MASS DETERMINATION OF TWO-PROTON RADIOACTIVE NUCLIDES*

K. MIERNIK

Physics Division, Oak Ridge National Laboratory, Oak Ridge TN 37830, USA
and
Faculty of Physics, University of Warsaw, Hoża 69, 00-681 Warszawa, Poland

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The masses of heavy two-proton emitters ($^{45}$Fe, $^{48}$Ni and $^{54}$Zn) are calculated, basing on experimentally measured two-proton decay energies. The results are compared with theoretical predictions and extrapolations.

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1. Introduction

The two-proton radioactivity is a phenomenon emerging at the proton-rich limit of existence of nuclides. The “true two-proton radioactivity” as defined by Goldansky [1] is a case when nuclide is bound against proton emission but, at the same time, unbound against two-proton emission ($2p$). This leads to rare decay mode, where two protons, and two protons only, can be emitted simultaneously from the nucleus. This situation may occur only for even $Z$ isotopes beyond proton drip line, and is a direct result of pairing character of nuclear forces. For heavier systems, we can expect half-lives sufficiently long [2] to establish a new radioactivity mode [3].

It is only thanks to the development of isotopes production techniques [4] that this rare decay can be studied experimentally. The early experiments [5–8] were focused on discovery of the phenomenon itself. The most recent experiments [9–11], introducing a new detection techniques based on gaseous detectors [12, 13] allowed to gain more insight into the correlation between protons and validated the three-body model of the $2p$ decay [14, 15]. The two-proton decay is also so far the only tool to learn some information

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about the nuclear structure of the emitters [9]. These nuclides are otherwise inaccessible experimentally due to extremely low production cross-sections [4, 16].

While these topics are covered in a detail elsewhere, e.g. in a most recent review work [17], this article will focus on the indirect determination of masses of $2p$ emitters based on experimentally measured observables and on comparison of the results with theoretical predictions and extrapolations.

2. Mass determination method

Figure 1 presents schematically the method used to find the masses of $2p$ emitters. The precursor $\frac{A}{Z}X$ decays by two-proton emission to the $\frac{A-2}{Z-2}Y$ nuclide with experimentally measured total decay energy $Q_{2p}$. The $2p$ decay daughter decays by $\beta^+$ to the $\frac{A-2}{Z-3}Z$ nuclide with significant branch of super-allowed Fermi transition to the Isobaric Analog State (IAS). These isotopes were studied in a very comprehensive work by Dossat et al. [18]. In all cases, the IAS is proton unbound and subsequently decays by an emission of a $\beta$-delayed proton(s). The total energy of this transition $E_{\beta np}$ is known experimentally. The mass of the IAS relative to the $\frac{A-2}{Z-3}Z$ ground state is determined from the relation

$$\Delta E = \Delta E_C - \Delta n_H,$$

where $\Delta E_C$ is Coulomb displacement energy and $\Delta n_H$ is a mass difference between neutron and hydrogen atom [19]. The mass of the final nuclide in the chain is known experimentally [19, 20].

This procedure could be applied to the $^{54}$Zn and the $^{45}$Fe case (note that the last decay in the chain is a $\beta$-delayed two-proton emission). In the case of $^{48}$Ni, the mass of $^{45}$Cr is not known from direct measurements, and it has to be connected to the mass of $^{44}$Ti by its $\beta$ decay properties. Moreover, both $\beta$-delayed protons in this chain were identified to be in coincidence with gamma radiation [18] indicating that the proton transitions were not proceeding directly to the ground state (see Fig. 1).

The Coulomb displacement energy is determined from extrapolation of compiled experimental data tables [21]. The Coulomb displacement energies depend on a mass number $A$, mean charge number of two nuclides of interest $\bar{Z}$ and their isospin $T$. In the reference [21] one may find needed fit coefficients for $T = 3/2$ and 2

$$\Delta E_C = 1411.1(3) \frac{\bar{Z}}{A^{1/3}} - 886.8(13) \text{ for } T = 3/2,$$

$$\Delta E_C = 1406.7(6) \frac{\bar{Z}}{A^{1/3}} - 872.8(32) \text{ for } T = 2.$$
However, for the decay of $^{43}$Cr and $^{46}$Fe isospins $T = 5/2$ and $3$ are needed respectively. The coefficients for these isospins values were determined by a fit to the selected dataset from tables [21] and yielded

$$\Delta E_C = 1412.0(7) \frac{\bar{Z}}{A^{1/3}} - 868.9(39) \quad \text{for } T = 5/2, \quad \Delta E_C = 1446.3(9) \frac{\bar{Z}}{A^{1/3}} - 1050.9(58) \quad \text{for } T = 3.$$ 

![Diagram of mass determination method](image)

Fig. 1. Schematic presentation of mass determination method. Figure is not to scale. See the text for details.

### 3. Results

The details of the calculations are presented in Table I. The case of $^{48}$Ni needed the determination of mass of $^{45}$Cr using Coulomb shift energy information. Note also that in the case of $^{45}$Cr and $^{48}$Ni, $E_{\beta p}$ values also
include the energy of gamma transitions, 494 + 12 keV and 1048 keV, in $^{45}$Cr and $^{44}$Ti, respectively. The 494 keV transition is expected to feed low energy excited state in $^{45}$Cr [18]. This assumption is based on properties of mirror nucleus $^{45}$Sc. Since the energy of this transition is not known, the value found in the mirror partner (12 keV) was used, with a large uncertainty of 100 keV to take into account possible level shifts between mirror nuclei.

### TABLE I

Details of mass calculations. All values given in keV. $E_{\beta p}$ for $^{48}$Ni and $^{45}$Cr is a sum of proton and gamma transition energy (see the text and Fig. 1). All masses of reference nuclides are taken from [20] except for $^{45}$Cr calculated here.

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>$Q_{2p}$</th>
<th>$\Delta E$</th>
<th>$E_{\beta p}$</th>
<th>Ref. nuclide</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{54}$Zn</td>
<td>1480(20)</td>
<td>8709(76)</td>
<td>1349(10)</td>
<td>$^{51}$Fe</td>
<td>−6797(79)</td>
</tr>
<tr>
<td>$^{45}$Fe</td>
<td>1152(12)</td>
<td>7820(85)</td>
<td>4363(19)</td>
<td>$^{41}$Sc</td>
<td>13848(88)</td>
</tr>
<tr>
<td>$^{48}$Ni</td>
<td>1280(60)</td>
<td>8460(123)</td>
<td>4745(106)*</td>
<td>$^{45}$Cr†</td>
<td>16410(176)</td>
</tr>
<tr>
<td>$^{45}$Cr</td>
<td>—</td>
<td>8436(33)</td>
<td>3170(10)*</td>
<td>$^{44}$Ti</td>
<td>−19436(34)</td>
</tr>
</tbody>
</table>

The $Q_{2p}$ value for $^{45}$Fe is a weighted average of three experimental results [5–7]. The $Q_{2p}$ of $^{54}$Zn decay was taken from [8], while one for $^{48}$Ni was reported in [16]. In the latter case, it is worth noting that a one decay event of energy 1.35(2) MeV determined by a silicon detector was reported in [7]. However, this event could also be a result of $\beta$-delayed proton emission (full energy or partial, due to escape) and as such was excluded in this work.

The $\beta$-decay protons and gamma transition energies are following values reported in the article [18]. All masses of reference nuclides, except for $^{45}$Cr calculated here, were taken from the most recent mass tables [20].

Figure 2 presents comparison of the results with the theoretical models by Ormand [22, 23], Cole [24], and Möller et al. [25]. Values obtained by Dossat et al. [18] with similar method as used in this work are also presented. In this case, the authors based their calculation on Isobaric Multiplet Mass Equation (IMME) for $^{48}$Ni and $^{54}$Zn, and Coulomb shift energy for $^{45}$Fe (however using a fit coefficients for $T = 2$). The extrapolated values from mass tables AME2003 [19] and AME2012 [20] are included in Fig. 2 as well.

Table II presents calculated root mean square (RMS) for aforementioned models as well as several other not included in the Fig. 2.

Clearly, the approach taken by Ormand [22, 23] and Cole [24], i.e., the extrapolations based on Coulomb energy shifts and IMME is the most successful method for predicting mass properties of $2p$ emitters. Another group of models with similar RMS consists of global mass models by Möller [25] and Goriely [26, 27], which present similar predictive power. It is worth noting that the RMS in both models are larger for $2p$ emitters than RMS
found globally [25]. The predictive power of method based on extrapolations [19, 20] is lower than aforementioned global mass models. This is especially seen in the case of $^{48}\text{Ni}$ which properties were unknown at the time of compilation [19] was published. The improvement of the results found in the latest mass tables [20] origins in inclusion of the experimentally measured $2p$ decay energies. The microscopic–macroscopic approach of Wang [28] and mass model by Myers and Swiatecki [29], even though give comparable results RMS for global calculations, seem less suitable for $2p$ emitters mass predictions.

![Fig. 2. Comparison of masses of two-proton emitters found in this work and values calculated within various theoretical models. The gray band represents the experimental uncertainty.](image)

**TABLE II**

Comparison of Root Mean Square (RMS) calculated for two-protons emitters for various mass models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Ref.</th>
<th>RMS [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>W.E. Ormand</td>
<td>[22, 23]</td>
<td>358</td>
</tr>
<tr>
<td>AME2012</td>
<td>[20]</td>
<td>435</td>
</tr>
<tr>
<td>P. Möller et al.</td>
<td>[25]</td>
<td>871</td>
</tr>
<tr>
<td>S. Goriely et al. HFB-17</td>
<td>[26]</td>
<td>1049</td>
</tr>
<tr>
<td>S. Goriely et al. HFB-21</td>
<td>[27]</td>
<td>1070</td>
</tr>
<tr>
<td>AME2003</td>
<td>[19]</td>
<td>1167</td>
</tr>
<tr>
<td>N. Wang et al.</td>
<td>[28]</td>
<td>1338</td>
</tr>
<tr>
<td>W.D. Myers et al.</td>
<td>[29]</td>
<td>2077</td>
</tr>
</tbody>
</table>
4. Summary

The experimental data were used to calculate mass excess of all three known two-proton radioactive nuclides. The results were compared with theoretical mass models and extrapolations. The estimation of their predictive powers may be useful for planning future experiments. The experimentally estimated masses of the most proton-rich nuclides may also serve as an anchoring point for the development of mass models.

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REFERENCES