TEMPERATURE DEPENDENCE OF UPSILON SUPPRESSION IN QUARK–GLUON PLASMA AND STUDY OF ITS CRITICAL PARAMETERS

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The critical values of screening parameters for $\Upsilon_1^S$ and $\Upsilon_2^S$ in Quark–Gluon Plasma (QGP) have been estimated, where the screened potential is represented by an optical potential of type $V(r, \lambda) = -\left[\frac{V_0+iW_0}{1+\exp((r-R)\lambda)}\right]$. It has been observed that the critical screening length possesses smaller value indicating a larger suppression compared to the value obtained when the screened potential is represented by real potential. The variation of two-particle energies of $\Upsilon_1^S$ and $\Upsilon_2^S$ with temperature has also been studied. It has been observed that below $T = 0.8T_c$, the two-particle energy remains almost invariant before approaching to deconfinement phase at $T = T_c$. The results obtained are compared with other existing estimates.

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1. Introduction

The study of quarkonia continues to play an important role in many aspects of QCD. It has been observed that QCD undergoes a rapid cross over from quark–hadron phase transition [1] into a de-confined state of matter known as the Quark–Gluon Plasma (QGP) at a critical temperature of about $T_c \approx 170$ MeV. In the vicinity of $T_c$, strong interactions among the coloured plasma constituents complicate the description of the medium dynamics. Therefore, the characterization of this region of the QCD phase diagram or

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the critical point has been the subject of intense interest in theoretical and experimental research. At the early stage of system evolution, the energy density of this hot and dense hadronic medium is sufficiently large to reach the deconfined phase [2–4]. From the experimental point of view, one of the important tools for determining the medium dynamics is its interactions with probes. Among those, an important one is the different heavy quarkonia states and their in-medium modifications [5]. These interactions even lead to their complete dissociation at sufficiently high temperature leading to the suppression of quarkonium. Although most studies have focused on $J/\Psi$ states due to the small $b$-quark production cross section at low-collision energies, the charm quarks are not sufficiently heavy for a reliable theoretical description. This causes significant uncertainties in their mechanism in nuclear collisions [6] as well as their interactions with deconfined matter [7]. Recently available data on bottomonium suppression both at the LHC [8] and at RHIC [9] are opening a much more theoretically-controlled channel.

In recent years, there has been a significant progress in the theoretical description of in-medium quarkonia properties. Potential models have been extensively used to describe the properties of finite temperature quarkonia states [10–13]. To extend the potential model to QGP, the effects of colour screening are needed to be considered. At high temperatures, colour screening occurs which is usually understood in terms of in-medium modification of inter-quark forces. Based on this concept, Matsui and Satz [5] argued that above the transition temperature $T_c$, screening effects are strong enough to lead to dissociation of the $J/\Psi$ state. Karsch et al. [10] have used a non-relativistic confinement potential model considering colour screening effects with an exponential damping factor. They have investigated the dependence of dissociation energies, the binding radii and the masses of heavy quark resonances on the colour screening length of the medium. They have also estimated the critical values of colour screening length for different resonances. Petreczky et al. [13] have studied quarkonium spectral function at high temperatures using a potential model with complex potential. Considering different quark binding potentials in a non-relativistic approximation, Liu et al. [14] have studied the binding and dissolution for heavy quarkonia bound states. They have calculated critical values of screening parameter and screening length by considering the Debye screening length. Their investigation indicates that the $T_c$ value for each bound state depends on the form of the binding potential. Fulcher et al. [15] have investigated the state of bottom quarks produced in heavy-ion collisions at RHIC using a flavour-independent potential model which includes colour screening effects. They have found that a sufficiently high number of $B_c$ meson will be produced to generate a detectable tri-lepton signal and the production rate of $B_c$ meson is highly sensitive to the properties of the deconfined source. The quarkonia properties using model spectral functions based on the potential
picture with screening have been studied in detail by Mocsy et al. [16]. They have found that the survival of the \( \Upsilon_{1S} \) state can be reproduced by potential models. Using the two-component model, Song et al. [17] have studied the effect of medium modifications of the binding energies and radii of bottomonium on their production in heavy-ion collisions. They have found that the contribution to bottomonia production from regeneration is small and the inclusion of medium effects is generally helpful for understanding the observed suppression of bottomonia production in experiments. Kakade et al. [18] have investigated the dissociation of quarkonia states in a deconfined medium of quarks at large baryon chemical potential and small temperature region by correcting both the short- and long-distance terms of Cornell potential. They have found that \( J/\psi \) is dissociated approximately at \( 2\mu_c \) (critical chemical potential) in the temperature range of 20–50 MeV which can indirectly help to locate the point on QCD phase diagram at large chemical potential and low temperature zone. Cugnon et al. [19] have studied the influence of the internal motion of \( c \bar{c} \) and the evolution of this pair by means of a Schrödinger model, and they have introduced the coupling of the \( c \bar{c} \) internal motion to the inelastic \( D \bar{D} \) channels through an imaginary potential. Beraudo et al. [20] have interpreted that the imaginary part in the real-time potential arises due to the collisions of the heavy quarks with the light fermions of the thermal bath, whereas Laine et al. [21] have measured the thermal imaginary part of real-time static potential from classical lattice gauge theory simulations. Margotta et al. [22] have investigated quarkonium states in a complex-valued potential and determined the dissociation temperature of the ground state and first excited states considering both real and imaginary part of the binding energy of quarkonia when immersed in QGP, whereas Pena et al. [23] have discussed the role of static screening and absorption/regeneration kinetics in the quarkonium time evolution by employing the time-dependent harmonic oscillator model and the temperature dependence of the real and imaginary parts of the oscillator frequency. Solana et al. [24] have shown that the complex potential is much more effective in suppressing quarkonia states than the real one. They have studied the suppression of heavy mesons by focussing on the effect a temperature-dependent potential has on the production of quarkonia states in a deconfined hadronic matter. Considering Yukawa potential, they have investigated the variation of binding energy of \( \Upsilon_{1S} \) and \( \Upsilon_{2S} \) states as a function of Debye screening length.

In the present work, we have investigated the dependence of masses and binding energies of \( \Upsilon_{1S} \) and \( \Upsilon_{2S} \) states on the colour screening parameter when the latter is immersed in QGP considering optical type of screened potential. We have also studied the variations of two-particle energies of \( \Upsilon \) states with temperature with suitable parametrization of string tension \( \sigma \).
2. Method

The Hamiltonian for a $q\bar{q}$ bound state in a non-relativistic approach can be described by

$$H(r, T) = \frac{p^2}{2m_{re}} + V(r, T),$$

where $m_{re}$ is the reduced mass of the $q\bar{q}$ system, $p$ is the relative momentum and the potential $V(r, T)$ has been represented by Cornell potential [25] which runs as

$$V(r, T = 0) = \sigma(T = 0) r - \frac{\alpha}{r},$$

where $\sigma(T)$ is a temperature-dependent string tension and $\alpha$ is a dimensionless adjustable parameter. To represent the screening effect, a number of potentials have been proposed by a number of authors [10–13]. Following the work of Cugnon et al. [19], we have assumed that this screening can be described by a complex in-medium potential as

$$V_c(r, \lambda) = -[v(r) + i\omega(r)] = -\left[\frac{V_0 + iW_0}{1 + \exp((r-R)\lambda)}\right]$$

in analogy with the optical potential conventionally used for nuclear scattering processes. $V_0$ and $W_0$ are the strength parameters representing the real and imaginary part of the potential, $R$ is the typical size of the plasma radius, $r$ is a variable denoting the transverse distance from the center of the plasma with the assumption that $\mathcal{T}$ is formed at the center of the plasma, and $\lambda$ is the screening parameter. Bencze [26] has shown that the Schrödinger equation has an exact analytical solution for the scattering states of S-wave neutrons. With the potential expressed in (3), the Schrödinger equation runs as

$$\frac{d^2u}{dr^2} + \left[K_n^2 + \frac{p^2}{1 + \exp((r-R)\lambda)}\right] u = 0,$$

where $K_n^2 = m_qE_n$ ($m_q$ being the quark mass, $E_n$ the eigenenergy for the $n^{th}$ state). The corresponding bound state wave function may be expressed as [26]

$$\Phi_{nl}(r) = \frac{U(r)}{r} = \frac{C_{nl}}{r} Y^\alpha(1 + Y)^n \ _2F_1\left(-n, -n-1, -3 - \frac{2K_n}{\lambda}; \frac{Y}{1+Y}\right),$$

where $Y = \exp((r-R)\lambda)$, $C_{nl}$ is a constant, $n = 1, 2, 3\ldots$ etc., $\alpha^2 = \frac{1}{\lambda^2}(K_n^2 - p^2)$, and $_2F_1$ is the hypergeometric function of four arguments. Extracting the zero’s of the scattering matrix $S_0(K)$ on the negative imaginary axis of the complex $K$-plane and solving the resulting equation [26], we have

$$\alpha + K_na \simeq -n$$
which gives us the bound state energy eigenvalues $E_n(= E_{BE})$ as (for $l = 0$ states)

$$E_n = \frac{n^2 + \left( \frac{m_q}{\hbar^2 \lambda^2} \right) (V_0 + iW_0) \lambda^2}{4m_q n^2}$$

(7)

with $V_0 = 0.6 \lambda$, $m_q = m_b = 4.2$ GeV [27] and $W_0 = 0.38$ GeV [19], we have evaluated the bound state energies $E_n$ by using the above relation.

In a thermodynamic environment of quarks and gluons, at temperature $T$, the in-medium potential will be modified due to the Debye screening effects. The temperature-dependent potential has been parameterized in the form by adding a term $f(\lambda)$ to $V_c(r, \lambda)$ such as

$$V(r, \lambda) = f(\lambda) + V_c(r, \lambda)$$

(8)

with [9]

$$f(\lambda) = \frac{\sigma(T)}{\lambda} (1 - \exp(-\lambda \mu)),$$

(9)

where $\mu$ is an adjustable parameter. We have evaluated $\mu$ by adjusting the observed values of the masses $\Upsilon_{1S}, \Upsilon_{2S}$ [28], respectively, in the mass formula which runs as

$$M = 2m_b + f(\lambda) + E_{BE}$$

(10)

which yields $\mu = 6.49$ GeV$^{-1}$ and 9.03 GeV$^{-1}$ for $\Upsilon_{1S}$ and $\Upsilon_{2S}$ states, respectively. We have used $m_b = 4.2$ GeV [27] and constant value of $\sigma$ (at $T = 0$) as 0.192 GeV$^2$ from the work of Nendzig et al. [29]. The masses of the $1S$ and $2S$ states of $\Upsilon$ meson for different values of the screening parameter $\lambda$ have been estimated and variations are displayed in Fig. 1 and Fig. 2, respectively, along with the continuum $[2m_b + f(\lambda)]$. The variation of binding energies with $\lambda$ for $1S$ and $2S$ states of $\Upsilon$ meson have also been studied and plotted in Fig. 3 and Fig. 4. The critical values of screening parameter $\lambda_c$ for $\Upsilon_{1S}$ and $\Upsilon_{2S}$ at which the binding energy of the particles ceases to exist have been extracted from the graph and this indicates a transition from the hadronic phase to the plasma phase.

To study the temperature dependence of two-particle energies of heavy quarkonia, we have used the parametrization of the string tension $\sigma(T)$ in the closed analytical form with respect to temperature as in Hashimoto et al. [30] near $T_c$ which runs as

$$\sigma(T) = \sigma(0) \left(1 - \frac{T}{T_c}\right)^{0.2}.$$
We have studied the variation of masses of $\Upsilon_{1S}$ and $\Upsilon_{2S}$ states with the temperature using expression (10). The variations are displayed in Fig. 5. Figure shows the slow falling of the masses with temperature for both the systems well below the critical value $T_c$, whereas around $T_c$, rapid fall in masses for both states have been observed. Similar type of observations have also been noted by Satz [31] for heavy quarkonia. They have discussed the spectral analysis of quarkonium states in a hot medium of deconfined
Fig. 3. Variation of binding energy of $\Upsilon_{1S}$ state with the screening parameter.

Fig. 4. Variation of binding energy of $\Upsilon_{2S}$ state with the screening parameter.

quarks and gluons, and have showed that this analysis provides a way to determine the thermal properties of QGP. Röpke et al. [32] have studied the effect of temperature variation on the dissociation of heavy quarkonia in QGP by solving numerically the Schrödinger equation using the linear superposition of exponential and the screened Coulomb type of potential. They also report similar-type observation.
3. Results and discussions

In our present investigation, we have used the optical type of potential which simulates screening as well as the absorption effect in the process of the dissolution of $\Upsilon$ in the QGP. We have studied the dependence of masses and the binding energies of heavy quarks with the colour screening parameter $\lambda$. Results are displayed in Fig. 1 and Fig. 3 for $\Upsilon_{1S}$, and Fig. 2 and Fig. 4 for $\Upsilon_{2S}$ states. We have estimated the critical value of $\lambda$ at which two-particle binding energy ceases to exist indicating the melting of quarkonia bound states to its constituent. The critical value of screening parameter ($\lambda_c$) is obtained from Fig. 1 and Fig. 3 as 0.332 GeV for $\Upsilon_{1S}$ state, whereas for $\Upsilon_{2S}$ state $\lambda_c$, from Fig. 2 and Fig. 4, we obtain 0.11 GeV and 0.112 GeV, respectively. Karsch et al. [10] have probed the variation of mass with respect to the screening parameter and suggested the value of $\lambda_c$ as 1.565 GeV and 0.671 GeV for $\Upsilon_{1S}$ and $\Upsilon_{2S}$ states, respectively. Using a flavour-independent potential model, Fulcher et al. [15] have observed $\lambda_c(\Upsilon_{1S}) = 0.84$ GeV, almost 200 MeV higher than the corresponding quantity for $J/\Psi$ and $\lambda_c(\Upsilon_{2S}) = 0.37$ GeV. Liu et al. [14] have also investigated critical value of screening parameter for different quark binding potentials by considering the Debye screening effect. We have obtained smaller value of $\lambda_c$ for both $\Upsilon_{1S}$ and $\Upsilon_{2S}$ compared to the values obtained by [10, 14, 15].

Complex potential has been widely used to incorporate the suppression effect. Nendzig et al. [29] have suggested a three-step model with complex potential for suppression of $\Upsilon(nS)$ state. They have pointed out that the
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\( \Upsilon (1S) \) suppression is primarily due to gluodissociation but for the higher state, the screening effect with collisional damping reduced feed down process making the suppression larger. Rothkopf [33] has studied the heavy quarkonia in QGP with complex potential and has shown that the role of imaginary part can be described as a spatial decoherence with the introduction of stochastic potential. A time-dependent complex potential between heavy quarks is suggested by Hayata et al. [34] in the strongly coupled QGP (SQGP) on the basis of gauge gravity duality. They have shown that the imaginary part of the potential started to dominate over a distance which is comparable to the distance between \( Q - \overline{Q} \) low-lying quarkonia. The relevance of imaginary part of the potential is the sequential melting of heavy quarkonia in SQGP. Solana [24] has compared the suppression pattern of in-medium brick which is characterized by real quarkonium potential to that of imaginary part with the same screening parameter and has shown that complex potential introduces more suppression. Digal et al. [35] have showed that in a hot medium quarkonia, higher states are dissociated at lower temperature than the ground states leading to a sequential suppression pattern. The CMS Collaboration [36, 37] data supports the increased suppression of less strongly bound states \( i.e. \, \Upsilon (1S) \) is the least suppressed and the \( \Upsilon (3S) \) is the most suppressed of the three states. In the present work (Fig. 5), it has been observed that the suppression of \( \Upsilon (2S) \) states is more pronounced than of the \( \Upsilon (1S) \) states and it is less suppressed. The difference may be attributed to the parametrization of \( \mu \) and \( \sigma (t) \). Closer to \( T_c \), the masses drop sharply. Satz et al. [31] have also studied the variation of heavy quarkonia mass as a function of temperature and they have noted similar type of observation.

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