THEORETICAL DESCRIPTION OF THE DECAY
CHAIN OF THE NUCLEUS $^{289}_{115}$\textsuperscript{*}

A. Sobieczewski

National Centre for Nuclear Research
Hoża 69, 00-681 Warszawa, Poland
and
GSI Helmholtzzentrum für Schwerionenforschung GmbH
64291 Darmstadt, Germany
adam.sobiczewski@fuw.edu.pl

(Received January 12, 2015)

A theoretical analysis of the $\alpha$-decay chain of the superheavy nucleus $^{289}_{115}$ is performed. The study is done in two variants using two realistic nuclear-mass models (HN and WS3+). The experimental $\alpha$-transition energies $Q^\alpha$ are reproduced by the calculations with the average accuracy of 200 keV and 220 keV in the HN and WS3+ variants, respectively. The experimental half-lives are reproduced, on the average, within the factors 3.5 and 4.6, when the HN and WS3+ mass models are used, respectively. The effect of the odd proton in the nucleus $^{281}$Rg, which decays mainly by spontaneous fission, on its spontaneous-fission half-life $T_{sf}$ is estimated. The effect is the elongation of $T_{sf}$ by a large factor of 140.

DOI:10.5506/APhysPolB.46.551
PACS numbers: 21.60.–n, 23.60.+e

1. Introduction

The superheavy element 115 (i.e. the element with the atomic number $Z = 115$) has an exceptionally large number (few tens) of observed $\alpha$-decay chains, as for such heavy one. Most of the chains were observed at JINR-Dubna [1–7], but some of them were obtained at GSI-Darmstadt [8–10]. Three isotopes of 115 with mass number $A = 287$, 288 and 289 were observed. Some of them were obtained directly in the reaction $^{48}$Ca + $^{243}$Am, and the other as the decay products of the element 117 obtained in the reaction $^{48}$Ca + $^{249}$Bk. A large statistics of the chains results in the accurate data, which supplies us with a good test for theoretical models describing these nuclei.

\textsuperscript{*} Presented at the Zakopane Conference on Nuclear Physics “Extremes of the Nuclear Landscape”, Zakopane, Poland, August 31–September 7, 2014.
A quite long decay chain (5 α decays) of the nucleus $^{287}\text{115}$ was studied theoretically in Ref. [11]. A comparison of the results with the experimental ones of both Dubna and GSI was done. The GSI results are close to the earlier Dubna observations, confirming the latter ones. Concerning the comparison of the theory with experiment, the calculated transition energies $Q^t_\alpha$ reproduced the Dubna data with the average accuracy of 314 keV and 487 keV, corresponding to two variants of the calculations. The respective values in the case of the GSI results were 248 keV and 330 keV. The theoretical half-lives reproduced the experimental Dubna results $T^\text{exp}_\alpha$ with the following average ratio (of the larger of $T^\text{ph}_\alpha$ and $T^\text{exp}_\alpha$ to the smaller one): 6.0 and 10.8 in the two variants of the calculations. In the case of the GSI results, the corresponding numbers are: 2.6 and 5.7. Thus, the reproduction is very good, especially of the GSI measurements.

The objective of the present paper is to describe theoretically the decay chains of the nucleus $^{289}\text{115}$. The chains are shorter (three α decays) than in the case of $^{287}\text{115}$. However, the number of the observed decays are quite large, which results in a larger accuracy of the data and, thus, in a better test for the model used in the description.

Approaches used in the calculations are presented in Sect. 2 and the results in Sect. 3. The summary of the paper is given in Sect. 4.

### 2. Approaches used in the calculations

#### 2.1. Alpha-decay energy

To get possible realistic α-decay energies for considered superheavy nuclei, we try to select models which best describe masses of heaviest nuclei. Table I shows rms (root-mean-square) values of the discrepancies between measured values [12] and those calculated with the use of the indicated models for the region of nuclei with $Z \geq 100$. This region contains 36 nuclei with measured mass. A wider discussion of the accuracy of available nuclear-mass models is given in Ref. [13].

<table>
<thead>
<tr>
<th>Model</th>
<th>DZ</th>
<th>FRDM</th>
<th>LSD</th>
<th>WS3+</th>
<th>HN</th>
</tr>
</thead>
<tbody>
<tr>
<td>rms [keV]</td>
<td>826</td>
<td>676</td>
<td>227</td>
<td>126</td>
<td>118</td>
</tr>
</tbody>
</table>

The rms values of the discrepancies between measured values and those calculated with the use of the indicated models for heaviest nuclei ($Z \geq 100$). Here, DZ is the Duflo–Zuker model [14] known for a good description of masses in the global region of nuclei ($Z \geq 8$, $N \geq 8$, where $Z$ and $N$ are proton and neutron numbers, respectively). The FRDM is the widely
Theoretical Description of the Decay Chain of the Nucleus

used Finite-Range Droplet Model of Möller et al. [15]. The LSD (Lublin–Strasbourg drop) is the model of Pomorski and Dudek [16]. The WS3+ model of Ning Wang and Min Liu [17] (often also denoted as WS3+RBF or WS3.3) is the Weizsäcker–Skyrme model applying the radial basis function (RBF) approach, which is a general mathematical method of extrapolation of known data of some quantity to predict unknown values for it. Finally, the HN (Heavy Nuclei) model [18] (see also Ref. [19]) is the Warsaw local model specially constructed to describe masses of heavy nuclei \((Z \geq 82, N \geq 126)\).

Four of the considered models are of the macroscopic–microscopic nature. Only the DZ model is of a different kind.

2.2. Alpha-decay half-life

The calculations of the \(\alpha\) half-lives are based on the phenomenological model of \(\alpha\) decay elaborated in Ref. [20]. The formula for the logarithm of the \(\alpha\) half-life, \(T_{\alpha}^{\text{ph}}\), obtained within this model, reads

\[
\log_{10} T_{\alpha}^{\text{ph}}(Z, N) = aZ \left[ Q_{\alpha}(Z, N) - \bar{E}_i \right]^{-1/2} + bZ + c, \quad (1)
\]

where \(Q_{\alpha}\) is the \(\alpha\)-decay energy (the ground-state to the ground-state transition) and \(\bar{E}_i\) is the average excitation energy of the daughter nucleus with odd number of protons or neutrons or both. For this reason, the effective \(\alpha\)-transition energy in these nuclei is \(Q_{\alpha}^t \equiv Q_{\alpha} - \bar{E}_i\), where \(\bar{E}_i = 0\) for an even–even nucleus.

The parameters \(a, b, c\) were fitted to experimental values of \(T_{\alpha}\) of even–even nuclei [21] with the use of experimental values of \(Q_{\alpha}\) of these nuclei [22, 23]. The result is

\[
a = 1.5372, \quad b = -0.1607, \quad c = -36.573. \quad (2)
\]

With these values fixed, the \(\bar{E}_i\) parameters were fitted to the data of o–e (odd-\(Z\), even-\(N\)), e–o, and o–o nuclei. For o–e nuclei, in which we are interested here, the result is: \(\bar{E}_i \equiv \bar{E}_p = 0.113\) MeV.

The model used is based on the natural assumption that the \(\alpha\) transition happens between the states of the same structure (i.e. of the same quantum numbers). Such transition does not usually happen between the ground states of the mother and the daughter nuclei of the odd-\(A\) and odd–odd kinds, as they generally have different structure. Most often, this is the transition from the ground state of the mother nucleus to an excited state of the daughter one. Thus, \(\bar{E}_i\) is the average excitation energy of the daughter nucleus.
3. Results and discussion

Results of the calculations are presented in Table II. These are mainly the $\alpha$-transition energies $Q_{t\alpha}$ and the half-lives $T_{\alpha}$ calculated in two variants of $Q_{t\alpha}^L$: WS3+ and HN. Theoretical estimations of the spontaneous-fission half-lives $T_{sf}$ are also given. Available experimental values for all these three quantities are shown as well. The difference between theoretical and experimental values of $Q_{t\alpha}^L$, $\delta Q_{t\alpha}^L$, is also shown for both variants of the calculations.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$^{289}\text{115}$</th>
<th>$^{285}\text{113}$</th>
<th>$^{281}\text{Rg}$</th>
<th>$^{277}\text{Mt}$</th>
<th>$^{273}\text{Bh}$</th>
<th>$^{269}\text{Db}$</th>
<th>$^{265}\text{Lr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{t\alpha}^L$</td>
<td>10.19</td>
<td>9.98</td>
<td>9.87</td>
<td>9.58</td>
<td>8.80</td>
<td>8.08</td>
<td>6.74</td>
</tr>
<tr>
<td>$Q_{t\alpha}^L$</td>
<td>10.63</td>
<td>10.10</td>
<td>10.37</td>
<td>9.73</td>
<td>8.78</td>
<td>8.06</td>
<td>6.51</td>
</tr>
<tr>
<td>$Q_{t\alpha}^L$</td>
<td>10.48</td>
<td>9.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta Q_{t\alpha}^L$ (WS3+)</td>
<td>−0.29</td>
<td>0.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta Q_{t\alpha}^L$ (HN)</td>
<td>0.15</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{\alpha}$ (WS3+)</td>
<td>2.1 s</td>
<td>1.8 s</td>
<td>0.78 s</td>
<td>1.1 s</td>
<td>48 s</td>
<td>35 m</td>
<td>2.4 y</td>
</tr>
<tr>
<td>$T_{\alpha}$ (HN)</td>
<td>0.15 s</td>
<td>0.86 s</td>
<td>38 ms</td>
<td>0.43 s</td>
<td>56 s</td>
<td>44 m</td>
<td>28 y</td>
</tr>
<tr>
<td>$f$(WS3+)</td>
<td>6.8</td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$(HN)</td>
<td>2.0</td>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{\alpha}$ (exp)</td>
<td>0.30 s</td>
<td>4.3 s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{sf}^{th}$</td>
<td>$1.4 \times 10^3$ s</td>
<td>2.8 s</td>
<td>42 ms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{sf}$ (exp)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16 s</td>
</tr>
</tbody>
</table>

The transition energies $Q_{t\alpha}^L$ and the half-lives $T_{\alpha}$ are calculated for a long artificial chain of seven consecutive decays to see the difference in the predictions for them, when two different, but both realistic mass models (WS3+ and HN) are used.

The experimental values of $Q_{t\alpha}^L$ and $T_{\alpha}$ are measured with a good statistics only for two nuclei: $^{289}\text{115}$ and $^{285}\text{113}$. The third nucleus, $^{281}\text{Rg}$, decays mainly by spontaneous fission and we consider only the spontaneous-fission half-life $T_{sf}$ for this nucleus.

One can see in Table II that the theoretical description of the experimental $Q_{t\alpha}^L$ is slightly better in the HN variant than in the WS3+ one. The average of the absolute values of the discrepancies, $|\bar{\delta}Q_{t\alpha}^L|$, is 0.20 MeV for the HN variant and 0.22 MeV for the WS3+ one.

Concerning the quality of the description of the experimental half-life $T_{\alpha}^{exp}$ by the theoretical one $T_{\alpha}^{th}$, this may be characterized by the factor $f$, which is the ratio of the larger of $T_{\alpha}^{th}$ and $T_{\alpha}^{exp}$ to the smaller of them. It is
seen in Table II that this factor does not exceed 5 in the case of HN and 6.8 in the case of WS3+. The average values of them are 3.5 and 4.6 for HN and WS3+, respectively.

Thus, the description of the experimental results for both $Q_\alpha^t$ and $T_\alpha$ may be considered as quite good.

In the graphical form, the results are presented in Fig. 1 for $Q_\alpha^t$ and in Fig. 2 for $\log_{10}T_\alpha$. It is seen in the figures that the results calculated in the variants HN and WS3+ are quite close to each other along the whole chain of seven decays. A larger discrepancy appears only for the nucleus $^{281}$Rg ($Z = 111$).

![Fig. 1. (Color on-line) The \(\alpha\)-transition energies \(Q_\alpha^t\) calculated within the models HN and WS3+, as compared with the experimental values.](image1)

![Fig. 2. (Color on-line) Logarithm of the \(\alpha\)-transition half-lives \(T_\alpha\) (given in seconds) calculated with the use of HN and WS3+ masses, as compared with the measured values.](image2)
An interesting result is the estimation of the effect of the odd proton in the nucleus $^{281}\text{Rg}$ on the spontaneous-fission half-life $T_{\text{sf}}$ of this nucleus. The theoretical values $T_{\text{sf}}^{\text{th}}$ given in Table II are the averages of two realistic values calculated in Ref. [24] for the neighboring even–even nuclei. Thus, the calculated values $T_{\text{sf}}^{\text{th}}$ do not contain any effect of an odd particle, while the $T_{\text{sf}}^{\text{exp}}$ of the odd-A nucleus naturally contains it. So, the ratio $T_{\text{sf}}^{\text{exp}} / T_{\text{sf}}^{\text{th}}$ is this effect. It amounts to 140 for the nucleus $^{281}\text{Rg}$, i.e. the effect is quite large.

4. Summary

The short $\alpha$-decay chain (two $\alpha$ decays) of the nucleus $^{289}\text{115}$ is studied theoretically. Two variants of the calculations, basing on two nuclear-mass models (HN and WS3+), are discussed.

The following conclusions may be drawn from the study:

1. The theoretical results for the $\alpha$-transition energy $Q_{\alpha}^t$ reproduce the experimental ones quite accurately. The average of the absolute values of the discrepancies between the calculated and measured results are: 200 keV and 220 keV in the HN and WS3+ variants, respectively.

2. The theoretical half-lives $T_{\alpha}^{\text{th}}$ reproduce the experimental ones $T_{\alpha}^{\text{exp}}$ with the average ratio: 3.5 in the HN case and 4.6 in the case of WS3+.

3. An interesting result is the estimation of the odd-proton effect on the spontaneous-fission half-life $T_{\text{sf}}$ of the nucleus $^{281}\text{Rg}$. The effect is the elongation of $T_{\text{sf}}$ by a factor of 140, i.e. quite large. The estimation may be expected to be quite accurate because the $T_{\text{sf}}^{\text{th}}$, not containing the odd-proton effect, is based on the realistic calculations of $T_{\text{sf}}$ for even–even nuclei, and the $T_{\text{sf}}^{\text{exp}}$, including the effect, is obtained from the experimental value for the nucleus $^{281}\text{Rg}$, averaged over 24 measurements.

The author would like to thank Yuri Oganessian and Vladimir Utyonkov for helpful discussions. Support by the GSI-Helmholtzzentrum, Helmholtz-Institut Mainz (HIM), and the Polish–JINR(Dubna) Cooperation Programme is gratefully acknowledged.
REFERENCES