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ILLUSTRATION OF ACCURACY OF PRESENTLY USED NUCLEAR-MASS MODELS

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A detailed study of the accuracy of the description of nuclear mass by theoretical models is performed. Seven models of a different nature are taken for the study. The discrepancy between the theoretical masses and the recently evaluated experimental ones is calculated for each nucleus in the region of heavy nuclei (with the proton number $Z \geq 82$). Main results are presented in the form of maps (colored online), one for each model. It is found that for one of the very recent models (WS4+RBF), a very small discrepancy, $| \delta m | \leq 250$ keV, is obtained for most of the considered nuclei.

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1. Introduction

Many important quantities and processes in nuclear physics and in astrophysics depend essentially on nuclear mass. This is the reason for the continuing big efforts in increasing the accuracy of measuring it and in the extension of the region of nuclei with measured mass to more and more exotic ones (*e.g.* Refs. [1–13]).

There is also a big effort on the theoretical side to improve the models describing the mass and in elaborating the new ones, as well as in the development of the methods to use the mass for the extension of our knowledge on the properties of nuclei and on the nuclear processes (*e.g.* Refs. [14–21]).

The accuracy of the description of mass by a given model is usually characterized by the root-mean-square (rms) of the discrepancies between the calculated and experimental masses obtained for all or almost all nuclei

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with measured mass. This very average value, however, although useful, is a rather poor characteristic of the model. It is due to the strong dependence of the accuracy on the region of nuclei considered. This dependence has been studied and illustrated in Ref. [22] for 10 nuclear-mass models used recently, by dividing the whole region of nuclei (usually with $Z, N \geq 8$, where Z is the proton and N the neutron numbers) into four subregions.

Still, the best characterization and illustration of the accuracy of a model is to calculate and illustrate its accuracy for each nucleus separately, without any averaging. Such a detailed study and illustration is the objective of the present paper. Seven different nuclear-mass models are taken for the study. Their accuracy is tested in the region of heavy nuclei ($Z \geq 82$). This is the region, where intensive investigations of new nuclei are being done recently (*e.g.* Refs. [23–29]).

2. Mass models selected for the illustration

As already stated above, seven models are selected for a detailed illustration of their accuracy. Five of them are of the macroscopic–microscopic nature, one is something between macroscopic–microscopic and purely microscopic types, and one of the purely microscopic character.

The macroscopic–microscopic models are: the Finite-Range Droplet Model (FRDM) [30] and four recently proposed Weizsäcker–Skyrme models: WS3 [16] and WS4 [20], and their improvements: (WS3+RBF) [17] and WS4+RBF [20].

The FRDM model is an extension of usual liquid-drop model by inclusion of higher-order terms in $A^{-1/3}$ and $(N-Z)/A$, and takes into account the finite range of the nuclear force. Here, A is the mass number of a nucleus. This model, widely used for a long time, may be considered as a reference point for other models.

The two recently proposed WS3 and WS4 models use a modified Bethe–Weizsäcker mass formula for the macroscopic part and the Strutinsky shell correction, based on the Woods–Saxon single-particle potential, as the microscopic part. Some care is taken for the consistency between the parameters of the macroscopic and microscopic parts, inspired by the Skyrme energy-density functional approach. The difference between the WS3 and WS4 models is that in the latter one, the surface diffuseness is taken into account for unstable nuclei.

The WS3+RBF and WS4+RBF models are the improvements of the WS3 and WS4 models, respectively, by applying to them the radial basis function (RBF) approach, which is a general mathematical method of extrapolation of known data of some quantity to predict unknown values for it. A detailed description of the method and the application of it to nuclear masses is given in Ref. [17].

The model of the type between the macroscopic–microscopic and pure microscopic is the model of Duflo and Zuker (DZ) [31] (see also Ref. [15]), known for a long time for its good accuracy. It uses a large number (28) of parameters fitted directly to experimental masses.

The purely microscopic, self-consistent model (HFB21) [14] is of the Hartree–Fock–Bogoliubov type with the BSK21 Skyrme interaction. This is the 21^{st} of a long series of models using the HFB approach, each being an improvement and a generalization of previous ones.

When comparing the quality of description of presently known masses [32] by the considered models, one should take into account dates of their publication. The old models are based on a much more modest data than the recent models. Here, only two models (WS4 and WS4+RBF) are based on the recent mass evaluation of 2012 [32].

3. Results and discussion

As already stated in the introduction, the region of heavy nuclei ($Z \geq 82$) is selected for a detailed illustration of the accuracy of the models in their description of nuclear mass. The region contains 473 nuclei with measured and recently evaluated [32] masses.

Table I gives the characteristic features of the considered models for this region. These are: rms, and the average, $\bar{\delta}$, values of the discrepancies between the calculated and measured [32] masses, the lowest value of the discrepancy (negative number), $l\delta m$, the highest value (positive number), $h\delta m$, and the difference between them, $\Delta\delta m = h\delta m - l\delta m$. The nuclei, for which the $l\delta m$ or the $h\delta m$ appear, are specified by the proton Z and the mass number A just below these values. The number of nuclei with both calculated and evaluated values of mass, N_{nucl}, is also shown for each model. The values of rms, $\bar{\delta}$ and N_{nucl} for the FRDM, HFB21, DZ, WS3 (WS3.6) and WS3+RBF (WS3.3) models are taken from Ref. [22]. The year of the publication of each model is specified as well.

One can see in Table I that after the DZ model, rms systematically decreases along the sequence of the WS3, WS4, WS3+ and WS4+ models established in the table. In effect, the best description of experimental masses (smallest rms and $\Delta\delta m$) is obtained for the WS4+ model. This will be also clearly seen in the figures given below.

A detailed, and simultaneously very compact, information on the accuracy of a model in the considered region of nuclei is the map of the discrepancies between the calculated and experimental masses given for each nucleus in this region. Such maps are presented in Figs. 1 to 7, one map for each model. Each figure has a full value only when taken in its colored variant, as given online.

TABLE I

The characteristic features (see the text) calculated for the region of heavy nuclei ($Z \geq 82$) with the use of seven nuclear-mass models. Here, WS3+ and WS4+ denote the WS3+RBF and WS4+RBF models, respectively.

Model (Year)	FRDM (1995)	HFB21 (2010)	DZ (1995)	WS3 (2011)	WS4 (2014)	WS3+ (2010)	WS4+ (2014)
N_{nucl}	473	473	473	473	473	473	473
rms	0.448	0.458	0.376	0.255	0.235	0.179	0.133
$\bar{\delta}$	0.006	0.073	−0.032	−0.033	−0.023	−0.004	−0.009
$l\delta m$	−2.00	−1.60	−3.06	−0.87	−0.77	−0.69	−0.43
Z, A	110,270	83,186	110,270	82,197	85,224	85,224	83,186
$h\delta m$	1.12	1.03	1.00	0.60	0.64	0.88	0.33
Z, A	84,222	91,226	88,234	88,234	95,240	84,186	90,215
$\Delta\delta m$	3.12	2.63	4.06	1.47	1.41	1.57	0.76

One can see in Fig. 1 (the FRDM model) that the accuracy changes quite much and quite fast with changes of the proton Z and the neutron N numbers. The discrepancy between the calculated and measured mass, $\delta m = m_{\mathrm{th}} - m_{\mathrm{exp}}$, changes in the considered region from −2.00 MeV for the nucleus ^{270}Ds ($Z = 110$, $A = 270$) to 1.12 MeV for the nucleus ^{222}Po ($Z = 84$, $A = 222$), *i.e.* by 3.12 MeV (see Table I), which is quite much. The number of nuclei with masses reproduced with a low discrepancy, $\mid \delta m \mid \leq 0.25$ MeV, is relatively small.

The accuracy of the self-consistent model HFB21 is illustrated in Fig. 2. Here, the similar rms as in the case of the FRDM model, finds its reflection in the similar number of nuclei with a low discrepancy of masses, $\mid \delta m \mid \leq 0.25$ MeV. The difference between the highest and the lowest discrepancies, $\Delta\delta m$, is lower (2.63 MeV) than in the FRDM case.

Figure 3 shows the map for the DZ model. Here, the number of nuclei with masses reproduced with a low discrepancy, $\mid \delta m \mid \leq 0.25$ MeV, is significantly larger than in the FRDM and HFB21 cases. However, the difference $\Delta\delta m$ is exceptionally large, 4.06 MeV.

The number of the nuclei with low discrepancy of mass, $\mid \delta m \mid \leq 0.25$ MeV, is systematically increasing along the sequence of Figs. 4 to 7, in accordance with the decreasing rms (see Table I) along this sequence in the table. In effect, for the model WS4+RBF, the low discrepancy of masses, $\mid \delta m \mid \leq 0.25$ MeV, is obtained for almost all considered nuclei (see Fig. 7).

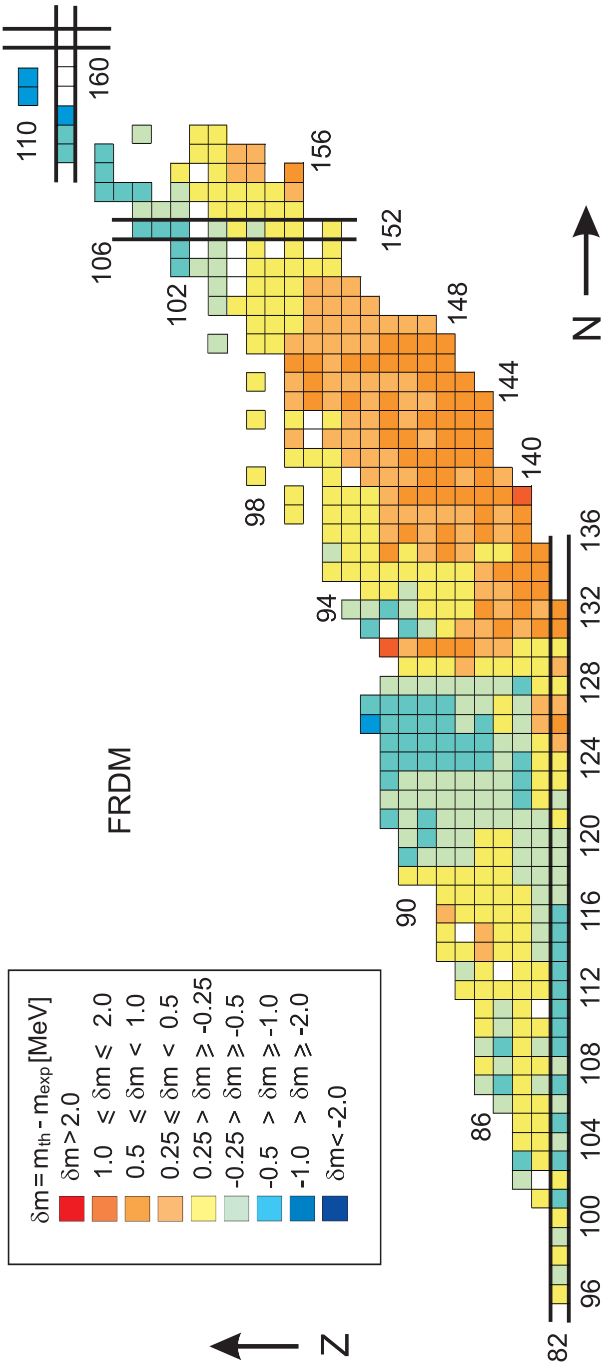

Fig. 1. Detailed map of the discrepancies of mass, δm, for the FRDM model.

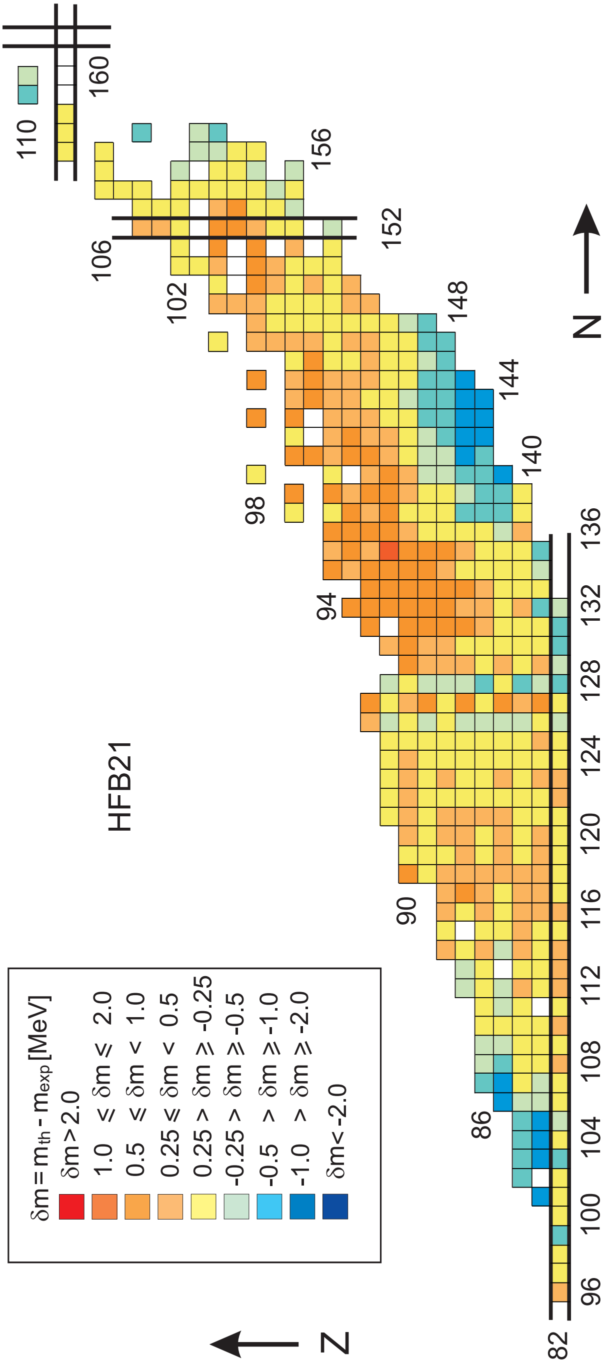

Fig. 2. The same as in Fig. 1, but for the HFB21 model.

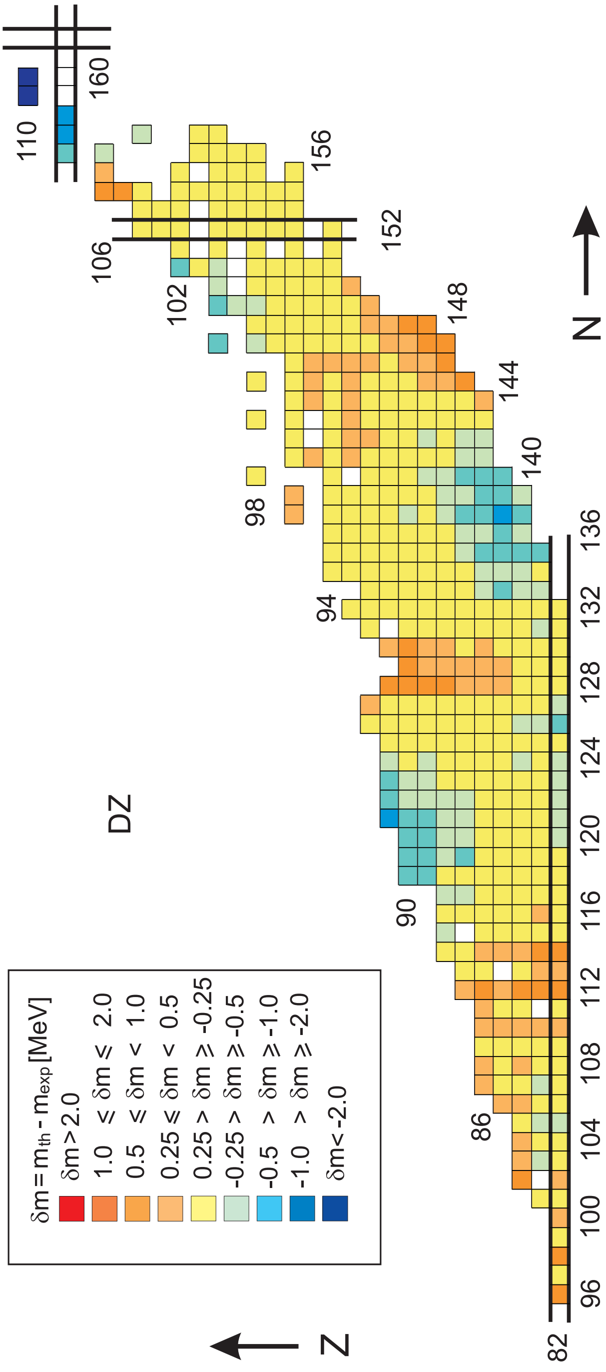

Fig. 3. The same as in Fig. 1, but for the DZ model.

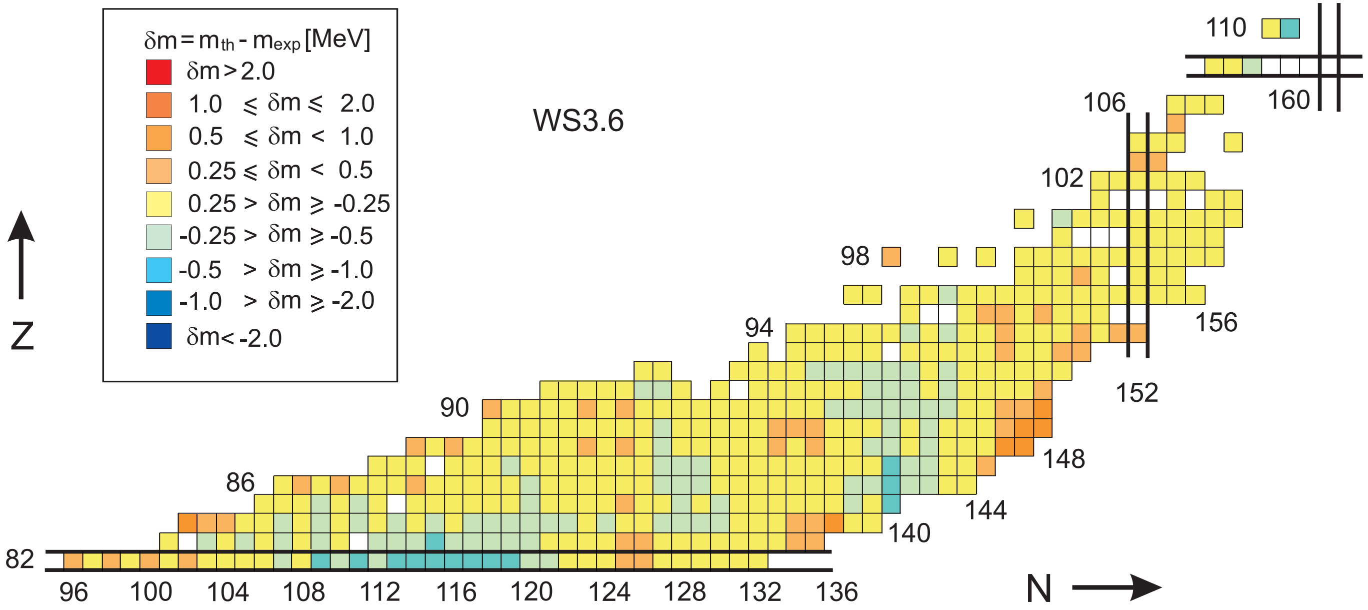

Fig. 4. The same as in Fig. 1, but for the WS3 (WS3.6) model.

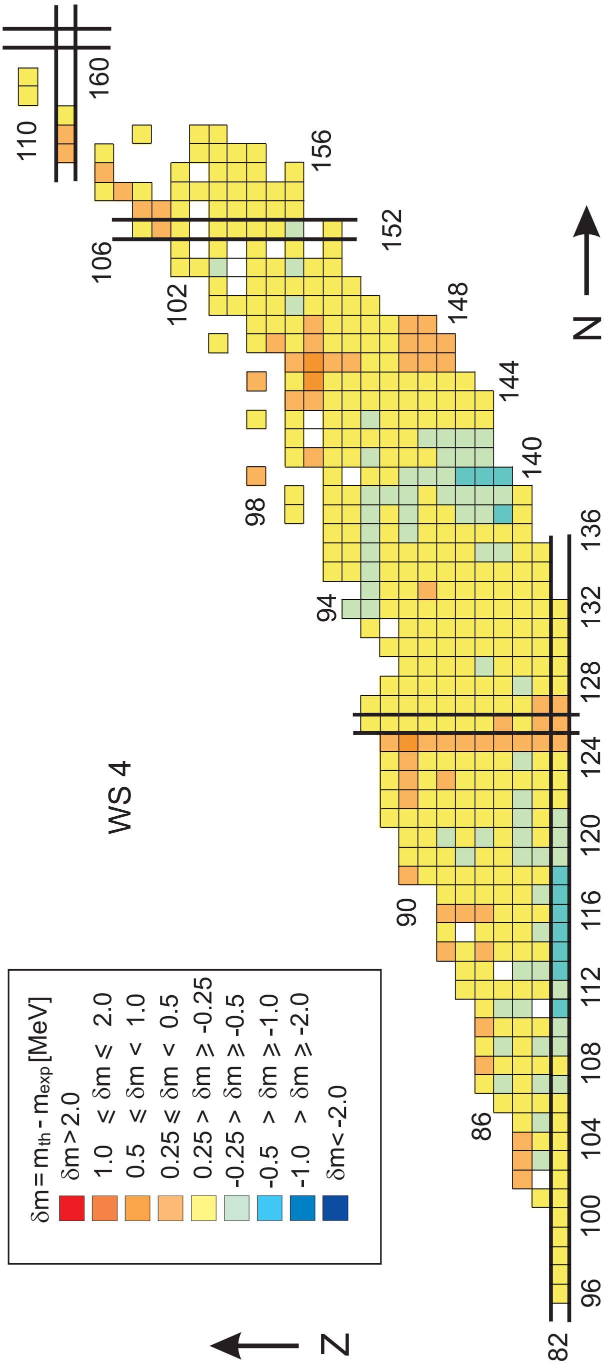

Fig. 5. The same as in Fig. 1, but for the WS4 model.

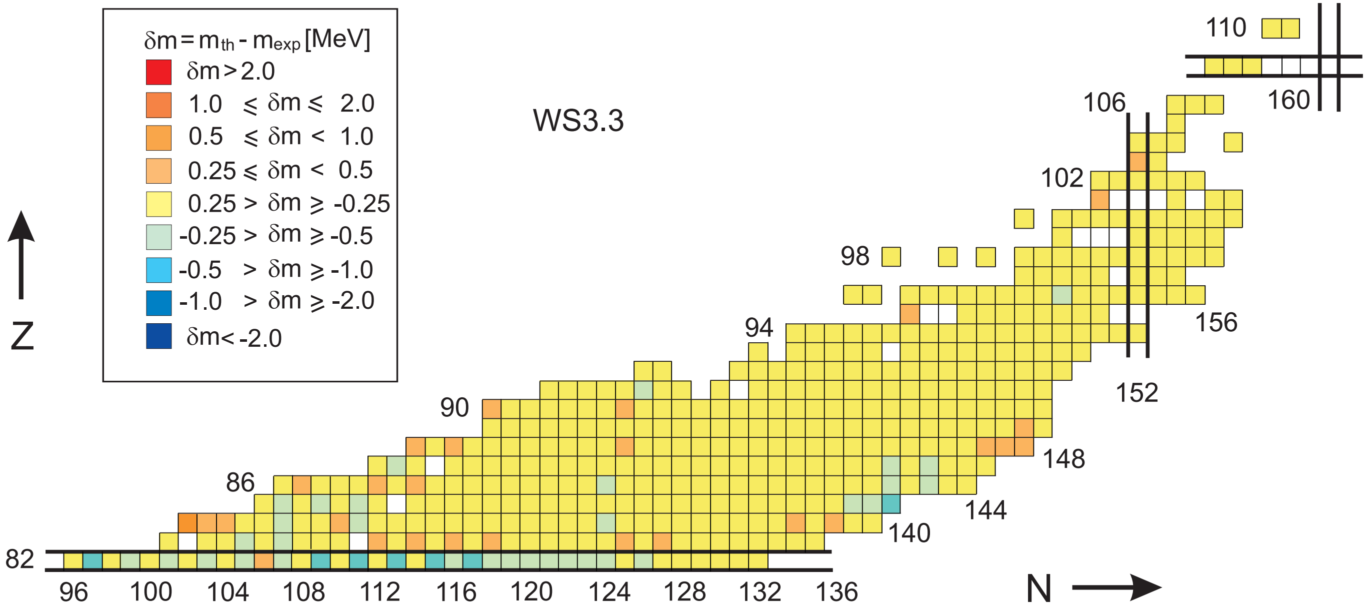

Fig. 6. The same as in Fig. 1, but for the WS3+RBF (WS3.3) model.

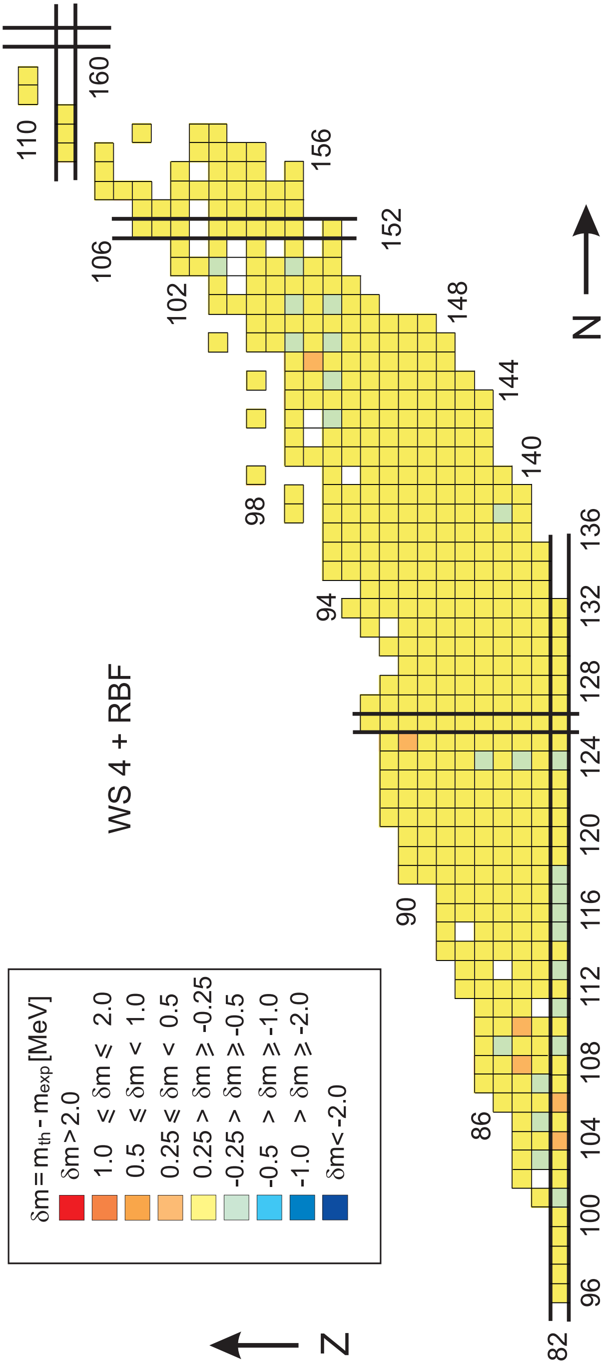

Fig. 7. The same as in Fig. 1, but for the WS4+RBF model.

4. Summary

Seven nuclear-mass models of a different nature are selected for a detailed study and illustration of their accuracy in the description of measured masses, evaluated recently. The region of heavy nuclei has been chosen for the study.

The following conclusions may be drawn from this study:

1. The discrepancy between the calculated and experimental masses, δm, changes quite much with changes of Z and N along the considered region.
2. The changes, $\Delta\delta m$, are from 0.76 MeV for the WS4+RBF model up to 4.06 MeV for the DZ model, which cannot be learned from the average values (rms) usually given in literature.
3. A comparison between the results for WS3 and WS4 models shows that the inclusion of the surface-diffusion correction improves importantly the WS3 model.
4. A comparison between the results for the WS3 and WS3+RBF, and also between WS4 and WS4+RBF, shows that the inclusion of the radial basis function (RBF) corrections (see Ref. [17]) increases importantly the accuracy of the WS model.
5. Among the seven models tested in our study, the best accuracy is obtained for the WS4+RBF model.

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