

COMPETING OF SZNAJD AND VOTER DYNAMICS
IN THE WATTS-STROGATZ NETWORK*

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We investigate the Watts–Strogatz network with the clustering coefficient C dependent on the rewiring probability. The network is an area of two opposite contact processes, where nodes can be in two states, S or D. One of the processes is governed by the Sznajd dynamics: if there are two connected nodes in D-state all their neighbors become D with probability p . For the opposite process, it is sufficient to have only one neighbor in state S; this transition occurs with probability 1. The concentration of S-nodes changes abruptly at given value of the probability p . The result is that for small p , in clusterized networks the activation of S nodes prevails. This result is explained by a comparison of two limit cases: the Watts–Strogatz network without rewiring, where $C = 0.5$, and the Bethe lattice, where $C = 0$.

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1. Introduction

Our aim here is to study the results of competition of two types of contact processes (CP) [1] in the Watts–Strogatz network. The idea is as follows. The network nodes are assumed to be in one of two states, say S and D. One of CP is the voter dynamics [2], where a randomly selected S node changes the state of one of his neighbor from D to S. The other CP is the unifying step of the Sznajd dynamics [3, 4], where a randomly selected pair of neighboring nodes, once both D, change the state of all their neighbors from S to D.

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In a recent text [5], a similar competition has been investigated on the Watts–Strogatz (WS) and the Erdős–Rényi (ER) networks. Also there, one process of activating nodes has been ruled by the voter dynamics, and the other was triggered by a pair of mutually connected nodes. The difference between that approach and the present work is that in paper [5], a node was activated by a pair of D nodes only if it was a nearest neighbor of both nodes of the pair. On the contrary, in the Sznajd dynamics used here all neighbors of each member of the D pair are activated [3]. We note that the third model of this family is the bootstrap percolation, where nodes of the activating pair are not necessary neighbors of each other [6, 7]; the latter model is not investigated here.

Our motivation is as follows. In [5], the clustering coefficient C was used as a control parameter. In WS, C was controlled by rewiring, while in ER, it was controlled by adding new links, along the method proposed by Holme and Kim [8, 9]. The results of [5] indicate that in WS, tuning of C can lead to switching the D process off and on. Namely, the D process is blocked if the clustering coefficient is below some critical value. This effect is a consequence of the model assumption, that both nodes of the activating pair must be neighbors of the activated node. The density of triangles is controlled by C . If the triangles in the network are rare, the D process is stopped. To check this conclusion, we are going to investigate the same process with the Sznajd dynamics, where triangles are not necessary. Consequently, the clustering coefficient C should be less important.

It is worth to add that the competing CP's enables a more precise measurement of intensity of the investigated process by its comparison with the voter dynamics. In a finite network, in particular a small-world network, the process can reach the whole lattice in a few steps. To switch another process on allows to find their mutual intensity when both processes balance each other. In [5], the voter process was more intense and, therefore, it was applied with probability p , while the D process was applied with probability 1. Here, the situation is opposite; the Sznajd dynamics is applied with the probability p , and the voter dynamics with the probability 1. We look for the values of p_c where the stationary percentage of both kinds of nodes is 50–50.

2. Algorithm

Fifty WS networks of $N = 1000$ nodes and degree $k = 4$ are prepared by gradual rewiring of randomly selected links, and they are stored when their clustering coefficient C is in the range $(C_k, C_k + 0.01)$. The demanded C_k are from 0.5 down to 0.1. Next, we select randomly a given percentage of nodes and assign them to be in the state S; other nodes are in the state D.

The simulation is performed as follows. A node is selected randomly; if it is in the state S, we check all its neighbors. If there is a node D, it is converted to be S. If the randomly selected node is in the state D, we continue with the probability p : namely, we check all his neighbors. If there is a node D, all its neighbors and all the neighbors of the initially selected node (which appeared to be D) are converted to be D. Having done this, we select another node and continue. The change is done immediately.

In each time step, N nodes are selected. There is 2500 time steps. The results are averaged over 50 networks, except in the case of $C = 0.5$, when there is only one network and the simulation is run 50 times.

3. Results

The influence of the clustering coefficient C on the population of S nodes is most apparent in the range of small initial population of these nodes, then the results to be shown here are obtained in this range. Also, their dependence on the initial state is visible there.

In Figs. 1 and 2, we show the time dependence of the population of S nodes for two values of the clustering coefficient C , 0.1 and 0.5. We observe that for $C = 0.1$ (Fig. 1) the time of simulation is long enough to get the stationary state. Although we have not obtained this state for $C = 0.5$ (Fig. 2), the results clearly show that in the latter case the relatively large probability p of the D process is not enough to reduce the population

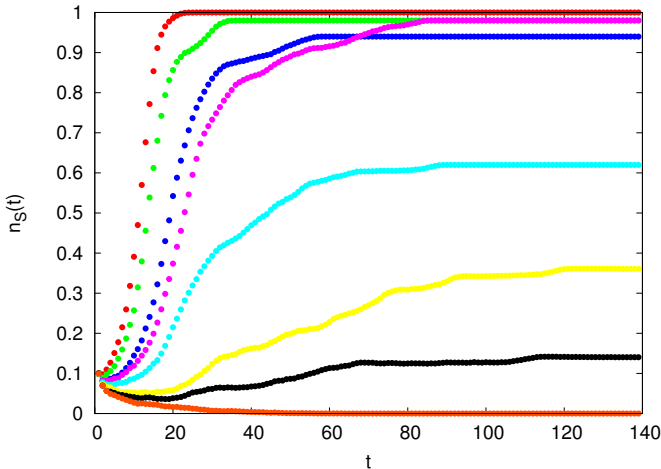


Fig. 1. Time dependence of the S-nodes participation in the WS network (10 percent of all 1000 nodes are initially S nodes) with $C = 0.1$. The probability p is from 0.2 to 0.27 with step 0.01, in ascending order from the top to the bottom of the graph.

of S nodes as strongly, as it is found for $C = 0.1$. Namely, while $p = 0.27$ is enough to eliminate S nodes for $C = 0.1$, we find that for $C = 0.5$ S nodes still prevail for $p = 0.38$.

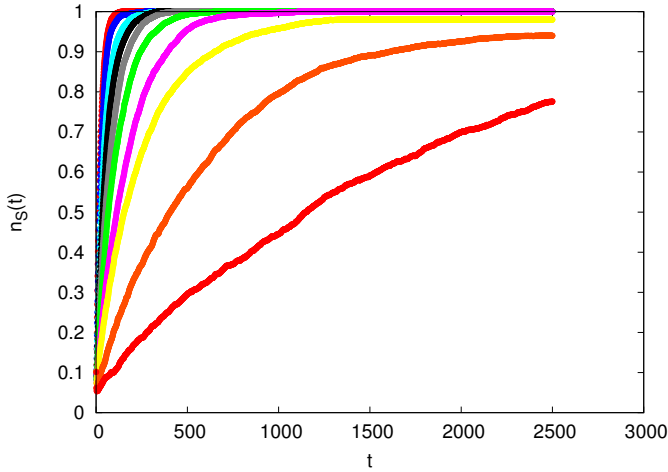


Fig. 2. Time dependence of the S-nodes participation in the WS network (10 percent of all 1000 nodes are initially S nodes) with $C = 0.5$. The probability p is from 0.20 to 0.38 with step 0.02, in ascending order from the top to the bottom of the graph.

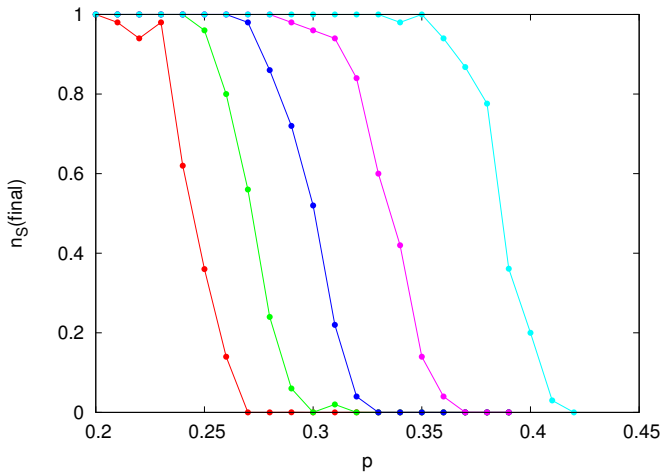


Fig. 3. The final S-nodes participation *vs* the probability p for the WS network (10 percent of all 1000 nodes are initially S nodes) for different clustering coefficient $C = 0.1, 0.2, 0.3, 0.4$ and 0.5 (curves from left to right, respectively).

The results on the population of S nodes after 2500 time steps are shown in Figs. 3 and 4 for different values of C , and two different values of the initial population of S nodes, 0.1 and 0.25. The results show how the fall of the observed population of S nodes with the probability p depend on the clustering coefficient C . Clearly, the larger C , the stronger is the process S, as larger values of p are necessary to damp the population of S nodes.

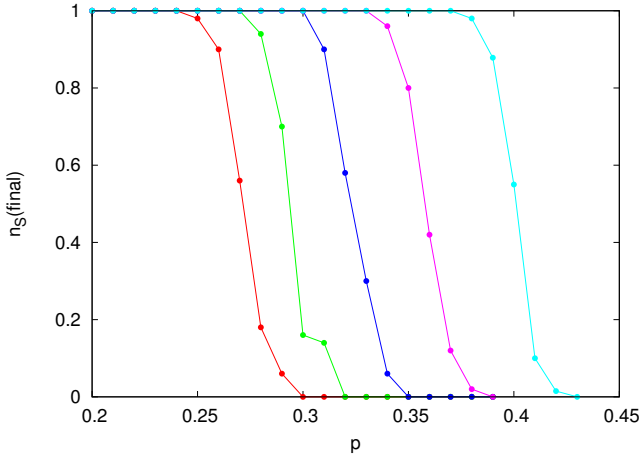


Fig. 4. The final S-nodes participation *vs* the probability p for the WS network (25 percent of all 1000 nodes are initially S nodes) for different clustering coefficient $C = 0.1, 0.2, 0.3, 0.4$ and 0.5 (curves from left to right, respectively).

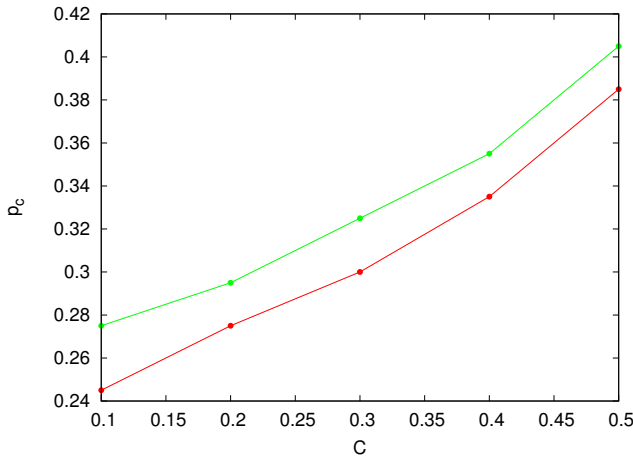


Fig. 5. The critical probability p_c against the clustering coefficient C for the WS network for different initial S-nodes participations equal to 10 percent (bottom curve) and 25 percent (top curve).

From these results, we can evaluate the probability p_c when the calculated population of S nodes is 0.5 after 2500 time steps. As shown in Fig. 5, the initial population of S nodes does not change these results qualitatively. In both cases, the observed probability p_c increases with C from about 0.26 to about 0.4 in the investigated range of C .

4. Discussion

The results indicate, that the S process is relatively more active for WS networks with large clustering coefficient C . In particular, for the case when the initial amount of S nodes is 0.1, the critical value p_c is about 0.39 for $C = 0.5$, but only 0.24 for $C = 0.1$. When the initial amount of nodes is 0.25, the same numbers for p_c are 0.39 and 0.27. This means that in clusterized networks a larger probability of the D process is necessary to balance the S process, than for the case when the clustering is small.

This result is opposite to the one obtained in our previous calculations [5], where the activation of D nodes was possible only if a node was simultaneously a nearest neighbor of both nodes of the activating pair of D nodes. This detail appears to be the cause of the observed different role of the clustering coefficient. While it seems natural that in our case the variation of C is simply not relevant, the reversed influence of the clustering calls for an interpretation.

To accomplish this, let us compare two limit cases: the WS network without rewiring, where the clustering coefficient is maximal, and the Bethe lattice. For the mean degree $\langle k \rangle = 4$, as assumed here, the respective values of C are 0.5 and 0.0. Although in our case, the WS network with large amount of rewiring is not equivalent to the Bethe lattice, it is anyway close to an Erdős–Rényi network, which could be approximated by a regular tree. For the purpose of our explanation, it is only important that in both cases the clustering coefficient is small. Let us consider the simplified case of the velocity of spread of the D phase from a pair of neighbors, with the S process switched off and all nodes simultaneously updated. In the WS network, this velocity is constant: for $k = 4$, two nodes are switched from S to D at each step. On the contrary, in the Bethe lattice with the same degree the number of nodes switched from S to D along one tree branch is multiplied by $k - 1$ at each time step. We should add that for the process investigated in [5], the above multiplication does not occur; in trees the D process considered there does not work at all.

The list of applications of the Sznajd model in politics and economy can be found in [4]. Here, we add that the comparison of the results presented here and of those in [5] reveals that a subtle modification of the mechanism seriously alters the intensity of the Sznajd dynamics in clusterized

networks. In the literature on the Sznajd model, we often find a reference to the threshold effect introduced to sociology by Granovetter [10, 11]. In this effect, people are inclined to imitate the others' behavior if more than a given number of persons behave in a given way. This means, that the imitators see all the imitated persons, and not only one. Accordingly, this sociological effect is described more accurately by the model described in [5] or even by the bootstrap percolation, than by the Sznajd dynamics.

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