{\mathrm{E}}M\mu/a$ and period $P_b = 2\pi a^{3/2}[G_{\mathrm{E}}M]^{-1/2}$ over an orbit with the result:

$$\begin{aligned} \langle dE/dt\rangle \;&=\; -(K(z)/\pi+1)^{-1}(16(1-z))^{-5/2}G^4\mu^2M^3c^{-5}a^{-5} \\ &\quad\times\left(1-\varepsilon^2\right)^{-7/2}\left(1+11/2\,\varepsilon^2+9/4\,\varepsilon^4\right)\,, \end{aligned} \tag{121}$$

$$\begin{aligned} P_b^{-1}dP_b/dt \;&=\; -3(K(z)/\pi+1)^{-2}(16(1-z))^{-5/2}G^3\mu M^2c^{-5}a^{-4} \\ &\quad\times\left(1-\varepsilon^2\right)^{-7/2}\left(1+11/2\,\varepsilon^2+9/4\,\varepsilon^4\right)\,. \end{aligned} \tag{122}$$

This change in period should be compared to the GR prediction (see for example (Will 1993)):

$$P_b^{-1}dP_b/dt = -96/5\; G^3\mu M^2c^{-5}a^{-4}\left(1-\varepsilon^2\right)^{-7/2}\left(1+73/24\,\varepsilon^2+37/96\,\varepsilon^4\right)\,. \tag{123}$$

Assuming an identical pulsar mass m_1 and eccentricity ε, and a mass ratio $m_2/m_1 \approx 0.963$ in Eq. (123), the changes in period are equal if the mass ratio $m_2/m_1 \approx 0.768$ in Eq. (122). With this value, the gravity theory predicts the same change in period for the binary pulsar PSR 1913 + 16 as does GR. However, the change in period results from the circular motion of the center of g-mass rather than unobserved gravitational radiation.

We can then use Eqs. (117) and (118) to obtain the radius $r_{\rm CM} \approx 1.6 \times 10^3$ m and period $P_{\rm CM} \approx 3.1 \times 10^{-5}$ s of the circular motion of the center of g-mass. Since orbital energy is transferred to the motion of the system as a whole as a result of gravity applied by the FS units, the energy loss is transferred to the FS units.

10. Derivation of the Coulomb force

The gravity theory makes possible a derivation of the Coulomb force. We hypothesize that the Coulomb force exerted by a charged object A with electric charge $q_{\rm A}$, mass $m_{\rm A}$, and gravity wavelength $\lambda_{\rm A}$ on a charged object B with electric charge $q_{\rm B}$ and mass $m_{\rm B}$ results from the interaction of the mass density fields of object A without a kinetic energy term and an equivalent mass Coulomb photon B in place of object B, integrated over the entire fabric of space (FS). In the same way as the gravity force is applied by the units of the FS, the Coulomb force is also applied by the units of the FS.

The mass density field $D_{\rm G}(r_{\rm B})$ of *Coulomb photon* B is obtained from Eq. (91):

$$D_{\rm G}(r_{\rm B}) = m_{\rm B}/4\pi\lambda_{\rm B}\ J_0(r_{\rm B}/\lambda_{\rm B})/r_{\rm B}^2\,, \tag{124}$$

where $m_{\rm B}$ is its effective mass, $\lambda_{\rm B}$ is its gravity wavelength, J_0 is the 0th order Bessel function of the first kind, and $r_{\rm B}$ is the distance from object B. The gravity wavelength $\lambda_{\rm B}$ of *Coulomb photon* B is obtained from Eq. (92):

$$\lambda_{\rm B} = m_{\rm B}c^2(N_{\rm A}/\kappa)^2/[GJ_0(1)]\,, \tag{125}$$

where $N_{\rm A}$ is Avogadro's number, κ is the FS atomic mass linear density, G is the gravitational constant, and $J_0(1)$ is the value of the J_0 Bessel function at unit argument.

We hypothesize that the coupling constant between the mass density fields of object A and *Coulomb photon* B is $C4\pi\lambda_{\rm B}$, where C is given by:

$$C = -(k/2)(q_{\rm A}/m_{\rm A})(q_{\rm B}/m_{\rm B})\,, \tag{126}$$

and where k is the Coulomb constant. The Coulomb force $F_{\rm C}(r_{\rm B})$ exerted by object A on object B is then:

$$\begin{aligned} F_{\rm C}(r_{\rm B}) &= (C4\pi\lambda_{\rm B})(m_{\rm A}/4\pi\lambda_{\rm A})(m_{\rm B}/4\pi\lambda_{\rm B}) \\ &\quad \times \int_0^\infty dr_{\rm A} r_{\rm A}^2 \int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\varphi J_0(r_{\rm A}/\lambda_{\rm A})/r_{\rm A}^2 \\ &\quad \times J_0\left(\left(r_{\rm B}^2 + r_{\rm A}^2 - 2r_{\rm B}r_{\rm A}\cos\theta\right)^{1/2}/\lambda_{\rm B}\right) \\ &\quad \times (r_{\rm B} - r_{\rm A}\cos\theta)/\left(r_{\rm B}^2 + r_{\rm A}^2 - 2r_{\rm B}r_{\rm A}\cos\theta\right)^{3/2}\,. \end{aligned} \tag{127}$$

As indicated previously, a photon experiencing gravity and hence a charged object experiencing the Coulomb force experiences only the Type I force and not the Type II force. Since the *Coulomb photon* gravity wavelength λ_B is very much larger than the gravity wavelength λ_A of object A (*i.e.* $\lambda_B/\lambda_A \gg 1$), the Coulomb force receives equal contributions from the $A(s)$ and $B(s)$ terms in Eqs. (35) and (49):

$$F_C(r_B) \approx 2Cm_Am_BJ_0(r_B/\lambda_B)/r_B^2 = -kq_Aq_BJ_0(r_B/\lambda_B)/r_B^2\,, \qquad (128)$$

where $\lambda_B/\lambda_A \gg 1$.

In the region where $r_B/\lambda_B \ll 1$, $J_0(r_B/\lambda_B) \approx 1$ and we obtain the classical Coulomb force:

$$F_{C\ \mathrm{Classical}}(r_B) = -kq_Aq_B/r_B^2\,, \qquad (129)$$

where $\lambda_B/\lambda_A \gg 1$, $r_B/\lambda_B \ll 1$.

For example, the gravity wavelength of a *Coulomb photon* corresponding to an electron is $\lambda_B = 5.8 \times 10^{44}$ m or 6.1×10^{28} light-years. As a result, electrons and protons experience the classical Coulomb force. Since the singularities in the gravity force act on the FS to effect any changes and interactions in the mass density fields instantaneously, the Coulomb force also acts instantaneously. This derivation of the Coulomb force suggests that elementary particles do not have an electric charge density since the electric charge appears only in the coupling constant between the two mass density fields.

The authors gratefully acknowledge Dr. H. Pierre Noyes, Professor Emeritus at Stanford Linear Accelerator Center (SLAC) for his advice, patience, and willingness to serve as a sounding board for the theories in this paper.

Appendix A

Gravity from density of space

We evaluate the first integral that is the contribution to gravity arising from the density of space:

$$\begin{aligned} F_{G1}(r_B) \ =\ & Gm_Am_B/2\lambda_A r_B^2 \int_0^\infty dr_A J_0(r_A/\lambda_A)\cos(v_Ar_A/c\lambda_A) \\ & \times[J_0((r_B+r_A)/\lambda_B)\cos(v_B(r_B+r_A)/c\lambda_B) \\ & + J_0((r_B-r_A)/\lambda_B)\cos(v_B(r_B-r_A)/c\lambda_B)]\,. \end{aligned} \qquad (A.1)$$

We use the integral representation of the Bessel function, expand the cosine functions in terms of exponential functions, and collect terms to obtain:

$$J_n(x) = i^{-n}/\pi \int_0^\pi d\theta \exp(ix\cos\theta)\cos(n\theta)\,, \tag{A.2}$$

$$\begin{aligned} F_{\mathrm{G1}}(r_{\mathrm{B}}) = {} & Gm_{\mathrm{A}}m_{\mathrm{B}}/4\lambda_{\mathrm{A}}r_{\mathrm{B}}^2 \int_0^\infty dr_{\mathrm{A}}\, J_0(r_A/\lambda_{\mathrm{A}})\; 1/\pi \int_0^\pi d\theta \\ & \times\{\exp(iyr_{\mathrm{B}}/\lambda_{\mathrm{B}})[\cos(r_{\mathrm{A}}(y/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c\lambda_{\mathrm{A}})) \\ & + \cos(r_{\mathrm{A}}(y/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))] \\ & + \exp(izr_{\mathrm{B}}/\lambda_{\mathrm{B}})[\cos(r_{\mathrm{A}}(z/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c\lambda_{\mathrm{A}})) \\ & + \cos(r_{\mathrm{A}}(z/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))]\}\,, \end{aligned} \tag{A.3}$$

where

$$\begin{aligned} y &= \cos\theta + v_{\mathrm{B}}/c\,, \\ z &= \cos\theta - v_{\mathrm{B}}/c\,. \end{aligned}$$

These terms with "$\cos(r_{\mathrm{A}}\ldots)$" contribute to Type I gravity.

Appendix B

Gravity from change due to rest mass

We evaluate the second integral that is the contribution to gravity arising from the change in the density of space due to the rest mass:

$$\begin{aligned} F_{\mathrm{G2}}(r_{\mathrm{B}}) = {} & -Gm_{\mathrm{A}}m_{\mathrm{B}}/4\lambda_{\mathrm{A}}\lambda_{\mathrm{B}}r_{\mathrm{B}}^2 \int_0^\infty dr_{\mathrm{A}}\; J_0(r_{\mathrm{A}}/\lambda_{\mathrm{A}})\cos(v_{\mathrm{A}}r_{\mathrm{A}}/c\lambda_A) \\ & \times \int_{r_{\mathrm{B}}-r_{\mathrm{A}}}^{r_{\mathrm{B}}+r_{\mathrm{A}}} dx/x\left(r_{\mathrm{A}}^2 - r_{\mathrm{B}}^2 + x^2\right)/r_{\mathrm{A}}\, J_0'(x/\lambda_{\mathrm{B}})\cos(v_{\mathrm{B}}x/c\lambda_{\mathrm{B}})\,. \end{aligned} \tag{B.1}$$

We use the Bessel function identities $J_0'(x/\lambda_{\mathrm{B}}) = -J_1(x/\lambda_{\mathrm{B}})$ and $J_1(x/\lambda_{\mathrm{B}})/(x/\lambda_{\mathrm{B}}) = 1/2\;(J_0(x/\lambda_{\mathrm{B}}) + J_2(x/\lambda_{\mathrm{B}}))$, and the integral representation of the Bessel functions to obtain:

$$F_{\mathrm{G2}}(r_{\mathrm{B}}) = Gm_{\mathrm{A}}m_{\mathrm{B}}/8\lambda_{\mathrm{A}}\lambda_{\mathrm{B}}^2 r_{\mathrm{B}}^2 \int_0^\infty dr_{\mathrm{A}}\, J_0(r_{\mathrm{A}}/\lambda_{\mathrm{A}})\cos(v_{\mathrm{A}}r_{\mathrm{A}}/c\lambda_{\mathrm{A}})$$

$$\times 1/\pi \int_0^\pi d\theta\,(1 - \cos\ 2\theta) \int_{r_\mathrm{B}-r_\mathrm{A}}^{r_\mathrm{B}+r_\mathrm{A}} dx \exp(i\cos\theta\ x/\lambda_\mathrm{B})$$
$$\times \left(r_\mathrm{A}^2 - r_\mathrm{B}^2 + x^2\right)/r_\mathrm{A}\cos(v_\mathrm{B}x/c\lambda_\mathrm{B})\,. \tag{B.2}$$

We now expand the cosine terms, perform the integration over x, and collect terms in r_A:

$$\begin{aligned}
F_{\mathrm{G2}}(r_\mathrm{B}) = {} & Gm_\mathrm{A}m_\mathrm{B}/4\lambda_\mathrm{A}\lambda_\mathrm{B}^2 r_\mathrm{B}^2 \int_0^\infty dr_\mathrm{A}\, J_0(r_\mathrm{A}/\lambda_\mathrm{A})1/\pi \int_0^\pi d\theta\left(1-\cos^2\theta\right)\\
& \times\Big\{ \exp(iyr_\mathrm{B}/\lambda_\mathrm{B})\Big[i\sin(r_\mathrm{A}(y/\lambda_\mathrm{B}+v_\mathrm{A}/c\lambda_\mathrm{A}))r_\mathrm{A}\lambda_\mathrm{B}/iy\\
& + i\sin(r_\mathrm{A}(y/\lambda_\mathrm{B}-v_\mathrm{A}/c\lambda_\mathrm{A}))r_\mathrm{A}\lambda_\mathrm{B}/iy\\
& + \cos(r_\mathrm{A}(y/\lambda_\mathrm{B}+v_\mathrm{A}/c\lambda_\mathrm{A}))\left(r_\mathrm{B}\lambda_\mathrm{B}/iy-\lambda_\mathrm{B}^2/(iy)^2\right)\\
& + \cos(r_\mathrm{A}(y/\lambda_\mathrm{B}-v_\mathrm{A}/c\lambda_\mathrm{A}))\left(r_\mathrm{B}\lambda_\mathrm{B}/iy-\lambda_\mathrm{B}^2/(iy)^2\right)\\
& + i\sin(r_\mathrm{A}(y/\lambda_\mathrm{B}+v_\mathrm{A}/c\lambda_\mathrm{A}))\left(\lambda_\mathrm{B}^3/(iy)^3-r_\mathrm{B}\lambda_\mathrm{B}^2/(iy)^2\right)/r_\mathrm{A}\\
& + i\sin(r_\mathrm{A}(y/\lambda_\mathrm{B}-v_\mathrm{A}/c\lambda_\mathrm{A}))\left(\lambda_\mathrm{B}^3/(iy)^3-r_\mathrm{B}\lambda_\mathrm{B}^2/(iy)^2\right)/r_\mathrm{A}\Big]\\
& + \exp(izr_\mathrm{B}/\lambda_\mathrm{B})\Big[i\sin(r_\mathrm{A}(z/\lambda_\mathrm{B}+v_\mathrm{A}/c\lambda_\mathrm{A}))r_\mathrm{A}\lambda_\mathrm{B}/iz\\
& + i\sin(r_\mathrm{A}(z/\lambda_\mathrm{B}-v_\mathrm{A}/c\lambda_\mathrm{A}))r_\mathrm{A}\lambda_\mathrm{B}/iz\\
& + \cos(r_\mathrm{A}(z/\lambda_\mathrm{B}+v_\mathrm{A}/c\lambda_\mathrm{A}))\left(r_\mathrm{B}\lambda_\mathrm{B}/iz-\lambda_\mathrm{B}^2/(iz)^2\right)\\
& + \cos(r_\mathrm{A}(z/\lambda_\mathrm{B}-v_\mathrm{A}/c\lambda_\mathrm{A}))\left(r_\mathrm{B}\lambda_\mathrm{B}/iz-\lambda_\mathrm{B}^2/(iz)^2\right)\\
& + i\sin(r_\mathrm{A}(z/\lambda_\mathrm{B}+v_\mathrm{A}/c\lambda_\mathrm{A}))\left(\lambda_\mathrm{B}^3/(iz)^3-r_\mathrm{B}\lambda_\mathrm{B}^2/(iz)^2\right)/r_\mathrm{A}\\
& + i\sin(r_\mathrm{A}(z/\lambda_\mathrm{B}-v_\mathrm{A}/c\lambda_\mathrm{A}))\left(\lambda_\mathrm{B}^3/(iz)^3-r_\mathrm{B}\lambda_\mathrm{B}^2/(iz)^2\right)/r_\mathrm{A}\Big]\Big\}\,,
\end{aligned} \tag{B.3}$$

where

$$\begin{aligned}
y &= \cos\theta + v_\mathrm{B}/c\,,\\
z &= \cos\theta - v_\mathrm{B}/c\,.
\end{aligned}$$

The terms with "$\cos(r_\mathrm{A}\ldots)$" and "$\sin(r_\mathrm{A}\ldots)r_\mathrm{A}$" contribute to Type I gravity while the terms with "$\sin(r_\mathrm{A}\ldots)/r_\mathrm{A}$" contribute to Type II gravity.

Appendix C

Gravity from change due to kinetic energy

We evaluate the third integral that is the contribution to gravity arising from the change in the density of space due to kinetic energy:

$$F_{\mathrm{G3}}(r_{\mathrm{B}}) = v_{\mathrm{B}}/c\, Gm_{\mathrm{A}}m_{\mathrm{B}}/4\lambda_{\mathrm{A}}\lambda_{\mathrm{B}}r_{\mathrm{B}}^2 \int_0^\infty dr_{\mathrm{A}}\ J_0(r_{\mathrm{A}}/\lambda_{\mathrm{A}})\cos(v_{\mathrm{A}}r_{\mathrm{A}}/c\lambda_{\mathrm{A}}) \times \int_{r_{\mathrm{B}}-r_{\mathrm{A}}}^{r_{\mathrm{B}}+r_{\mathrm{A}}} dx/x\left(r_{\mathrm{A}}^2 - r_{\mathrm{B}}^2 + x^2\right)/r_{\mathrm{A}}\ J_0(x/\lambda_{\mathrm{B}})\sin(v_{\mathrm{B}}x/c\lambda_{\mathrm{B}})\,. \quad (\mathrm{C}.1)$$

We use the integral representation of the sine function and reverse the order of integration to obtain:

$$F_{\mathrm{G3}}(r_{\mathrm{B}}) = v_{\mathrm{B}}^2/c^2\ Gm_{\mathrm{A}}m_{\mathrm{B}}/4\lambda_{\mathrm{A}}\lambda_{\mathrm{B}}^2r_{\mathrm{B}}^2 \int_0^\infty dr_{\mathrm{A}}\ J_0(r_{\mathrm{A}}/\lambda_{\mathrm{A}})\cos(v_{\mathrm{A}}r_{\mathrm{A}}/c\lambda_{\mathrm{A}}) \times \int_0^1 dt \int_{r_{\mathrm{B}}-r_{\mathrm{A}}}^{r_{\mathrm{B}}+r_{\mathrm{A}}} dx\left(r_{\mathrm{A}}^2 - r_{\mathrm{B}}^2 + x^2\right)/r_{\mathrm{A}}\ J_0(x/\lambda_{\mathrm{B}})\cos(v_{\mathrm{B}}xt/c\lambda_{\mathrm{B}})\,. \quad (\mathrm{C}.2)$$

This is similar to the integral in Appendix B. We make the substitution $s = r_{\mathrm{B}}/\lambda_{\mathrm{B}}$ so that:

$$\begin{aligned} F_{\mathrm{G3}}(r_{\mathrm{B}}) = {} & v_{\mathrm{B}}^2/c^2\ Gm_{\mathrm{A}}m_{\mathrm{B}}/4\lambda_{\mathrm{A}}r_{\mathrm{B}}^2 \int_0^\infty dr_{\mathrm{A}}\, J_0(r_{\mathrm{A}}/\lambda_{\mathrm{A}})1/\pi \int_0^\pi d\theta \int_0^1 dt \\ & \times\Big\{ \exp(iy_ts)\Big[i\sin(r_{\mathrm{A}}(y_t/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))r_{\mathrm{A}}/iy_t\lambda_{\mathrm{B}} \\ & + i\sin(r_{\mathrm{A}}(y_t/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))r_{\mathrm{A}}/iy_t\lambda_{\mathrm{B}} \\ & + \cos(r_{\mathrm{A}}(y_t/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\left(s/iy_t - 1/(iy_t)^2\right) \\ & + \cos(r_{\mathrm{A}}(y_t/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\left(s/iy_t - 1/(iy_t)^2\right) \\ & + i\sin(r_{\mathrm{A}}(y_t/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\left(\lambda_{\mathrm{B}}/(iy_t)^3 - s\lambda_{\mathrm{B}}/(iy_t)^2\right)/r_{\mathrm{A}} \\ & + i\sin(r_{\mathrm{A}}(y_t/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\left(\lambda_{\mathrm{B}}/(iy_t)^3 - s\lambda_{\mathrm{B}}/(iy_t)^2\right)/r_{\mathrm{A}}\Big] \\ & + \exp(iz_ts)\Big[i\sin(r_{\mathrm{A}}(z_t/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))r_{\mathrm{A}}/iz_t\lambda_{\mathrm{B}} \\ & + i\sin(r_{\mathrm{A}}(z_t/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))r_{\mathrm{A}}/iz_t\lambda_{\mathrm{B}} \\ & + \cos(r_{\mathrm{A}}(z_t/\lambda_{\mathrm{B}} + v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\left(s/iz_t - 1/(iz_t)^2\right) \\ & + \cos(r_{\mathrm{A}}(z_t/\lambda_{\mathrm{B}} - v_{\mathrm{A}}/c\lambda_{\mathrm{A}}))\left(s/iz_t - 1/(iz_t)^2\right) \end{aligned}$$

$$+i\sin(r_{\rm A}(z_t/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))\left(\lambda_{\rm B}/(iz_t)^3-s\lambda_{\rm B}/(iz_t)^2\right)/r_{\rm A}$$
$$+i\sin(r_{\rm A}(z_t/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))\left(\lambda_{\rm B}/(iz_t)^3-s\lambda_{\rm B}/(iz_t)^2\right)/r_{\rm A}\Big]\Big\}, \quad \text{(C.3)}$$

where

$$y_t = \cos\theta + v_{\rm B}t/c\,,$$
$$z_t = \cos\theta - v_{\rm B}t/c\,.$$

For the terms that contribute to Type I gravity, we evaluate the integrals over t by taking the derivative with respect to s and then integrating by parts. For example:

$$d/ds\left\{v_{\rm B}^2/c^2\int_0^1 dt\exp(iy_ts)\Big[i\sin(r_{\rm A}(y_t/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))r_{\rm A}/iy_t\lambda_{\rm B}\right.$$
$$\left.+\cos(r_{\rm A}(y_t/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))\left(s/iy_t-1/(iy_t)^2\right)\Big]\right\}$$
$$=-iv_{\rm B}/c\Big\{\exp(iys)\cos(r_{\rm A}(y/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))$$
$$-\exp(is\cos\theta)\cos(r_{\rm A}(\cos\theta/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))\Big\}\,. \quad \text{(C.4)}$$

The lower limit of the integral is cancelled by the lower limit of the corresponding term in z_t and we then integrate with respect to s to obtain:

$$F_{\rm G3}(r_{\rm B}) = Gm_{\rm A}m_{\rm B}/4\lambda_{\rm A}r_{\rm B}^2\int_0^\infty dr_{\rm A}J_0(r_{\rm A}/\lambda_{\rm A})\;1/\pi\int_0^\pi d\theta$$
$$\times\Big\{-iv_{\rm B}/c\Big\{\exp(iys)\Big[\cos(r_{\rm A}(y/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))$$
$$+\cos(r_{\rm A}(y/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))\Big]/iy-\exp(izs)$$
$$\times\Big[\cos(r_{\rm A}(z/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))+\cos(r_{\rm A}(z/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))\Big]/iz\Big\}$$
$$+iv_{\rm B}^2/c^2\int_0^1 dt\Big\{\exp(iy_ts)\left(\lambda_{\rm B}/(iy_t)^3-s\lambda_{\rm B}/(iy_t)^2\right)$$
$$\times\Big[\sin(r_{\rm A}(y_t/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))+\sin(r_{\rm A}(y_t/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))\Big]/r_{\rm A}$$
$$+\exp(iz_ts)\left(\lambda_{\rm B}/(iz_t)^3-s\lambda_{\rm B}/(iz_t)^2\right)\Big[\sin(r_{\rm A}(z_t/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))$$
$$+\sin(r_{\rm A}(z_t/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))\Big]/r_{\rm A}\Big\}\Big\}. \quad \text{(C.5)}$$

The terms with "$\cos(r_{\rm A}\ldots)$" contribute to Type I gravity while the terms with "$\sin(r_{\rm A}\ldots)/r_{\rm A}$" contribute to Type II gravity.

Appendix D

Type I gravity cancellation

We evaluate the "$\sin(r_{\rm A}\ldots)r_{\rm A}$" integrals and show that they cancel the Type I gravity that would arise from the "$\cos(r_{\rm A}\ldots)$" integrals when the Weber arguments are imaginary. If we define $s = r_{\rm B}/\lambda_{\rm B}$ and the function $D(s)$, we have:

$$F_{\rm G21}(r_{\rm B}) = Gm_{\rm A}m_{\rm B}\lambda_{\rm B}/4\lambda_{\rm A}r_{\rm B}^2\, D(s)\,, \tag{D.1}$$

$$\begin{aligned} D(s) = {} & 1/\lambda_{\rm B}^2 \int_0^\infty dr_{\rm A} J_0(r_{\rm A}/\lambda_{\rm A})\, 1/\pi \int_0^\pi d\theta \left(1-\cos^2\theta\right) \\ & \times\Big\{ \exp(iys)/y\Big[\sin(r_{\rm A}(y/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A})) \\ & + \sin(r_{\rm A}(y/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))\Big] r_{\rm A} \\ & + \exp(izs)/z\Big[\sin(r_{\rm A}(z/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A})) \\ & + \sin(r_{\rm A}(z/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))\Big] r_{\rm A}\Big\}\,. \end{aligned} \tag{D.2}$$

At this point, if we reverse the order of integration and evaluate the integrals over $r_{\rm A}$, we find that the integrals are Weber discontinuous integrals but with a sine instead of a cosine argument. As a result, the four integrals are respectively non-zero when:

$$\begin{aligned} (y/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}) &> 1/\lambda_{\rm A} \quad \text{or} \quad (y/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}) < -1/\lambda_{\rm A}\,, \\ (y/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}) &> 1/\lambda_{\rm A} \quad \text{or} \quad (y/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}) < -1/\lambda_{\rm A}\,, \\ (z/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}) &> 1/\lambda_{\rm A} \quad \text{or} \quad (z/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}) < -1/\lambda_{\rm A}\,, \\ (z/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}) &> 1/\lambda_{\rm A} \quad \text{or} \quad (z/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}) < -1/\lambda_{\rm A}\,. \end{aligned} \tag{D.3}$$

Rather than put the limits on each integral, we leave the limits 0 and π in place, but with the understanding that the integrals are really over an annular region bounded by 0 and π whose width is determined by the above conditions. To evaluate the integrals, we first take the derivative of $D(s)$ to remove the $1/y$ and $1/z$ factors and then integrate by parts:

$$\begin{aligned} d/dsD(s) = {} & i/\pi\, 1/\lambda_{\rm B} \int_0^\infty dr_{\rm A}\; J_0(r_{\rm A}/\lambda_{\rm A}) \\ & \times\{\sin\theta[\exp(iys)[\cos(r_{\rm A}(y/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A})) \\ & + \cos(r_{\rm A}(y/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))] \end{aligned}$$

$$
\begin{aligned}
&+\exp(izs)[\cos(r_{\rm A}(z/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))\\
&+\cos(r_{\rm A}(z/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))]]|_{\theta=0}^{\pi}\\
&+is\int_0^{\pi} d\theta(1-\cos^2\theta)\\
&\times[\exp(iys)[\cos(r_{\rm A}(y/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))\\
&+\cos(r_{\rm A}(y/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))]\\
&+\exp(izs)[\cos(r_{\rm A}(z/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))\\
&+\cos(r_{\rm A}(z/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))]]\\
&-\int_0^{\pi} d\theta\cos\theta\\
&\times[\exp(iys)[\cos(r_{\rm A}(y/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))\\
&+\cos(r_{\rm A}(y/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))]\\
&+\exp(izs)[\cos(r_{\rm A}(z/\lambda_{\rm B}+v_{\rm A}/c\lambda_{\rm A}))\\
&+\cos(r_{\rm A}(z/\lambda_{\rm B}-v_{\rm A}/c\lambda_{\rm A}))]]\}\,. \qquad \text{(D.4)}
\end{aligned}
$$

The first term is zero. We then use that $y = \cos\theta + v_{\rm B}/c$ and $z = \cos\theta - v_{\rm B}/c$ to rewrite the last integral and note that the integrals are the same integrals as arose in the "$\cos(r_{\rm A}\ldots)$" terms in Type I gravity, except for the integration limits:

$$
\begin{aligned}
d/dsD(s) &= -\{d/dsB(s)+d/dsA(s)+d/dsC(s)\}\,, &\text{(D.5)}\\
D(s) &= -\{B(s)+A(s)+C(s)\}\,. &\text{(D.6)}
\end{aligned}
$$

Thus the $D(s)$ term cancels the integration region in Type I gravity for which the Weber argument is imaginary.

Appendix E

Remaining kinetic energy term cancellation

The remaining kinetic energy term in the second derivative of $A(s)$ is:

$$
\begin{aligned}
{\rm Rem}_{\rm KE} = -v_{\rm B}/c\Bigg\{&1/\pi\int_0^{\pi} d\theta\exp(iys)\cos\theta\\
&\times\Big[\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(y+v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}\\
&+\left(\lambda_{\rm B}^2/\lambda_{\rm A}^2-(y-v_{\rm A}\lambda_{\rm B}/c\lambda_{\rm A})^2\right)^{-1/2}\Big]
\end{aligned}
$$

$$
\begin{aligned}
&-1/\pi \int_0^\pi d\theta \exp(izs) \cos\theta \\
&\times\Big[\left(\lambda_B^2/\lambda_A^2 - (z + v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \\
&+ \left(\lambda_B^2/\lambda_A^2 - (z - v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \Big]\Bigg\}. \qquad \text{(E.1)}
\end{aligned}
$$

If the solution is a Bessel function of order zero, we can replace the terms in s in the remainder integrals by $J_0(s)$ to obtain:

$$
\begin{aligned}
\mathrm{Rem}_{\mathrm{KE}} &= -v_B/c\ J_0(s)\ 1/\pi \int_0^\pi d\theta \cos\theta \\
&\times\Big\{ \left(\lambda_B^2/\lambda_A^2 - (y + v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \\
&+ \left(\lambda_B^2/\lambda_A^2 - (y - v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \\
&- \left(\lambda_B^2/\lambda_A^2 - (z + v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \\
&- \left(\lambda_B^2/\lambda_A^2 - (z - v_A\lambda_B/c\lambda_A)^2\right)^{-1/2} \Big\}. \qquad \text{(E.2)}
\end{aligned}
$$

The first and fourth integrals and the second and third integrals cancel as they are mirror images with respect to the integration interval and occur with opposite sign. Thus the remaining kinetic energy term integrates to zero and the solution is indeed a Bessel function of order zero.

REFERENCES

Bergmann, P.G., *Introduction to the Theory of Relativity*, Prentice-Hall, New York 1942.

Einstein, A., *On the Electrodynamics of Moving Bodies*, *Ann. Phys.* **17**, 891 (1905).

Erdelyi, W., Magnus, F., Tricomi, F.G., *Tables of Integral Transforms*, Vol. II, in *Bateman Manuscript Project*, McGraw-Hill, New York 1954.

Massachusetts Institute of Technology (MIT), 2008 MIT Haystack Observatory Incoherent Scatter Radar (ISR),
`http://www.haystack.mit.edu/atm/mho/instruments/isr`

Penzias, A.A., Wilson, R.W., *Astrophys. J. Lett.* **142**, 1149 (1965).

Rohringer, G., The Yellow–Green-Infrared Glow Following Nuclear Detonations, Report, General Electric Company, Santa Barbara, CA 1–7, 1968.

Will, C.M., *Theory and Experiment in Gravitational Physics*, Cambridge University Press, New York 1993.

Wolfram 1996–2006 The Wolfram Integrator, Wolfram Research Inc., http://integrals.wolfram.com

Wolfram 1998–2006 The Wolfram Functions Site, Wolfram Research Inc., http://functions.wolfram.com