

ASPECTS OF ANOMALOUS GLUE*

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Non-perturbative glue associated with gluon topology plays a vital role in determining the mass of the η' meson. We give an introduction to axial U(1) physics and explain how this non-perturbative glue also contributes to η' interactions with other hadrons. We concentrate on resonant $\eta'\pi$ production in partial waves with exotic quantum numbers not compatible with a quark–antiquark state and on the η' in nuclear media where there are new results from the COMPASS and CBELSA/TAPS experiments, respectively.

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1. Introduction

The physics of the η' is associated with OZI violation. While pions and kaons are would-be Goldstone bosons associated with chiral symmetry, the isosinglet η and η' mesons are too massive by about 300–400 MeV for them to be pure Goldstone states. They receive extra mass from non-perturbative gluon dynamics associated with the QCD axial anomaly; for recent reviews, see [1, 2]. This non-perturbative glue is also expected to influence the η' -nucleon interaction [3], the behaviour of η and η' mesons in nuclear media [4] as well as η' production and decay processes [5–9]. In this paper, we focus on the η' and non-perturbative gluon dynamics in η' phenomenology with emphasis on the η' in the nuclear medium and resonant exclusive $\eta'\pi^-$ production where there are fresh data from experiments at ELSA in Bonn and COMPASS at CERN respectively. New experiments on the η' in nuclei are planned or underway at ELSA and GSI/FAIR.

Without the gluonic mass contribution the η' and η mesons would be strange and light-quark systems, like the flavour structure of the isoscalar ϕ and ω vector mesons. To the extent that interactions of the η' with

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nucleons and nuclei are induced by light-quark components in the η' , then any observed η' -nucleon scattering length or η' mass shift in nuclei is induced by the same non-perturbative glue that generates the large η' mass. Interesting new data from the CBELSA/TAPS Collaboration suggest an η' mass shift of about -37 MeV at nuclear matter density [10], very close to the prediction of the Quark Meson Coupling model [4]. The first measurement of the η' -nucleon scattering length in free space has recently been obtained by the COSY-11 experiment [11]. The η' has also been suggested to play a prominent feature in constituent quark model calculations [12] for stabilising the baryon spectra.

In $\eta'\pi$ rescattering processes, one finds a strong OZI violating contribution to the odd L partial wave contribution with exotic quantum numbers L^{-+} [9], a resonant contribution that cannot be described by a simple quark-antiquark state. A factor of 5–10 enhancement is observed for $\eta'\pi^-$ production over $\eta\pi^-$ production in the $L = 1, 3, 5$ partial waves in exclusive production at COMPASS [13]. Large branching ratios for B and D_s meson decays to η' final states observed at B -factories are believed to be mediated by strong coupling to gluonic intermediate states [6, 7].

Further, the physics of the flavour-singlet QCD axial anomaly is important in understanding the spin structure of the proton [14–18]. The nucleon's flavour-singlet axial-charge which measures the quark spin content of the proton also picks up a polarised gluon contribution $-3\frac{\alpha_s}{2\pi}\Delta g$ [16] as well as a possible topological contribution with support only at Bjorken x equal to zero associated with a possible subtraction constant in the dispersion relation for the nucleon's g_1 spin dependent structure function measured in polarised deep inelastic scattering [14, 18]. The quest to measure this polarised glue has inspired vast experimental activity at CERN, DESY and RHIC. The present status is that the gluon polarisation at the scale of the experiments, about $3\text{--}10\text{ GeV}^2$, is finite [19] and probably less than about 0.5, consistent with the expectation from gluon radiation from valence quarks in the nucleon [20].

The plan of this paper is as follows. In Section 2, we give a brief introduction to the η' mass problem. Section 3 describes the QCD axial anomaly and ideas about non-perturbative anomalous glue (glue connected to the anomaly) that might explain the η' mass. Then, in Section 4 we explain how this anomalous glue enters in low-energy QCD processes involving the η' in meson production and in the nuclear medium. Finally, we summarise and conclude in Section 5 with an outlook to future developments.

2. The η' mass problem

Low energy QCD is characterised by confinement and dynamical chiral symmetry breaking. The absence of parity doublets in the hadron spectrum tells us that the near-chiral symmetry for light u and d quarks is spontaneously broken. Scalar confinement implies dynamical chiral symmetry breaking. For example, in the bag model, the bag wall connects left- and right-handed quarks leading to quark–pion coupling and the pion cloud of the nucleon [21]. Spontaneous chiral symmetry breaking in QCD is associated with a non-vanishing chiral condensate

$$\langle \text{vac} | \bar{\psi}\psi | \text{vac} \rangle < 0. \quad (1)$$

This spontaneous symmetry breaking induces an octet of Goldstone bosons associated with SU(3) and also (before extra gluonic effects in the singlet channel) a flavour-singlet Goldstone boson. The Goldstone bosons P couple to the axial-vector currents which play the role of Noether currents through

$$\langle \text{vac} | J_{\mu 5}^i | P(p) \rangle = -i f_P^i p_\mu e^{-ip \cdot x} \quad (2)$$

with f_P^i the corresponding decay constants and satisfy the Gell-Mann–Oakes–Renner relation

$$m_\pi^2 f_\pi^2 = -m_q \langle \bar{\psi}\psi \rangle. \quad (3)$$

The pion and kaon fit well in this picture.

The isosinglet η and η' masses are about 300–400 MeV too heavy to be pure Goldstone states. One needs extra mass in the flavour-singlet channel associated with non-perturbative topological gluon configurations [1, 22], related, perhaps, to confinement [23–27] or instantons [28]. SU(3) breaking generates mixing between the octet and singlet states yielding the massive η and η' bosons.

To see the effect of the gluonic mass contribution, consider the η – η' mass matrix for free mesons (at leading order in the chiral expansion)

$$M^2 = \begin{pmatrix} \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2 & -\frac{2}{3}\sqrt{2}(m_K^2 - m_\pi^2) \\ -\frac{2}{3}\sqrt{2}(m_K^2 - m_\pi^2) & [\frac{2}{3}m_K^2 + \frac{1}{3}m_\pi^2 + \tilde{m}_{\eta_0}^2] \end{pmatrix}. \quad (4)$$

Here, $\tilde{m}_{\eta_0}^2$ is the flavour-singlet gluonic mass term.

The masses of the physical η and η' mesons are found by diagonalising this matrix, *viz.*

$$\begin{aligned} |\eta\rangle &= \cos\theta |\eta_8\rangle - \sin\theta |\eta_0\rangle, \\ |\eta'\rangle &= \sin\theta |\eta_8\rangle + \cos\theta |\eta_0\rangle, \end{aligned} \quad (5)$$

where

$$\eta_0 = \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s}) , \quad \eta_8 = \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s}) . \quad (6)$$

One obtains values for the η and η' masses

$$m_{\eta',\eta}^2 = (m_K^2 + \tilde{m}_{\eta_0}^2/2) \pm \frac{1}{2} \sqrt{(2m_K^2 - 2m_\pi^2 - \frac{1}{3}\tilde{m}_{\eta_0}^2)^2 + \frac{8}{9}\tilde{m}_{\eta_0}^4} . \quad (7)$$

The gluonic mass term is obtained from summing over the two eigenvalues in Eq. (7) to give the Witten–Veneziano mass formula [29, 30]

$$m_\eta^2 + m_{\eta'}^2 = 2m_K^2 + \tilde{m}_{\eta_0}^2 . \quad (8)$$

Substituting the physical values of m_η , $m_{\eta'}$ and m_K gives $\tilde{m}_{\eta_0}^2 = 0.73 \text{ GeV}^2$. The gluonic mass term has a rigorous interpretation in terms of the Yang–Mills topological susceptibility, see Eqs. (10)–(12) below.

In the OZI limit of no gluonic mass term, the η would be approximately an isosinglet light-quark state ($\frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d\rangle$) with mass $m_\eta \sim m_\pi$ degenerate with the pion and the η' would be a strange-quark state $|\bar{s}s\rangle$ with mass $m_{\eta'} \sim \sqrt{2m_K^2 - m_\pi^2}$ — mirroring the isoscalar vector ω and ϕ mesons.

Phenomenological studies of various decay processes give a value for the η – η' mixing angle between -15° and -20° [6, 31–33]. This mixing means that non-perturbative glue through axial U(1) dynamics plays an important role in both the η and η' and their interactions. Treating the η as an octet pure would-be Goldstone boson risks losing essential physics. Recent lattice calculations give values for the mixing angle between about -10° and -20° [34].

3. Non-perturbative glue and $\tilde{m}_{\eta_0}^2$

The gluonic mass term is related to the QCD axial anomaly in the divergence of the flavour-singlet axial-vector current [35]. While the non-singlet axial-vector currents are partially conserved (they have just mass terms in the divergence), the singlet current $J_{\mu 5} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s$ satisfies the anomalous divergence equation

$$\partial^\mu J_{\mu 5} = 6Q + \sum_{k=1}^3 2im_k \bar{q}_k \gamma_5 q_k , \quad (9)$$

where $Q = \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$ is the topological charge density, $G_{\mu\nu}$ is the gluon field tensor, and $\tilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}$. The axial anomaly derives from a clash

of symmetries. In classical field theory, the axial-vector currents are always partially conserved. In quantum field theory, the flavour-singlet axial-vector current can couple through gluon intermediate states. Here the axial-vector current, two vector quark–gluon-vertex triangle diagram is essential. When we regularise the ultraviolet behaviour of momenta in the loop, we find that we can preserve current conservation at the quark–gluon-vertices (necessary for gauge invariance and renormalisability) or partial conservation of the axial-vector current but not both simultaneously. Current conservation wins and induces the gluonic anomaly term in the singlet divergence equation (9) from the point-like ultraviolet part of the triangle loop.

Besides adding the extra term to the RHS of the divergence equation (9), the anomaly opens a window to non-perturbative gluon dynamics. The integral over space $\int d^4z Q = n$ measures the gluonic winding number [22] as a measure of non-local topological structure which is an integer for (anti-)instantons and which vanishes in perturbative QCD. The QCD topological charge plays a vital role in the η' .

The Witten–Veneziano mass formula [29, 30] for the η' mass follows from the flavour-singlet Ward identities and relates the gluonic mass term for the singlet boson to the topological susceptibility of pure Yang–Mills (glue with no quarks), *viz.*

$$\tilde{m}_{\eta_0}^2 = -\frac{6}{f_\pi^2} \chi(0)|_{\text{YM}}, \quad (10)$$

where

$$\chi(k^2)|_{\text{YM}} = \int d^4z i e^{ikz} \langle \text{vac} | T Q(z) Q(0) | \text{vac} \rangle |_{\text{YM}}. \quad (11)$$

Understanding the QCD dynamical origin of this topological susceptibility (which gluon configurations saturate it) and how it contributes to hadron phenomenology are at heart of axial U(1) physics. In the QCD limit of large number of colours, N_c , if we assume that the topological winding number remains finite independent of the value of N_c then

$$\tilde{m}_{\eta_0}^2 \sim 1/N_c. \quad (12)$$

The topological susceptibility $\chi(0)$ in full QCD (with quarks) vanishes in the limit of massless quarks where the pure Yang–Mills contribution is cancelled against a pole term involving the massive η' . QCD lattice calculations give values $\chi^{1/4}(0)|_{\text{YM}} = 191 \pm 5$ MeV [36] and $\chi^{1/4}(0)|_{\text{YM}} = 193(1)(8)$ MeV [37], very close to the value 180 MeV which follows from taking $\tilde{m}_{\eta_0}^2 = 0.73$ GeV² in the Witten–Veneziano formula Eq. (8).

The topological charge density Q is a total divergence. The anomaly equation can also be written

$$\partial^\mu J_{\mu 5} = 6\partial^\mu K_\mu + \sum_{k=1}^3 2im_k \bar{q}_k \gamma_5 q_k, \quad (13)$$

where

$$K_\mu = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu \left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma \right) \right] \quad (14)$$

is the gluonic Chern–Simons current and $\alpha_s = g^2/4\pi$ is the QCD coupling.

Equation (13) allows us to define a partially conserved current

$$J_{\mu 5} = J_{\mu 5}^{\text{con}} + 2fK_\mu \quad (15)$$

with

$$\partial^\mu J_{\mu 5}^{\text{con}} = \sum_{i=1}^3 2im_i \bar{q}_i \gamma_5 q_i. \quad (16)$$

When we make a gauge transformation U , the gluon field transforms as

$$A_\mu \rightarrow U A_\mu U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1} \quad (17)$$

and the operator K_μ transforms as

$$\begin{aligned} K_\mu \rightarrow & K_\mu + i \frac{g}{8\pi^2} \epsilon_{\mu\nu\alpha\beta} \partial^\nu \left(U^\dagger \partial^\alpha U A^\beta \right) \\ & + \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} \left[\left(U^\dagger \partial^\nu U \right) \left(U^\dagger \partial^\alpha U \right) \left(U^\dagger \partial^\beta U \right) \right], \end{aligned} \quad (18)$$

where the third term on the RHS is associated with gluon gauge field topology [22]. Before possible confinement considerations [25], finiteness of the QCD action requires that $x^2 G_{\mu\nu} \rightarrow 0$ as $x_\mu \rightarrow \infty$ in almost every direction so the field A_μ should tend to a pure gauge configuration $gA_\mu \rightarrow iG^{-1} \partial_\mu G$. With this constraint

$$\nu = \int d^4x \, Q = \int d^4x \, \partial^\mu K_\mu = \int \sigma^\mu K_\mu = \text{integer} \quad (19)$$

for instanton topological charge [22]. A finite topological charge comes here from the requirement that the gluon field is pure gauge at infinity and that the gauge group is topologically non-trivial. The topological winding number is determined by the gluonic boundary conditions at “infinity” (a large

surface with boundary which is spacelike with respect to the positions z_k of any operators or fields in the physical problem). It is insensitive to local deformations of the gluon field $A_\mu(z)$ or of the gauge transformation $U(z)$.

The current $J_{\mu 5}$ in QCD is multiplicatively renormalised and picks up a two-loop anomalous dimension from the axial anomaly [22, 38]. (Partially) conserved currents are not renormalised. It follows that $J_{\mu 5}^{\text{con}}$ is renormalisation scale invariant and the scale dependence of $J_{\mu 5}$ is carried entirely by K_μ . Gauge transformations shuffle a scale invariant operator quantity between the two operators $J_{\mu 5}^{\text{con}}$ and K_μ whilst keeping $J_{\mu 5}$ invariant.

In general, matrix elements of K_μ are gauge dependent. This means that one has to be careful writing matrix elements of $J_{\mu 5}$ as the sum of (measurable) “quark” and “gluonic” contributions. For the η' , if we consider the singlet boson version of Eq. (2) and write $J_{\mu 5}$ as the sum of $J_{\mu 5}^{\text{con}}$ and K_μ contributions, then the two matrix elements corresponding to the gauge dependent currents are, in general, separately each gauge dependent. Isolating a gluonic leading Fock component from the η' involves subtle issues of gauge invariance and only makes sense with respect to a particular renormalisation scheme like the gauge invariant scheme $\overline{\text{MS}}$ [39].

Since the topological charge density is a total divergence, it follows that the gluonic Chern–Simons current (14) should couple to a massless pole. This is the Kogut–Susskind pole [24]. The fact that there is no massless flavour-singlet Goldstone boson in the physical spectrum implies that this pole cancels against a second massless pole term which couples to the partially conserved current $J_{\mu 5}^{\text{con}}$ with equal residue and opposite sign. This second pole term stays massless when quark masses are turned on to cancel the pole term coupled to K_μ .

Corresponding to the two currents $J_{\mu 5}$ and $J_{\mu 5}^{\text{con}}$, we can define the two operator charges $X(t) = \int d^3z J_{05}(z)$ and $Q_5 = \int d^3z J_{05}^{\text{con}}(z)$. The charge $X(t)$ is manifestly gauge invariant whereas Q_5 is invariant only under “small” gauge transformations which are topologically deformable to the identity. Choosing the gauge $A_0 = 0$, the charge Q_5 transforms as $Q_5 \rightarrow Q_5 - 2fn$, where n is the winding number associated with the gauge transformation U . Although Q_5 is gauge dependent, we can define a gauge invariant chirality q_5 for a given operator \mathcal{O} through the gauge-invariant eigenvalues of the equal-time commutator $[Q_5, \mathcal{O}]_- = -q_5 \mathcal{O}$. The gauge invariance of q_5 follows because this commutator appears in gauge invariant Ward Identities despite the gauge dependence of Q_5 [22]. One finds that the time derivative of spatial components of the gluon field have zero chirality q_5 (following from the non-renormalisation of the partially conserved current $J_{\mu 5}^{\text{con}}$) but non-zero X charge, which is induced by the anomaly. The analogous situation in QED is discussed in Refs. [35, 40, 41]. If one requires that chirality is

renormalisation group invariant and that the time derivative of the spatial components of the gluon field have zero chirality, then one is led to using q_5 associated with $J_{\mu 5}^{\text{con}}$ to define chirality.

When topological effects are taken into account, the QCD vacuum $|\theta\rangle$ is understood to be a coherent Bloch superposition of states characterised by different topological winding number m [42]

$$|\text{vac}, \theta\rangle = \sum_m e^{im\theta} |m\rangle, \quad (20)$$

where the QCD θ angle is experimentally less than 10^{-10} [43]. For integer values of the topological winding number m , the states $|m\rangle$ contain mf quark–antiquark pairs with non-zero Q_5 chirality $\sum_l \chi_l = -2fm$, where f is the number of light-quark flavours. Relative to the $|m = 0\rangle$ state, the $|m = +1\rangle$ state carries topological winding number $+1$ and f quark–antiquark pairs with Q_5 chirality equal to $-2f$. Each state $|m\rangle$ carries zero net axial-charge as measured by $J_{\mu 5}$ and $X(t)$.

Invariance under “large” gauge transformations which change the topological winding number is like saying the physics is invariant under the choice of which state $|m\rangle$ we choose as “zero” when we set up a “ruler” to label the θ vacuum states $|m\rangle$. A “large” gauge transformation which changes the topological charge m by k units just changes the phase of the θ -vacuum by amount $e^{-ik\theta}$.

Instanton tunnelling processes between different $|m\rangle$ states differing by topological winding number one, $|m\rangle \rightarrow |m \pm 1\rangle$, connect left- and right-handed fermions [28]. A flavour-singlet combination of quarks incident on an instanton “vertex” annihilate (fill up vacant levels) with the delocalised quark–antiquark pairs in the neighbouring state, thus liberating a flavour-singlet combination of quarks with opposite chirality into the final state. Energy-momentum is conserved between incoming and outgoing quarks with net axial charge conserved by q_5 chirality being absorbed into the “vacuum”, that is shifted into a zero-mode. This process can induce a topological $x = 0$ contribution to the proton’s flavour-singlet axial-charge or “quark spin content” [18] depending on whether instantons spontaneously [22] or explicitly [44] break axial $U(1)$ symmetry. The valence quarks can thus act as a source for polarising the θ vacuum inside a proton relative to the vacuum outside by shifting some fraction of the spin of moving partons into a topological zero-mode contribution [14, 18]. The proton matrix element of the Abelian part (first term) of the gluonic Chern–Simons current in light-cone co-ordinate, K_+ , in $A_+ = 0$ gauge measures the gluon polarisation in the nucleon [16].

Instantons are an important source of finite topological charge and come with large N_c behaviour $\sim e^{-N_c}$, decaying faster with increasing N_c than $\tilde{m}_{\eta_0}^2 \sim 1/N_c$ in Eq. (12). Witten argued that with quark confinement the vacuum no longer requires gluon fields to be pure gauge at infinity. One might also get non-vanishing topological charge from non-instanton effects. Additional gluonic configurations with the topological charge, suggested in the literature, involve calorons [45] and centre-vortices [46, 47]. Large N_c and more quark-model-like approaches to understanding the η' wavefunction involving non-perturbative gluon intermediate states [23] might here be reconciled if *e.g.* confinement processes associated with topology induce a dynamical scalar component in the non-perturbative quark gluon vertex connecting left- and right-handed quarks in the flavour-singlet channel in addition to effects associated with pion and kaon production in the non-singlet channels [27].

This formalism generalises readily to the definition of baryon number in the presence of electroweak gauge fields [48]. The vector current which measures baryon number is sensitive to the axial anomaly through the parity violating electroweak interactions. This vector current can be written as the sum of left- and right-handed currents

$$J_\mu = \bar{\Psi}\gamma_\mu\Psi = \bar{\Psi}\gamma_\mu\frac{1}{2}(1 - \gamma_5)\Psi + \bar{\Psi}\gamma_\mu\frac{1}{2}(1 + \gamma_5)\Psi. \quad (21)$$

In the Standard Model, the left-handed fermions couple to the SU(2) electroweak gauge fields W^\pm and Z^0 . This means that this baryon current is sensitive to the axial anomaly [28]. One finds the anomalous divergence equation

$$\partial^\mu J_\mu = n_f (-\partial^\mu K_\mu + \partial^\mu k_\mu), \quad (22)$$

where K_μ and k_μ are the SU(2) electroweak and U(1) hypercharge anomalous Chern–Simons currents. If one requires that baryon number is renormalisation group invariant and that the time derivative of the spatial components of the W boson field have zero baryon number, then one is led to using the conserved vector current analogy of q_5 to define the baryon number [48], *e.g.* in sphaleron induced electroweak baryogenesis in the early Universe [49].

4. The η' in low-energy QCD

Independent of the detailed QCD dynamics, one can construct low-energy effective chiral Lagrangians which include the effect of the anomaly and axial U(1) symmetry, and use these Lagrangians to study low-energy processes involving the η and η' .

The physics of axial U(1) degrees of freedom is described by the U(1)-extended low-energy effective Lagrangian [50]. In its simplest form, this reads

$$\begin{aligned}\mathcal{L} = & \frac{F_\pi^2}{4} \text{Tr} \left(\partial^\mu U \partial_\mu U^\dagger \right) + \frac{F_\pi^2}{4} \text{Tr} M \left(U + U^\dagger \right) \\ & + \frac{1}{2} i Q \text{Tr} \left[\log U - \log U^\dagger \right] + \frac{3}{\tilde{m}_{\eta_0}^2 F_0^2} Q^2.\end{aligned}\quad (23)$$

Here, $U = \exp i(\phi/F_\pi + \sqrt{\frac{2}{3}}\eta_0/F_0)$ is the unitary meson matrix, where $\phi = \sum \pi_a \lambda_a$ denotes the octet of would-be Goldstone bosons associated with spontaneous chiral $\text{SU}(3)_L \otimes \text{SU}(3)_R$ breaking and η_0 is the singlet boson. In Eq. (23), Q denotes the topological charge density; $M = \text{diag}[m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2]$ is the quark-mass induced meson mass matrix. The pion decay constant $F_\pi = 92.4$ MeV and F_0 is the flavour-singlet decay constant, $F_0 \sim F_\pi \sim 100$ MeV [31].

The flavour-singlet potential involving Q is introduced to generate the gluonic contribution to the η and η' masses and to reproduce the anomaly in the divergence of the gauge-invariantly renormalised flavour-singlet axial-vector current. The gluonic term Q is treated as a background field with no kinetic term. It may be eliminated through its equation of motion to generate a gluonic mass term for the singlet boson, *viz.*

$$\frac{1}{2} i Q \text{Tr} \left[\log U - \log U^\dagger \right] + \frac{3}{\tilde{m}_{\eta_0}^2 F_0^2} Q^2 \mapsto -\frac{1}{2} \tilde{m}_{\eta_0}^2 \eta_0^2. \quad (24)$$

The most general low-energy effective Lagrangian involves a $U_A(1)$ invariant polynomial in Q^2 . Higher-order terms in Q^2 become important when we consider scattering processes involving more than one η' [5]. In general, couplings involving Q give OZI violation in physical observables. The interactions of the η and η' with other mesons and with nucleons can be studied by coupling the Lagrangian Eq. (23) to other particles.

4.1. Light-mass exotic meson production

The OZI violating interaction

$$\mathcal{L}_{m2Q} = \lambda Q^2 \partial_\mu \pi_a \partial^\mu \pi_a \quad (25)$$

is needed to generate the leading (tree-level) contribution to the decay $\eta' \rightarrow \eta \pi \pi$ [5]. When iterated in the Bethe–Salpeter equation for meson–meson rescattering, this interaction yields a dynamically generated exotic state with quantum numbers $J^{\text{PC}} = 1^{-+}$ and mass about 1400 MeV in $\eta' \pi$ rescattering mediated by the OZI violating coupling of the η' [9].

This suggests a dynamical interpretation of the light-mass 1^{-+} exotics observed in experiments at BNL [51] and CERN [52]. These mesons are particularly interesting because the quantum numbers $J^{PC} = 1^{-+}$ are inconsistent with a simple quark–antiquark bound state.

Further, this OZI violating interaction will also play an important role in higher L -odd partial waves. Different partial wave contributions are projected out via integrating

$$T_L = \frac{1}{2} \int_{-1}^{+1} d(\cos \theta) P_L(\cos \theta) T \quad (26)$$

weighted by Legendre polynomials P_L with T the T matrix. The Legendre polynomials are either odd or even in $\cos \theta$ about zero. So if we get an effect in $L = 1$ (p -wave), it must come from something odd in $\cos \theta$ in T . This odd contribution will also integrate to finite values with $L = 3, 5, \dots$ (not $L = 2, 4, \dots$) with the key determining feature being whether the driving term is odd or even about zero in $\cos \theta$.

The COMPASS experiment at CERN has just measured exclusive production of $\eta'\pi^-$ and $\eta\pi^-$ in 191 GeV π^- collisions from a hydrogen target [13]. They find the interesting result that $\eta'\pi^-$ production is enhanced relative to $\eta\pi^-$ production by a factor of 5–10 in the exotic $L = 1, 3, 5$ partial waves with quantum numbers L^{-+} in the inspected invariant mass range up to 3 GeV. No enhancement was observed in the even L partial waves.

4.2. The η' in nuclear matter

Measurements of the η - and η' - (as well as pion and kaon) nucleon and nucleus systems promise to yield valuable new information about dynamical chiral and axial U(1) symmetry breaking in low energy QCD. The quark condensate is modified in the nuclear environment which leads to changes in the properties of hadrons in medium including the masses of the Goldstone bosons [53]. How does the gluonic mass contribution to the η and η' change in nuclei?

With increasing density, chiral symmetry is partially restored corresponding to a reduction in the value of the quark condensate and pion decay constant f_π . Experiments with pionic atoms give a value $f_\pi^{*2}/f_\pi^2 = 0.64 \pm 0.06$ at nuclear matter density ρ_0 [53, 54]. This implies changes in the meson masses in the medium and also the coupling of the Goldstone bosons to the constituent quarks and the nucleon. For reviews of medium modifications at finite density and the QCD phase diagram, see [55–57]. These medium modifications need to be understood self-consistently within the interplay of confinement, spontaneous chiral symmetry breaking and axial

U(1) dynamics. In the limit of chiral restoration, the pion should decouple from the physics, the pion decay constant f_π goes to zero and (perhaps) with scalar confinement the pion constituent-quark and pion nucleon coupling constants should vanish with dissolution of the pion wavefunction.

For pions, one finds a small mass shift of the order of a few MeV in nuclear matter [53] whereas kaons are observed to experience an effective mass drop for the K^- to about 270 MeV at two times nuclear matter density in heavy ion collisions [58].

The η - and η' -nucleon interactions are believed to be attractive suggesting that these mesons may form strong-interaction bound-states in nuclei. For the η , one finds a sharp rise at threshold in the cross section for η production from helium in photoproduction [59] and proton-deuteron collisions [60] which may hint at a reduced η mass in the nuclear medium. The η' is very interesting. As we have seen in Section 2, without glue this would be a strange quark state. To the extent that coupling to nucleons and nuclear matter (*e.g.* via the σ correlated two-pion mean field in the nucleus) is induced by light-quark components in the meson, any observed η' -nucleon scattering length and η' mass shift in medium is induced by the QCD axial anomaly that generates part of the η' mass [2].

Meson mass shifts can be investigated through bound state searches in nuclei. Meson mass shifts can also be investigated through studies of excitation functions in photoproduction experiments from nuclear targets. The production cross section is enhanced with the lower effective mass in the nuclear medium. When the meson leaves the nucleus, it returns on-shell to its free mass with the energy budget conserved at the expense of the kinetic energy so that excitation functions and momentum distributions can provide essential clues to the meson properties in medium [61]. Using this physics, a first (indirect) estimate of the η' mass shift has recently been deduced by the CBELSA/TAPS Collaboration [10]. The η' -nucleus optical potential $V_{\text{opt}} = V_{\text{real}} + iW$ deduced from these photoproduction experiments is

$$\begin{aligned} V_{\text{real}}(\rho_0) &= m^* - m = -37 \pm 10(\text{stat.}) \pm 10(\text{syst.}) \text{ MeV}, \\ W(\rho_0) &= -10 \pm 2.5 \text{ MeV} \end{aligned} \tag{27}$$

at nuclear matter density. These numbers with small imaginary part [62] suggest that possible η' bound states in nuclei may be within reach of forthcoming experiments. For clean observation of a bound state, one needs the real part of the optical potential to be much bigger than the imaginary part. The mass shift, Eq. (27), is also very similar to the expectations of the Quark Meson Coupling model, see below.

There is presently a vigorous experimental programme to search for evidence of η and η' bound states with ongoing experiments at COSY to look for possible η bound states in helium [63] and new experiments in photo-production at ELSA [64], and using the (p, d) reaction at GSI/FAIR [65] to look for possible η' bound states in carbon. Eta bound states in helium require a large η -nucleon scattering length with real part greater than about 0.9 fm [66].

Medium modifications can be understood at the quark level through coupling of the scalar isoscalar σ (and also ω and ρ) mean fields in the nucleus to the light quarks in the hadron [57]. The binding energies and in-medium masses of the η and η' are sensitive to the flavour-singlet component in the mesons and hence to the non-perturbative glue associated with axial U(1) dynamics [4].

Within the effective Lagrangian approach of Eq. (23), the medium dependence of $\tilde{m}_{\eta_0}^2$ is introduced through coupling to the σ mean-field in the nucleus through the interaction term

$$\mathcal{L}_{\sigma Q} = g_{\sigma Q} Q^2 \sigma, \quad (28)$$

where $g_{\sigma Q}$ denotes coupling to the σ mean field. One finds the gluonic mass term decreases in-medium $\tilde{m}_{\eta_0}^{*2} < \tilde{m}_{\eta_0}^2$ independent of the sign of $g_{\sigma Q}$ and the medium acts to partially neutralise axial U(1) symmetry breaking by gluonic effects [4]. To estimate the size of the effect, we look to phenomenology and QCD motivated models.

Interesting results for the η and η' mass shifts with η - η' mixing are obtained within the Quark Meson Coupling model (QMC) of hadron properties in the nuclear medium [4]. Here, the large η and η' masses are used to motivate taking an MIT bag description for the meson wavefunctions. Gluonic topological effects are understood to be “frozen in”, meaning that they are only present implicitly through the masses and mixing angle in the model. The in-medium mass modification comes from coupling the light (up and down) quarks and antiquarks in the meson wavefunction to the scalar σ mean-field in the nucleus working in mean-field approximation [57, 67, 68]. The coupling constants in the model for the coupling of light-quarks to the σ (and ω and ρ) mean-fields in the nucleus are adjusted to fit the saturation energy and density of symmetric nuclear matter and the bulk symmetry energy. The strange-quark component of the wavefunction does not couple to the σ field and η - η' mixing is readily built into the model. Gluon fluctuation and centre-of-mass effects are assumed to be independent of density. The model results for the meson masses in medium and the real part of the meson-nucleon scattering lengths are shown in Table I for different values of the η - η' mixing angle, which is taken to be density independent in these

calculations [4]. The effective scattering length is calculated through

$$2m\Delta m = 4\pi\rho a (1 + m/M), \quad (29)$$

where m is the mass of the meson in free space, Δm is the mass shift in medium, ρ is the density of the nuclear medium, M is the nucleon mass and a is the scattering length [69]. Increasing the flavour-singlet component in the η at the expense of the octet component gives more attraction, more binding and a larger value of the η -nucleon scattering length, $a_{\eta N}$. The QMC model makes no claim about the imaginary part of the scattering length.

TABLE I

Physical masses fitted in free space, the bag masses in medium at normal nuclear-matter density, $\rho_0 = 0.15 \text{ fm}^{-3}$, and corresponding meson–nucleon scattering lengths.

	m [MeV]	m^* [MeV]	$\text{Re } a$ [fm]
η_8	547.75	500.0	0.43
η (-10°)	547.75	474.7	0.64
η (-20°)	547.75	449.3	0.85
η_0	958	878.6	0.99
η' (-10°)	958	899.2	0.74
η' (-20°)	958	921.3	0.47

For -20 degrees η – η' mixing angle, QMC predicts the η' mass shift to be -37 MeV at nuclear matter density ρ_0 , corresponding to the real part of the effective η' -nucleon scattering length being 0.5 fm. This value is very similar to the mass shift $-37 \pm 10 \pm 10$ MeV deduced from photoproduction data by the CBELSA/TAPS Collaboration [10]. Further, COSY-11 have recently determined the η' -nucleon scattering length in free space to be

$$\begin{aligned} \text{Re}(a_{p\eta'}) &= 0 \pm 0.43 \text{ fm}, \\ \text{Im}(a_{p\eta'}) &= 0.37^{+0.40}_{-0.16} \text{ fm} \end{aligned} \quad (30)$$

from studies of the final state interaction in η' production in proton–proton collisions close to threshold [11]. Larger mass shifts, downwards by up to 80–150 MeV, were found in Nambu–Jona-Lasinio model [70] and linear sigma model calculations [71], which, in general, also give a rising η mass at finite density. Each of these theoretical models prefers a positive sign for the real part of $a_{\eta'N}$ in medium. The energy and density dependence of the η' - (and also η -) nucleon scattering lengths is a open topic of investigation [66]. If one assumes no density and energy dependence of the η' -nucleon scattering length, then the value obtained in Eq. (30) is consistent with the QMC

result [4] and disfavours the expectations in [70, 71]. A chiral coupled channels calculation performed with possible scattering lengths with real part between 0 and 1.5 fm is reported in [72].

η - η' mixing with the phenomenological mixing angle -20° leads to a factor of two increase in the mass-shift and in the scattering length obtained in the model relative to the prediction for a pure octet η_8 . This result may explain why values of $a_{\eta N}$ extracted from phenomenological fits to experimental data where the η - η' mixing angle is unconstrained [73] give larger values than those predicted in theoretical coupled channels models where the η is treated as a pure octet state [74, 75]. We refer to Ref. [2] for comparison of different models and their predictions for the η and η' nucleus systems.

5. Summary and conclusions

Besides generating the large η' mass, non-perturbative anomalous glue plays an important role in driving η' interactions and production and decay processes with significant OZI violation. Glue associated with the QCD axial anomaly also plays an important role in the spin structure of the proton. In high-energy processes, B and D_s meson decays to final states involving the η' are enhanced through coupling to gluonic intermediate states. In low-energy reactions, the $\eta' \rightarrow \eta\pi\pi$ decay process, exclusive $\eta'\pi$ production in exotic odd- L partial waves and η' interactions with nuclear media are catalysed by OZI violation associated with the η' . QCD inspired models are used to describe these different processes and the interplay of confinement, chiral symmetry and axial U(1) dynamics. New data on possible η and η' bound states in nuclei is expected soon from running and planned experiments at COSY, ELSA and GSI, and will help to further pin down the dynamics of axial U(1) symmetry breaking in low-energy QCD.

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