

# ON THE ANOMALOUS $t$ -QUARK CHARGE ASYMMETRY AND NONCONTRACTIBILITY OF THE PHYSICAL SPACE

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Heavy flavour production at hadron colliders represents a very promising field to test perturbative QCD. The integrated forward–backward asymmetry of the top–antitop quark production is particularly sensitive to any deviation from the standard QCD calculations. The two Tevatron collaborations, CDF and D0, reported a much larger  $t$ -quark charge asymmetry than predicted by the theory. We show that the QCD in noncontractible space, where the minimal distance is fixed by weak interactions, enhances the asymmetry by more than a factor of 3 (5) at the parton level in leading order of the coupling for the Tevatron (LHC) center-of-mass energies. This result should not be a surprise since the asymmetry observable directly explores the far ultraviolet sector of the spacelike domain of the Minkowski spacetime.

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## 1. Introduction and motivation

The missing flavour-mixed light neutrinos, no lepton or baryon number violation and the absence of any candidate for a dark matter particle, call for a substantial improvement of the Standard Model (SM) of electroweak and strong interactions. Namely, the dominance of the baryon over antibaryon matter in the Universe suggests that baryon or lepton number be broken in particle physics [1]. It is well known, for example, that the broken lepton number can induce the breaking of the baryon number [2]. Any alternative to the presence of the cold dark matter in the Universe supposes drastic changes not only of the General Relativity, but also of the nonrelativistic Newtonian theory of gravity [3]. However, the direct or indirect detection of the dark matter particle is indispensable. The HESS source J1745-290

at the centre of our galaxy (for the most recent analysis see Ref. [4]) and the hints from the anomalous positron (or antiproton) abundance from the PAMELA mission [5] (see, for example, the analysis in Ref. [6]) suggest on the existence of the very heavy dark matter particle when searching for the dark matter annihilation products.

MOND (with its few relativistic generalizations) represents the alternative to the dark matter paradigm (see a discussion in Ref. [7]).

Besides the supersymmetric, grand unified or extra dimensional extensions of the SM, a very conservative alternative to the SM was proposed in [8], called the BY theory, resolving the ultraviolet singularity and the SU(2) global anomaly problems. Light and heavy Majorana neutrinos with flavour mixing and lepton CP violation could play a crucial role as hot and cold dark matter particles in the evolution of the expanding and rotating Universe [8, 9].

The noncontractible space of the BY theory, as an alternative symmetry-breaking mechanism to the Higgs one, introduces into the physical realm a new universal Lorentz and gauge invariant constant (UV cutoff in the space-like domain of the Minkowski spacetime, see Ref. [8])  $\Lambda = \frac{\hbar}{cd} = \frac{2}{g} \frac{\pi}{\sqrt{6}} M_W \simeq 326 \text{ GeV}$ .

The enhanced strong coupling at small distances and the absence of the asymptotic freedom in QCD are the immediate consequences of noncontractible space [10]. We show that electroweak quantum loops with heavy  $t$ -quark contributing to the CP violating processes of  $K$  and  $B$  mesons are affected by the UV cutoff [11]. The branching fraction for a rare decay  $B_s \rightarrow \mu\mu$  is lower by more than 30% in the BY theory compared with the SM owing to the modified short distance part of the amplitude [12].

ATLAS and CMS experiments at the LHC reported recently [13] a discovery of the 125 GeV resonance. It could be the Higgs boson of some theory beyond the SM, but it could be also some pseudoscalar or scalar meson with a substantial component of the pseudoscalar or scalar toponium [14]. Even the spin 1 boson cannot be excluded as an interpretation of the new 125 GeV resonance [15].

Anyhow, the Higgs mechanism does not solve the mass problem of particles. Eventually, the solutions of the coupled system of nonlinear integral Dyson–Schwinger equations of the UV nonsingular BY theory could resolve the mass problem of elementary particles [16].

In this paper, we study the implication of the UV cutoff to the leading QCD contribution for the forward–backward asymmetry in the top–antitop production. The large discrepancy between the theory and the experiment for this asymmetry observable is reported by the Tevatron collaborations CDF and D0 [17]. Let us quote the most recent results of the CDF Collaboration [18]: parton level asymmetry  $A_{\text{FB}}(M_{t\bar{t}} < 450 \text{ GeV})$  :

Data ( $\pm$  stat  $\pm$  syst) =  $0.084 \pm 0.046 \pm 0.026$  vs. SM expectation =  $0.047 \pm 0.014$ ;  $A_{\text{FB}}(M_{t\bar{t}} \geq 450 \text{ GeV})$ : Data =  $0.295 \pm 0.058 \pm 0.031$  vs. SM expectation =  $0.100 \pm 0.030$ .

In the next chapter, we present the main ingredients of the calculations while providing more details in the Appendix. Results and conclusions are given in the last chapter.

## 2. Charge asymmetry at the parton level

Almost invariably, various asymmetry observables of the electroweak or strong interactions are very sensitive to the details of the underlying processes. It appears that the  $t$ -quark pair charge asymmetry can test QCD loop corrections [19]. We shall study the dominant quark–antiquark annihilation channel whose structure equals the electron–positron annihilation amplitude modulo coupling and gauge group constant factors [20–22].

Let us define the asymmetric part of the differential cross sections [19]

$$\frac{d\sigma_{\text{A}}^{q\bar{q}}}{d\cos\theta} = (\sigma_{\text{A}}^{q\bar{q}})' \equiv \frac{1}{2} \left[ \frac{d\sigma(q\bar{q} \rightarrow QX)}{d\cos\theta} - \frac{d\sigma(q\bar{q} \rightarrow \bar{Q}X)}{d\cos\theta} \right].$$

Born cross section (symmetric part of the quark–antiquark annihilation to leading order  $\alpha_s^2$ ) is given by [19, 20]

$$\begin{aligned} \frac{d\sigma(q\bar{q} \rightarrow Q\bar{Q}; \text{Born})}{d\cos\theta} &= \alpha_s^2 \frac{T_{\text{F}} C_{\text{F}}}{N_c} \frac{\pi\beta}{2s} (1 + c^2 + 4m^2), \\ T_{\text{F}} &= \frac{1}{2}, \quad C_{\text{F}} = \frac{4}{3}, \quad N_c = 3, \quad \beta = \sqrt{1 - 4m^2}, \quad m^2 = \frac{m_Q^2}{s}, \\ s &= E_{\text{cm}}^2, \quad c = \beta \cos\theta, \quad \angle(\vec{p}(q), \vec{p}(Q)) = \theta. \end{aligned}$$

The asymmetric part to the leading  $\alpha_s^3$  order consists of the virtual, soft and hard gluon emission differential cross sections [19, 21, 22]

$$\begin{aligned} (\sigma_{\text{A}}^{q\bar{q}})' &= (\sigma_{\text{A}}^{q\bar{q}})'(\text{virtual}) + (\sigma_{\text{A}}^{q\bar{q}})'(\text{soft}) \\ &+ \int_{\text{(I)}} \frac{\partial^4 (\sigma_{\text{A}}^{q\bar{q}}(\text{hard}) - \sigma_{\text{A}}^{q\bar{q}}(\text{soft}))}{\partial \cos\theta \partial \Omega_\gamma \partial k} d\Omega_\gamma dk + \int_{\text{(II)}} \frac{\partial^4 \sigma_{\text{A}}^{q\bar{q}}(\text{hard})}{\partial \cos\theta \partial \Omega_\gamma \partial k} d\Omega_\gamma dk, \end{aligned} \tag{1}$$

$$\text{(I)} \quad 0 \leq k \leq k_1, \quad -1 \leq \cos\theta_\gamma \leq 1,$$

$$\text{(II)} \quad k_1 \leq k \leq k_2, \quad g_1(k, E_{\text{th}}) \leq \cos\theta_\gamma \leq g_2(k, E_{\text{th}}).$$

The equations for the virtual, hard and soft gluon radiation in the appendix of Ref. [19] are obtained from the equations in [21, 22] in the limit of the vanishing mass of incoming fermions.

The QCD in noncontractible space differs from the standard QCD when quantum loops are evaluated with the cutoff in the spacelike domain. Thus, one can find two possible sources of deviation from the standard QCD calculation for the asymmetry function  $A^\infty(\cos\theta) = \sigma'_A/\sigma'_{\text{Born}}$ : (1) calculation of the running coupling  $\alpha_s^A$  (see Ref. [10]), (2) box diagram contribution to the virtual correction [19, 21, 22]

$$\begin{aligned}(\sigma_A^A)' &= (\sigma_A^A)'(\text{virtual}, \alpha_s^A) + (\sigma_A^A)'(\text{soft}, \alpha_s^A) + (\sigma_A^A)'(\text{difference}, \alpha_s^A) \\ &\quad + (\sigma_A^A)'(\text{hard}, \alpha_s^A) = \left(\frac{\alpha_s^A}{\alpha_s^\infty}\right)^3 (\sigma_A^\infty)'(\alpha_s^\infty) + (\sigma_A^A)'(\text{virtual}, \alpha_s^A) \\ &\quad - (\sigma_A^\infty)'(\text{virtual}, \alpha_s^A), \\ (\sigma_{\text{Born}}^A)' &= \left(\frac{\alpha_s^A}{\alpha_s^\infty}\right)^2 (\sigma_{\text{Born}}^\infty)', \\ A^A &= \frac{(\sigma_A^A)'}{(\sigma_{\text{Born}}^A)'},\end{aligned}$$

$$\begin{aligned}A^A(\cos\theta) &= A^\infty + \delta A_\alpha^A + \delta A_{\text{box}}^A, \\ \delta A_\alpha^A &\equiv \frac{\alpha_s^A - \alpha_s^\infty}{\alpha_s^\infty} A^\infty, \\ \delta A_{\text{box}}^A &\equiv \frac{(\sigma_A^A)'(\text{virtual}, \alpha_s^A) - (\sigma_A^\infty)'(\text{virtual}, \alpha_s^A)}{(\sigma_{\text{Born}}^A)'},\end{aligned}\tag{2}$$

where  $A$  denotes quantity in the BY theory,  $\infty$  denotes quantity in the SM.

We mean that  $(\sigma_A^A)'(\text{virtual}, \alpha_s^A)$  is evaluated with  $\alpha_s^A$  coupling, *etc.* The calculation of the strong interaction running coupling in noncontractible space was performed in the momentum subtraction renormalization scheme to one-loop order in Ref. [10]. Hard and soft gluon radiations do not contain loop diagrams to leading  $\alpha_s^3$  order.

Our main task should be a reevaluation of the interference term in the cross section containing the box diagram in the virtual correction term. To accomplish this in noncontractible space, we have to reduce the amplitude into pieces that are manifestly translationally and Lorentz invariant.

We render light quark masses nonvanishing as a regulator of the collinear singularity that is canceled away in the asymmetric cross sections. Infrared singularity is controlled by the regulator gluon mass and is canceled away in both  $A^\infty$  and  $\delta A_{\text{box}}^A$  asymmetry parameters. The virtual corrections can

be represented with the following expression [22]

$$\frac{d\sigma_A(\text{virtual})}{d\cos\theta} = \alpha_s^3 \frac{d_{abc}^2 \beta_t}{32N_c^2 s} \left[ \sum_{j=1}^7 w_j I_j - (\theta \rightarrow \pi - \theta) \right], \quad (3)$$

$$d_{abc}^2 = \frac{40}{3}, \quad \beta_t = \sqrt{1 - 4m_t^2/s}.$$

Definitions are given in Appendix, as well as the procedure how to evaluate the integrals in noncontractible space to maintain translational and Lorentz invariance.

Now we can compare the  $t$ -quark charge asymmetries to the leading one-loop order in the standard QCD and the QCD in noncontractible space. The numerics and discussion can be found in the last chapter.

### 3. Results and conclusions

The difference between the  $t$ -quark charge asymmetries of the standard QCD and the QCD in noncontractible space lies in the additional two terms of Eq. (3)  $\delta A_\alpha^A$  and  $\delta A_{\text{box}}^A$ . The first additional term  $\delta A_\alpha^A$  can be evaluated using Table I derived from the formulae for  $\alpha_s^A$  in Ref. [10]. This correction can enhance the SM asymmetry by up to 47% for the largest parton  $E_{\text{cm}} = 14$  TeV. The strong coupling  $\alpha_s^A(\mu)$  is frozen at  $\mu \simeq 0.5$  TeV.

TABLE I

Running strong couplings at the scale  $\mu = E_{\text{cm}}/2$  assuming  $m_u = 2.5$  MeV,  $m_d = 5.0$  MeV,  $m_s = 100$  MeV,  $m_c = 1.6$  GeV,  $m_b = 4.8$  GeV,  $m_t = 172$  GeV and  $\alpha_s(\mu = M_Z) = 0.12$ .

$E_{\text{cm}}$ [TeV]	0.4	1.96	8	14
$\alpha_s^\infty(\mu)$	0.1077	0.08985	0.07886	0.0749
$\alpha_s^A(\mu)$	0.1104	0.110	0.110	0.110
$\frac{\alpha_s^A - \alpha_s^\infty}{\alpha_s^\infty}(\mu)$	0.0248	0.225	0.397	0.471

This is not enough to explain the asymmetry observed at the Tevatron [17]. Fortunately, the second additional term  $\delta A_{\text{box}}^A$  provides the necessary enhancement (see Fig. 1 and Table II).

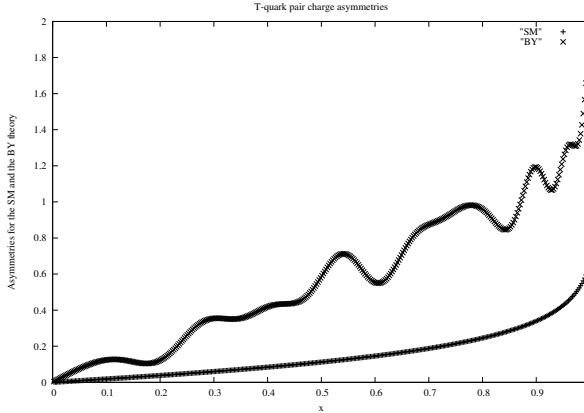


Fig. 1. Asymmetry parameters  $A^\infty$  and  $A^A$  as a function of  $x = \cos \theta$ ;  $E_{\text{cm}} = 1.96$  TeV,  $m_t = 172$  GeV,  $E_{\text{th}} = 0.9$  TeV.

TABLE II

Integrated  $t$ -quark charge asymmetries for parton  $E_{\text{cm}}$  evaluated with  $E_{\text{th}} = 0.9 \times E_{\text{cm}}/2$  and  $m_t = 172$  GeV.

$E_{\text{cm}}$ [TeV]	0.4	1.96	8	14
$A_{\text{int}}^\infty$	0.0740	0.1774	0.1519	0.1449
$A_{\text{int}}^A$	0.0939	0.661	0.805	0.874
$A_{\text{int}}^A/A_{\text{int}}^\infty$	1.27	3.73	5.30	6.03

We define the integrated charge asymmetry parameter as [19]

$$A_{\text{int}} \equiv \frac{\int_0^1 \sigma'_A d \cos \theta}{\int_0^1 \sigma'_{\text{Born}} d \cos \theta}. \tag{4}$$

One can conclude that the charge asymmetries at the parton level are enhanced in the BY theory by more than a factor of 3 (5) for Tevatron (LHC) center of mass energies. It is evident from Tables I and II that the deviation from the SM is larger for higher  $E_{\text{cm}}$  and the virtual correction (box diagram)  $\delta A_{\text{box}}^A$  dominates over the strong coupling correction  $\delta A_\alpha^A$ . It means that the box diagram explores the deep spacelike domain of the Minkowski spacetime to which the asymmetry observable is very sensitive and, in addition, there is no new negative compensation of the real hard and soft contributions (no quantum loops to this order of perturbation) except the new  $\alpha_s^A$  factor.

To find charge asymmetry for hadrons, one has to convolve parton cross sections with parton distributions. It is necessary to solve DGLAP and BFKL equations in noncontractible space. This work remains for the future. It is very unlikely that higher orders of perturbation in the strong coupling or new parton distributions can remove large deviation of the asymmetry from the standard QCD found at the parton level. If the LHC confirms the Tevatron results, it will be necessary to investigate the issue to higher perturbative order to reach higher accuracy, because to date, it is the largest discrepancy observed between the standard QCD and the experiment.

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### Appendix

Since the details for the QCD running coupling evaluations can be found in Ref. [10] and the equations for the SM asymmetries in Ref. [19], in the Appendix, we outline the equations for the virtual corrections in the SM and the BY theory using notations of Refs. [21, 22].

Let us define the energy unit  $E = E_{\text{cm}}/2$  and the dimensionless mass of the light quark by  $\bar{m}_u = m_u/E$  and the  $t$ -quark by  $\bar{m}_t = m_t/E$  [22]. With previously defined  $c = \beta_t \cos \theta$  the coefficients  $w_j$  in the sum  $\sum_{j=1}^7 w_j I_j$  of Eq. (3) are as follows [22]

$$\begin{aligned} w_1 &= 1 + c^2 - 2c^3 + (1 - 2c) (\bar{m}_u^2 + \bar{m}_t^2) , \\ w_2 &= 2c(1 - c) - \bar{m}_u^2 - c\bar{m}_t^2 , \\ w_3 &= 2c(1 - c) - \bar{m}_t^2 - c\bar{m}_u^2 , \\ w_4 &= 2 - c + c^2 + \bar{m}_t^2 + \bar{m}_u^2 , \\ w_5 &= -1 - c , \\ w_6 &= 1 , \\ w_7 &= 1 - c . \end{aligned}$$

We need further definitions to describe the process  $q(p_+) + \bar{q}(p_-) \rightarrow t(q_+) + \bar{t}(q_-)$  and its amplitude [21, 22]

$$\begin{aligned} P &= \frac{1}{2}(p_+ + p_-) , \\ \Delta &= \frac{1}{2}(p_+ - p_-) , \\ Q &= \frac{1}{2}(q_+ - q_-) , \\ \overline{\int}(f(k)) &\equiv \frac{4}{\pi^2} \Im \int \frac{f(k) d^4 k}{(\Delta)(Q)(+)(-)} , \end{aligned}$$

$$\begin{aligned}
 (\Delta) &= k^2 - 2k \cdot \Delta - P^2 + i\varepsilon, \\
 (Q) &= k^2 - 2k \cdot Q - P^2 + i\varepsilon, \\
 (\pm) &= k^2 \pm 2k \cdot P + P^2 - m_{\text{gluon}}^2 + i\varepsilon.
 \end{aligned}$$

Now we can define dimensionless integrals  $I_j$  of Eq. (3)

$$\begin{aligned}
 I_1 &= E^4 \overline{\int} (1), \\
 I_2 &= E^2 \overline{\int} (k \cdot \Delta), \\
 I_3 &= E^2 \overline{\int} (k \cdot Q), \\
 I_4 &= E^2 \overline{\int} (k^2), \\
 I_5 &= \overline{\int} ((k \cdot P)^2), \\
 I_6 &= \overline{\int} ((k \cdot \Delta)^2 + (k \cdot Q)^2), \\
 I_7 &= \overline{\int} ((k \cdot \Delta)(k \cdot Q)).
 \end{aligned}$$

These integrals can be evaluated by the integrals from Ref. [21]

$$[J; J_\mu; J_{\mu\nu}] = \int d^4k \frac{[1; k_\mu; k_\mu k_\nu]}{(\Delta)(Q)(+)(-)},$$

that are expressed in terms of nine functions:  $F, G, F_\Delta, F_Q, G_\Delta, G_Q, H_P, H_\Delta, H_Q$ .

Let us represent seven integrals  $I_j$  of Ref. [22] in terms of functions from [21]

$$\begin{aligned}
 I_1 &= \frac{4}{\pi^2} \frac{F + G}{2P^2} E^4, \\
 I_2 &= \frac{4}{\pi^2} (\Delta^2 J_\Delta + \Delta \cdot Q J_Q) E^2, \\
 I_3 &= \frac{4}{\pi^2} (Q^2 J_Q + \Delta \cdot Q J_\Delta) E^2, \\
 I_4 &= \frac{4}{\pi^2} (4K_O + P^2 K_P + \Delta^2 K_\Delta + Q^2 K_Q + 2\Delta \cdot Q K_X) E^2, \\
 I_5 &= \frac{4}{\pi^2} (K_O P^2 + K_P (P^2)^2),
 \end{aligned}$$



$$\begin{aligned}
 I_6 &= \frac{4}{\pi^2} \left( K_O (\Delta^2 + Q^2) + K_\Delta \left( (\Delta^2)^2 + (\Delta \cdot Q)^2 \right) + K_Q \left( (\Delta \cdot Q)^2 + (Q^2)^2 \right) \right. \\
 &\quad \left. + 2K_X \Delta \cdot Q (\Delta^2 + Q^2) \right), \\
 I_7 &= \frac{4}{\pi^2} \left( \Delta \cdot Q K_O + \Delta \cdot Q \Delta^2 K_\Delta + \Delta \cdot Q Q^2 K_Q + K_X \left( Q^2 \Delta^2 + (\Delta \cdot Q)^2 \right) \right), \tag{5}
 \end{aligned}$$

where  $J_\Delta$ ,  $J_Q$ ,  $K_O$ ,  $K_P$ ,  $K_\Delta$ ,  $K_Q$  and  $K_X$  functions are defined in terms of nine functions  $F, \dots, H_Q$  [21].

We use standard definitions for the scalar two, three and four point functions [23]

$$\begin{aligned}
 B_0(p; m_1, m_2) &= (i\pi^2)^{-1} \int d^4k [k^2 - m_1^2 + i\epsilon]^{-1} [(k+p)^2 - m_1^2 + i\epsilon]^{-1}, \\
 C_0(p_1, p_2; m_0, m_1, m_2) &= (i\pi^2)^{-1} \int d^4k [k^2 - m_0^2 + i\epsilon]^{-1} \\
 &\times [(k+p_1)^2 - m_1^2 + i\epsilon]^{-1} [(k+p_2)^2 - m_2^2 + i\epsilon]^{-1}, \\
 D_0(p_1, p_2, p_3; m_0, m_1, m_2, m_3) &= (i\pi^2)^{-1} \int d^4k [k^2 - m_0^2 + i\epsilon]^{-1} \\
 &\times [(k+p_1)^2 - m_1^2 + i\epsilon]^{-1} [(k+p_2)^2 - m_2^2 + i\epsilon]^{-1} [(k+p_3)^2 - m_3^2 + i\epsilon]^{-1}.
 \end{aligned}$$

In Ref. [21] expressions for all nine functions  $F, \dots, H_Q$  in the standard QCD can be found. The same functions have to be expressed by the previous scalar two, three and four point Green functions in noncontractible space in order to properly restore translational invariance [10–12].

Functions  $G, F_\Delta, F_Q$  have already a suitable form of the three point functions [21]

$$\begin{aligned}
 G &= \int d^4k (\Delta)^{-1} (Q)^{-1} (+)^{-1}, \\
 F_\Delta &= \int d^4k (\Delta)^{-1} (+)^{-1} (-)^{-1}, \\
 F_Q &= \int d^4k (Q)^{-1} (+)^{-1} (-)^{-1}.
 \end{aligned}$$

Note that all expressions in Ref. [21] are derived under the assumption of  $m_{\text{gluon}} \equiv \lambda \ll m_u, m_t, E_{\text{cm}}$ . From their definitions,  $G_\Delta$  and  $G_Q$  can be expressed as

$$\begin{aligned}
 \Im G_Q &= \frac{1}{\beta_t^2} \Im F_Q + \frac{2\pi^2}{s\beta_t^2} [\Re B_0(-2P; \lambda, \lambda) - \Re B_0(-Q - P; \lambda, m_t)], \\
 \Im G_\Delta &= \frac{1}{\beta_u^2} \Im F_\Delta + \frac{2\pi^2}{s\beta_u^2} [\Re B_0(-2P; \lambda, \lambda) - \Re B_0(-\Delta - P; \lambda, m_u)]. \tag{6}
 \end{aligned}$$

For functions  $F, H_P, H_\Delta, H_Q$ , we derive the equations that allow to put these functions in the alternative form expressed only through scalar  $n$ -point integrals.

The linear system for the  $F$  function looks as

$$\begin{aligned} p_1^2 \eta_1 + p_1 \cdot p_2 \eta_2 + p_1 \cdot p_3 \eta_3 &= R_1, \\ p_1 \cdot p_2 \eta_1 + p_2^2 \eta_2 + p_2 \cdot p_3 \eta_3 &= R_2, \\ p_1 \cdot p_3 \eta_1 + p_2 \cdot p_3 \eta_2 + p_3^2 \eta_3 &= R_3, \end{aligned} \tag{7}$$

$$\begin{aligned} R_1 &= \frac{1}{2} [\Re C_0(p_2, p_3; m_0, m_2, m_3) - \Re C_0(p_2 - p_1, p_3 - p_1; m_1, m_2, m_3) \\ &\quad - (p_1^2 - m_1^2 + m_0^2) \Re D_0(p_1, p_2, p_3; m_0, m_1, m_2, m_3)] , \\ R_2 &= \frac{1}{2} [\Re C_0(p_1, p_3; m_0, m_1, m_3) - \Re C_0(p_2 - p_1, p_3 - p_1; m_1, m_2, m_3) \\ &\quad - (p_2^2 - m_2^2 + m_0^2) \Re D_0(p_1, p_2, p_3; m_0, m_1, m_2, m_3)] , \\ R_3 &= \frac{1}{2} [\Re C_0(p_1, p_2; m_0, m_1, m_2) - \Re C_0(p_2 - p_1, p_3 - p_1; m_1, m_2, m_3) \\ &\quad - (p_3^2 - m_3^2 + m_0^2) \Re D_0(p_1, p_2, p_3; m_0, m_1, m_2, m_3)] , \end{aligned}$$

$$p_1 = 2P,$$

$$p_2 = P - \Delta,$$

$$p_3 = P - Q,$$

$$m_0 = m_1 = \lambda,$$

$$m_2 = m_u,$$

$$m_3 = m_t \Rightarrow \Im F = -\Im F_Q + 2\pi^2 (\Delta^2 \eta_2 + \Delta \cdot Q \eta_3) .$$

Similarly, we derive the linear system for  $H$  functions

$$\begin{aligned} p_1^2 \rho_1 + p_1 \cdot p_2 \rho_2 &= M_1, \\ p_1 \cdot p_2 \rho_1 + p_2^2 \rho_2 &= M_2, \end{aligned} \tag{8}$$

$$\begin{aligned} M_1 &= \frac{1}{2} [\Re B_0(p_2; \lambda, m_2) - \Re B_0(p_2 - p_1; m_1, m_2) + (-\lambda^2 + m_1^2 - p_1^2) \\ &\quad \times \Re C_0(p_1, p_2; \lambda, m_1, m_2)] , \\ M_2 &= \frac{1}{2} [\Re B_0(p_1; \lambda, m_1) - \Re B_0(p_2 - p_1; m_1, m_2) + (-\lambda^2 + m_2^2 - p_2^2) \\ &\quad \times \Re C_0(p_1, p_2; \lambda, m_1, m_2)] , \end{aligned}$$

$$p_1 = P - \Delta,$$

$$p_2 = P - Q,$$

$$m_1 = m_u,$$

$$m_2 = m_t \Rightarrow \Im H_P = \Im G + \pi^2 (\rho_1 + \rho_2) ,$$

$$\Im H_\Delta = -\pi^2 \rho_1 ,$$

$$\Im H_Q = -\pi^2 \rho_2 .$$

The validity of new forms for  $F, H_P, H_\Delta$  and  $H_Q$  is also checked numerically. The virtual corrections can be evaluated by Eq. (A.1) of Ref. [22] or by Eq. (12) of Ref. [21].

We are now prepared for the crucial step to calculate virtual corrections in noncontractible space defining scalar  $n$ -point integrals in noncontractible space.  $B_0^A$  function is outlined in Refs. [10–12]. The similar procedure should be applied to the three point function

$$\Re C_0^\infty = \Re C_0^A + \delta C_0^A(\text{symm}), \tag{9}$$

$$\delta C_0^A(\text{symm}) = \frac{1}{3} \left[ \delta C_0^A(p_1, p_2; m_0, m_1, m_2) + \delta C_0^A(-p_1, p_2 - p_1; m_1, m_0, m_2) + \delta C_0^A(-p_2, p_1 - p_2; m_2, m_0, m_1) \right],$$

$$\delta C_0^A(p_1, p_2; m_0, m_1, m_2) = \pi^{-2} \int_0^{1/\Lambda} dw w^{-5} \int_{-1}^{+1} dx \sqrt{1-x^2} \int_{-1}^{+1} dy$$

$$\times \int_0^{2\pi} d\phi [-k^2 - m_0^2]^{-1} [-k^2 + 2(k \cdot p_1) + p_1^2 - m_1^2]^{-1}$$

$$\times [-k^2 + 2(k \cdot p_2) + p_2^2 - m_2^2]^{-1} (k = w^{-1}),$$

where  $(k \cdot p_1) = ikx(p_1)^0 - \vec{k} \cdot \vec{p}_1$  and  $\vec{k} = k\sqrt{1-x^2}(\sqrt{1-y^2} \cos \phi, \sqrt{1-y^2} \sin \phi, y)$ .

All the imaginary parts of the subintegral function in  $\delta C_0^A$  are erased by integration as odd functions in variable  $x$ .

The same decomposition is possible for the four point function, although with four terms necessary for symmetrization in  $\delta D_0^A$ .

Multidimensional numerical integrations in virtual and real gluon radiations are performed by Suave routine from CUBA library [24] to the relative accuracy of  $\mathcal{O}(10^{-4})$  with up to 50 million of sampling points per integral.

### REFERENCES

[1] A.D. Sakharov, *J.E.T.P. Lett.* **5**, 24 (1967).  
 [2] E.W. Kolb, M.S. Turner, *The Early Universe*, Addison-Wesley, Redwood City 1990.  
 [3] J.A. Peacock, *Cosmological Physics*, Cambridge University Press, Cambridge 1999.

- [4] J.A.R. Cembranos, V. Gammaldi, A.L. Maroto, [arXiv:1204.0655](#) [hep-ph].
- [5] O. Adriani *et al.*, *Nature* **458**, 607 (2009).
- [6] M. Cirelli, M. Kadastik, M. Raidal, A. Strumia, *Nucl. Phys.* **B813**, 1 (2009).
- [7] B. Famaey, S. McGaugh, [arXiv:1112.3960](#) [astro-ph.CO].
- [8] D. Palle, *Nuovo Cim.* **A109**, 1535 (1996).
- [9] D. Palle, *Nuovo Cim.* **B111**, 671 (1996); **B114**, 853 (1999); **B115**, 445 (2000); **B118**, 747 (2003); *Eur. Phys. J.* **C69**, 581 (2010); *Entropy* **14**, 958 (2012).
- [10] D. Palle, *Hadronic J.* **24**, 87 (2001); *Hadronic J.* **24**, 469 (2001).
- [11] D. Palle, *Acta Phys. Pol. B* **43**, 1723 (2012).
- [12] D. Palle, [arXiv:1111.1639](#) [physics.gen-ph]; [arXiv:1210.4404](#) [physics.gen-ph].
- [13] CMS Collaboration, *Phys. Lett.* **B716**, 30 (2012); ATLAS Collaboration, *Phys. Lett.* **B716**, 1 (2012).
- [14] J.W. Moffat, [arXiv:1207.6015](#) [hep-ph]; [arXiv:1211.2746](#) [hep-ph]; P. Cea, [arXiv:1209.3106](#) [hep-ph].
- [15] J.P. Ralston, [arXiv:1211.2288](#) [hep-ph].
- [16] D. Palle, [arXiv:hep-ph/0703203](#).
- [17] T. Aaltonen *et al.* [CDF Collaboration], *Phys. Rev.* **D83**, 112003 (2011); V.M. Abazov *et al.* [D0 Collaboration], *Phys. Rev.* **D84**, 112005 (2011).
- [18] T. Aaltonen *et al.* [CDF Collaboration], [arXiv:1211.1003](#) [hep-ph].
- [19] J.H. Kühn, G. Rodrigo, *Phys. Rev.* **D59**, 054017 (1999).
- [20] F.A. Berends, K.J.F. Gaemers, R. Gastmans, *Nucl. Phys.* **B57**, 381 (1973).
- [21] F.A. Berends, K.J.F. Gaemers, R. Gastmans, *Nucl. Phys.* **B63**, 381 (1973).
- [22] F.A. Berends, R. Kleiss, S. Jadach, Z. Waż, *Acta. Phys. Pol. B* **14**, 413 (1983).
- [23] G.J. van Oldenborgh, *Comput. Phys. Commun.* **66**, 1 (1991).
- [24] T. Hahn, *Comput. Phys. Commun.* **168**, 78 (2005).