SOME NEW ASPECTS OF QUARKONIA PRODUCTION AT THE LHC*

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(Received April 3, 2017)

We discuss some different new aspects of $J/\psi$ meson production in exclusive, semiexclusive and inclusive processes. We finish with a short discussion of double $J/\psi$ production. We point out some new results obtained recently by our group and discuss some open issues.

DOI:10.5506/APhysPolB.48.1207

1. Introduction

The $J/\psi$ meson is interesting due to its simple structure ($c\bar{c}$ state). However, the mechanism of its production is not always well-understood. There are different subfields of high-energy physics that deal with the production of $J/\psi$ meson. Our group participated in different fields and it is the aim of this talk (presentation) to show some progress in the different fields.

The first topic is connected with the exclusive production of $J/\psi$ mesons in the $pp \rightarrow ppJ/\psi$ reaction. There the main production mechanism is a photon–pomeron fusion. The same type of process was studied some years ago in the context of $\gamma p \rightarrow J/\psi p$ reactions measured at HERA \cite{1}. The pomeron is a key word which is often by different authors understood and used very differently. At high energies, which correspond to small fractions of gluon longitudinal momentum fractions, a problem of gluon saturation may be important. The problem with the current experiment is that so far it was not possible to control the exclusivity of the process. Therefore, recently we have performed more detailed studies of semiexclusive processes that contribute in experimental environment but, strictly speaking, are not exclusive in a rigorous sense of the word.

\footnote{Presented at the Cracow Epiphany Conference “Particle Theory Meets the First Data from LHC Run 2”, Kraków, Poland, January 9–12, 2017.}
The inclusive production of $J/\psi$ mesons is a long-standing problem in spite that there are well-formulated nonrelativistic pQCD rules. Forward production of $J/\psi$ mesons may be also related to the phenomenon of gluon saturation. In recent years, the luminosity of the running experiments was increased and allowed to measure even two $J/\psi$ mesons in one event. Here, a new mechanism, called double parton (or multiple parton) scattering, may appear at high energies, where the density of partons, in particular of gluon, becomes large. In contrast to single parton scattering, the formalism of double (or multiple) scattering is not under similar theoretical precision. While there was a relatively good agreement for experiments where low-transverse momenta of $J/\psi$ mesons were involved (e.g. LHCb), the cases where cuts on transverse momenta are (were) more problematic and large disagreement of theoretical calculation with the data was observed unless the constant known as effective cross section was considerably lowered. Below, we wish to discuss some mechanisms that were neglected and may be important for a good understanding of the situation.

2. Exclusive $pp \rightarrow ppJ/\psi$ reaction

2.1. Sketch of the theoretical methods

The imaginary part of the forward amplitude, for transverse photons (T), sketched in Fig. 1, can be written as [1]

$$\Im M_T(W, \Delta^2 = 0, Q^2 = 0) = W^2 e_v \sqrt{4\pi\alpha_{\text{em}}} \frac{2}{4\pi^2} \int_0^1 \frac{dz}{z(1-z)}$$

$$\times \int_0^\infty d^2 k \psi_V(z, k^2) \int_0^\infty \frac{d^2 \kappa^2}{\kappa^4} \alpha_s(q^2) \mathcal{F}(x_{\text{eff}}, \kappa^2)$$

$$\times \left( A_0(z, k^2) W_0(k^2, \kappa^2) + A_1(z, k^2) W_1(k^2, \kappa^2) \right).$$

(1)

![Fig. 1. A sketch of dynamics of the $\gamma p \rightarrow J/\psi p$ process.](image-url)
The full amplitude at finite momentum transfer is given by
\[ M(W, \Delta^2) = (i + \rho) \Im M(W, \Delta^2 = 0, Q^2 = 0) \exp \left(-B(W)\Delta^2/2\right), \tag{2} \]
where the real part of the amplitude is restored from analyticity
\[ \rho = \frac{\Re M}{\Im M} = \tan \left(\pi \frac{\partial \log (\Im M/W^2)}{\partial \log W^2}\right). \tag{3} \]

Above \( B(W) \), there is a slope parameter which, in general, depends on the photon–proton center-of-mass energy and is parametrized in the present analysis as
\[ B(W) = b_0 + 2\alpha'_\text{eff} \log \left(\frac{W^2}{W_0^2}\right). \tag{4} \]

The full Born amplitude for the \( pp \to pVp \) process (see Fig. 2) can be written as
\[
\begin{align*}
\mathcal{M}_{\lambda_1 \lambda_2 \to \lambda'_1 \lambda'_2 \lambda V}(s, s_1, s_2, t_1, t_2) &= \mathcal{M}_{\gamma p} + \mathcal{M}_{p\gamma} \\
&= \langle p'_1, \lambda'_1 | J_\mu | p_1, \lambda_1 \rangle \epsilon^*_\mu(q_1, \lambda_V) \frac{\sqrt{4\pi\alpha_{\text{em}}}}{t_1} \mathcal{M}_{\gamma^* h_2 \to V h_2}(s_2, t_2, Q^2_1) \\
&\quad + \langle p'_2, \lambda'_2 | J_\mu | p_2, \lambda_2 \rangle \epsilon^*_\mu(q_2, \lambda_V) \frac{\sqrt{4\pi\alpha_{\text{em}}}}{t_2} \mathcal{M}_{\gamma^* h_1 \to V h_1}(s_1, t_1, Q^2_2). \tag{5}
\end{align*}
\]

Fig. 2. Two mechanisms of exclusive \( J/\psi \) meson production at the Born level.

The effects of absorption are illustrated in Fig. 3. The relevant formalism is described \textit{e.g.} in Ref. [3].
2.2. Selected results

Most of the calculations in the literature neglect the anomalous photon coupling to proton quantified by the Pauli coupling. In Fig. 4, we illustrate the role of the Pauli form factor on the transverse momentum distribution of $J/\psi$ mesons. Inclusion of the Pauli form factor considerably enhances the cross section at large transverse momenta of $J/\psi$ meson.

In Fig. 5, we show rapidity distributions for different unintegrated gluon distribution function (UGDF). The results strongly depend on UGDF used. The best result is achieved with UGDFs that include nonlinear effects (here, a nonlinear Kutak–Stasto UGDF was used). This may be connected with the onset of gluon saturation, a phenomenon expected to show up in processes with small gluon longitudinal momentum fraction but never unambiguously identified.
Fig. 5. Rapidity distributions of exclusively produced $J/\psi$ mesons for three different unintegrated gluon distributions at $\sqrt{s} = 7$ TeV.

The role of absorption effects is illustrated in Fig. 6. The absorption effects modify the shape of transverse momentum distributions. Therefore, a careful treatment of absorption effects is very important in the context of possible searches for odderon exchange discussed in the literature.

Fig. 6. Transverse momentum distributions of exclusively produced $J/\psi$ mesons for three different unintegrated gluon distributions. The shaded (green online) band represents typical uncertainties in calculating absorption effects as described in the text.

In Ref. [2], we have shown similar effects for $\psi'$ meson production. Here, in Fig. 7, we show only the ratios of the cross section for $\psi'$ to the cross section for $J/\psi$ meson production as a function of $J/\psi$ ($\psi'$) rapidity. A good agreement is achieved for the Gaussian wave functions. In contrast, approaches that do not use wave functions cannot give a reasonable description of the data.
3. Semiexclusive production of $J/\psi$ mesons with proton dissociation

3.1. Sketch of the theoretical methods

There are a few mechanisms of semiexclusive excitation of participating protons. Let us start from electromagnetic dissociation of one of the protons (see Fig. 8).

Fig. 8. Electromagnetic excitation of one of the photons in proton–proton collisions.

The cross section for such processes can be written as [4]

$$\frac{d\sigma(pp \rightarrow X\gamma p; s)}{dyd^2pdM_X^2} = \int \frac{d^2q}{\pi q^2} \mathcal{F}^{(\text{inel})}_{\gamma/p} (z_+, q^2, M_X^2) \frac{1}{\pi} \frac{d\sigma^{\gamma^*p \rightarrow Vp}}{dt} (z_+s, t = -(q-p)^2) + (z_+ \leftrightarrow z_-),$$

(6)

where $z_\pm = e^\pm y \sqrt{(p^2 + m_V^2)} / s$. In the kinematics of interest, the “fully unintegrated” flux of photons associated with the breakup of the proton is
calculable in terms of the structure function $F_2$ of a proton [5, 6]

$$
\mathcal{F}_{\gamma/p}^{(\text{inel})} (z, \mathbf{q}^2, M_X^2) = \frac{\alpha_{\text{em}}}{\pi} (1 - z) \theta (M_X^2 - M_{\text{thr}}^2) \\
\times \frac{F_2 (x_{\text{Bj}}, Q^2)}{M_X^2 + Q^2 - m_p^2} \left[ \frac{\mathbf{q}^2}{\mathbf{q}^2 + z (M_X^2 - m_p^2) + z^2 m_p^2} \right]^2.
$$

(7)

In distinction to electromagnetic dissociation, the diffractive excitation is highly model-dependent. We consider two mechanisms of the diffractive dissociation: resonance production and continuum excitation. A diagrammatic sketch of the reaction mechanism is shown in Fig. 9.

![Fig. 9. Diffractive excitation of resonances on one of the protons.](image)

The contribution of three positive-parity baryon resonances on the nucleon trajectory are taken into account:

1. $N(1680), J = \frac{5}{2}$,
2. $N(2220), J = \frac{9}{2}$,
3. $N(2700), J = \frac{13}{2}$.

They contribute to the $p\bar{p} \to X$ amplitude as

$$
\Im m A (M_X^2, t) = \sum_{n=1,3} [f(t)]^{2(n+1)} \frac{\Im m \alpha (M_X^2)}{(J_n - \Re e \alpha (M_X^2))^2 + (\Im m \alpha (M_X^2))^2}.
$$

(8)

Here, $J_n$ is the spin of the $n^{th}$ resonance, and the explicit form of the complex Regge trajectory $\alpha (M_X^2)$ as well as the form factor $f(t)$ (see [7, 8]).

In addition, we include the so-called Roper resonance $N^*(1440)$. More details are given in our original paper [4].

We include also the partonic continuum for the $\gamma p \to V X$ subreaction. Corresponding schematic diagrams are shown in Fig. 10.
In this approximation, the cross section can be written as
\[
\frac{d\sigma_{\text{diff, partonic}}^{pp\rightarrow Vj}}{dy_Vdy_Jd^2p_t} = \frac{1}{16\pi^2s^2}x_1q_{\text{eff}}(x_1, \mu_F^2) x_2\gamma(x_2)|M_{q\gamma\rightarrow Vq}|^2
\]
\[+ \frac{1}{16\pi^2s^2}x_1\gamma(x_1)x_2q_{\text{eff}}(x_2, \mu_F^2) |M_{q\gamma\rightarrow Vq}|^2. \tag{9}\]

We neglect transverse momenta of the photon and the initial parton. The two terms correspond to the two diagrams in Fig. 10. The effective parton distribution means
\[
q_{\text{eff}}(x, \mu_F^2) = \frac{81}{16}g(x, \mu_F^2) + \sum_f [q_f(x, \mu_F^2) + \bar{q}_f(x, \mu_F^2)]. \tag{10}\]

In Ref. [4], we use a simple formula for the two-gluon exchange
\[
\frac{d\sigma_{\gamma q\rightarrow Vq}}{dt} \propto \alpha_s^2(\bar{Q}_t^2) \alpha_s^2(|\hat{\ell}|) \frac{m_V^3\Gamma(V\rightarrow l^+l^-)}{(\bar{Q}_t^2)^4}, \tag{11}\]

where \(\bar{Q}_t^2 = m_V^2 + |\hat{\ell}|\). The normalization constant has been adjusted in [4] to the H1 HERA experimental data.

### 3.2. Selected results

In Ref. [4], we have shown many results for both full phase space as well as for particular experiments. In Fig. 11, we show the distribution in rapidity of the \(J/\psi\) meson.

In Fig. 12, we show transverse momentum distribution of \(J/\psi\) meson for different ranges of proton excitations (\(M_X < 2, 5\) and 10 GeV). The larger masses of the proton excitations, the larger slopes of the transverse...
momentum distribution of $J/\psi$. For comparison, we show distribution for the purely exclusive case. The inelastic contributions start to dominate over the purely elastic (exclusive) contribution for $p_t > 1$ GeV.

Fig. 11. Rapidity distribution of $J/\psi$ mesons for $\sqrt{s} = 7$ TeV for purely exclusive and for different contributions to semiexclusive processes.

Fig. 12. Dependence of the cross section on transverse momentum squared for the LHCb experiment.
It is also interesting to see how the cross sections for semiexclusive processes compare to those for the purely exclusive one. To quantify their contribution, we define the following ratio:

\[ R(y) = \frac{d\sigma_{pp \to pJ/\psi X}(M_X < M_{X,\text{max}})/dy}{d\sigma_{pp \to pJ/\psi p}/dy}. \]  

(12)

This ratio is shown in Fig. 13 for three different values of \( M_{X,\text{max}} \). We see that the magnitude as well as the shape of the ratio depend on the range of missing masses included in the calculation.

Fig. 13. The ratio of the semiexclusive-to-exclusive cross section as a function of rapidity of \( J/\psi \) meson for different ranges of proton excitation energies.

### 4. Inclusive production of \( J/\psi \) mesons

#### 4.1. Sketch of the theoretical methods

We now turn to the fully inclusive production of \( J/\psi \) mesons. In this presentation, we shall consider only the contributions of the colour-singlet mechanism. Here, at the perturbative stage of the process, a \( c\bar{c} \) system is produced in the colour-singlet state. In Fig. 14, we show two dominant colour-singlet mechanisms of \( J/\psi \) meson production. The first diagram represents the so-called direct contribution, while the second diagram represents an example of feed down from other quarkonia.

\[
\begin{align*}
\frac{d\sigma(pp \to J/\psi gX)}{dy_J/\psi dy_g d^2p_{J/\psi,t} d^2p_{g,t}} &= \frac{1}{16\pi^2 s^2} \int \frac{d^2q_{1t}}{\pi} \frac{d^2q_{2t}}{\pi} \left| \mathcal{M}^{\text{off-shell}}_{g^*g^* \to J/\psi g} \right|^2 \\
&\times \delta^2(q_{1t} + q_{2t} - \vec{p}_{H,t} - \vec{p}_{g,t}) \mathcal{F}_g(x_1, q_{1t}^2, \mu_F^2) \mathcal{F}_g(x_2, q_{2t}^2, \mu_F^2).
\end{align*}
\]  

(13)
The corresponding matrix element squared for the \( gg \to J/\psi g \) is proportional to the radial wavefunction at the origin
\[
|\mathcal{M}_{gg\to J/\psi g}|^2 \propto \alpha_s^3 |R(0)|^2.
\] (14)

In our calculations, the matrix elements obtained by Baranov [9] were used.

Another important mechanism is the production of \( \chi_c \) mesons and its subsequent decay \( \chi_c \to J/\psi + \gamma \). The \( \chi_c \) mesons are p-wave \( c\bar{c} \) states of positive \( C \)-parity. They can be produced by the gluon–gluon fusion mechanism. In the \( k_t \)-factorization approach, the leading-order cross section for the \( \chi_c \) meson production can be written in a similar way as for the \( J/\psi \) production.

The matrix element squared for the \( g^*g^* \to \chi_c \) subprocess is proportional to the derivative of the wave function at the origin
\[
|\mathcal{M}_{g^*g^*\to \chi_c}|^2 \propto \alpha_s^2 |R'(0)|^2.
\] (15)

\( R'(0) \) can be treated as a free parameter to get correctly relative contribution of \( J/\psi \) from \( \chi_c \). We used the matrix element taken from [10].

Another mechanism which we take into account is the production of \( \psi' \) meson and its decay. The corresponding ratio \( \text{Br}(\psi' \to J/\psi + X) = 0.574 \) is relatively large. The cross section for production of \( \psi' \) mesons is calculated using a formula analogous to Eq. (13). Of course, then the wave function of \( J/\psi \) is replaced by a wave function of \( \psi' \): \( |R_{\psi'}(0)|^2 \approx 5/8|R_{J/\psi}(0)|^2 \). The difference may be understood qualitatively as due to the fact that \( \psi' \) is a 2S state, while \( J/\psi \) is 1S state (different radial excitations).

4.2. Selected results

Here, we show only some selected results. More detailed discussion will be presented elsewhere [11]. In Fig. 15, we show rapidity distributions obtained with the KMR UGDF. This gluon distribution gives a relatively good
description of the open charm inclusive production and correlations. At low energies, a relatively good agreement is obtained. When increasing energy, the agreement becomes worse and worse. The disagreement may be connected with the onset of nonlinear effects, leading eventually to gluon saturation which is an asymptotic (small-$x$) notion.

![Graph of inclusive rapidity distributions of $J/\psi$ mesons for three different energies obtained with the KMR UGDFs.](image)

Fig. 15. Inclusive rapidity distributions of $J/\psi$ mesons for three different energies obtained with the KMR UGDFs.

For the LHCb configuration, typically one longitudinal momentum fraction is small and the other much larger. Replacing the KMR UGDF by the Kutak–Stasto UGDF for the low-$x$ values improves the situation, see Fig. 16. In our opinion, it seems precocious to draw too definite conclusion in the moment.

In [11], we will discuss many other aspects of $J/\psi$ meson production.

![Graph of inclusive rapidity distributions of $J/\psi$ mesons for two energies obtained with one KMR and one KS (nonlinear) UGDF.](image)

Fig. 16. Inclusive rapidity distributions of $J/\psi$ mesons for two energies obtained with one KMR and one KS (nonlinear) UGDF.
5. Production of two $J/\psi$ mesons

5.1. Sketch of the theoretical methods

The colour-singlet mechanisms used so far in the literature are shown in Fig. 17. The first type of diagrams will be called here “box” for brevity. The circle inside the box represents different gluon insertions. There are 20 diagrams (see e.g. [12]). These contributions are of the order of $O(\alpha_s^4)$. In addition, they depend on the value of the wave function at the origin $\sigma \propto |R(0)|^4$. The matrix elements are too complicated to be shown here explicitly. The second diagram, of the order of $O(\alpha_s^6)$, represents double parton scattering mechanism. Although of higher order than the previous box contributions it may be enhanced by higher powers of gluon distributions.

For the presentation at the Epiphany 2017 workshop, the box contribution was calculated only in the collinear approach\(^1\). Then the inclusive differential cross section can be written as

$$
\frac{d\sigma(pp \to J/\psi J/\psi)}{dy_{V1} dy_{V2} d^2p_t} = \frac{1}{16\pi^2 s^2} \left| M_{\text{on-shell}}^{gg \to J/\psi J/\psi} \right|^2 \times g \left( x_1, \mu_F^2 \right) g \left( x_2, \mu_F^2 \right) . \tag{16}
$$

In our calculations for the presentation, we used the MSTW08 gluon distributions. We have checked that the somewhat artificial matrix element gives a good representation of experimental data for inclusive $J/\psi$ production.

The calculation of double parton scattering is simplified. Instead of including all single scattering mechanism discussed in the previous section, we parametrize the single $J/\psi$ production using the following formula:

$$
\frac{d\sigma(pp \to J/\psi g)}{dy_{J/\psi} dy_X d^2p_t} = \frac{1}{16\pi^2 s^2} \left| M_{\text{eff}}^{gg \to J/\psi X} \right|^2 \times g \left( x_1, \mu_F^2 \right) g \left( x_2, \mu_F^2 \right) . \tag{17}
$$

\(^1\) Now a full $k_t$-factorization approach is available but will be not shown here.
We take a parametrization of the effective matrix element by Kom–Kulesza–Stirling [13] using the MSTW08 PDF.

In the present approach, we do calculations using a factorized Ansatz. Then the cross section for double parton scattering can be written as

$$
\frac{d\sigma}{dy_1 d^2p_1t dy_2 d^2p_2t} = \frac{1}{2\sigma_{\text{eff}}} \frac{d\sigma}{dy_1 d^2p_1t} \frac{d\sigma}{dy_2 d^2p_2t}.
$$

(18)

The $\sigma_{\text{eff}}$ parameter is, in principle, a free parameter responsible for the overlap of partonic densities of colliding protons. $\sigma_{\text{eff}} = 15 \text{ mb}$ is the world average for different reactions. We shall use this value having in mind that a departure from a factorized Ansatz is possible. A complete calculation of the single parton scattering contributions should show how much room is left for the DPS contribution, which by itself is rather difficult to be calculated from first principle.

In Fig. 18, we show mechanisms not included routinely in the calculation of simultaneous two $J/\psi$ meson production. The first one (16 diagrams) was included before in [12] but there it was not crucial. The second one was not discussed in the presentation at Epiphany 2017, but will be presented soon elsewhere (see also [14]).

5.2. Selected results

Here, we show some preliminary results obtained for very recent ATLAS data [15]. Both cuts on $J/\psi$ and muon kinematical variables have been imposed. The details will be given elsewhere [16].

In Fig. 19, we present distribution in rapidity difference between the two $J/\psi$ mesons. Both the leading-order and two-gluon exchange contributions are shown for two different factorization scales. Summing the different contributions, one can almost describe the experimental data except of very small rapidity differences. We ask the reader to note a similar shape of distributions for DPS and two-gluon exchange.

Fig. 18. New mechanisms included or being included by us recently.
In Fig. 20, we show distribution in dimeson invariant mass. In the collinear-factorization approach and with the ATLAS experimental cuts on transverse momenta of $J/\psi$ mesons, there is a sharp cut off in meson invariant mass ($M > 20$ GeV). There is no such a sharp cut off for the DPS contributions where both $J/\psi$ mesons are not correlated in azimuthal angle. Although we do not show the sum of the contributions, it is obvious that no good description of the data is possible, especially for low invariant masses. This will be discussed in detail in our forthcoming paper [16].
In the collinear-factorization approach, only $d\sigma/dy_{\text{diff}}$ and $d\sigma/dM_{VV}$ can be obtained. For DPS mechanism, also $p_{t,\text{sum}} = |\vec{p}_{1t} + \vec{p}_{2t}|$ distribution can be obtained as shown in Fig. 21. Clearly, the DPS contribution is not sufficient, as can be seen by comparison of the cross-section normalizations and, in particular, the shapes of the theoretical and experimental distributions.

![Fig. 21. Distribution of $p_{t,\text{sum}}$. Only the DPS contribution is shown and compared to the ATLAS data.](image)

6. Conclusions

In the present paper, we have discussed different reactions with the $J/\psi$ meson in the final state.

We have started from exclusive $pp \rightarrow ppJ/\psi$ process. In contrast to other groups, we include interference effect between the photon–pomeron and pomeron–photon amplitudes and present distributions in $J/\psi$ transverse momentum. We quantify the effect of including the anomalous photon coupling to nucleon and those due to absorption effects, both often neglected in the literature. Our calculation suggest that the LHCb data show an onset of nonlinear effects.

However, some caution is required when interpreting the present experimental results. We have discussed that the present measurements were not fully exclusive as protons were not measured and only rapidity gap conditions were imposed experimentally. We have discussed the reaction in which one of the protons is dissociated, but large rapidity gaps around $J/\psi$ are present. We called them semiexclusive reactions. In general, there are two classes of the dissociative processes: associated with photon exchange and associated with pomeron exchange. We have presented a novel method to
include electromagnetic dissociation. In this method, the $p \to X$ vertex is expressed in terms of $F_2$ structure function. By using parametrizations valid in the full phase space, one can include both resonance and continuum excitations. Diffractive excitations are more difficult to model. We have discussed both resonance contributions using a model from the literature and partonic continuum caused by the exchange of colour singlet two-gluon exchange with parameters adjusted to HERA data. We have shown that, in disagreement with naive expectation in the field, the electromagnetic excitation gives larger contribution than the diffractive one. The effect of the semiexclusive processes on rapidity and transverse momenta of the $J/\psi$ mesons have been presented.

Paradoxically, the inclusive production of $J/\psi$ mesons is less understood than the exclusive one. Different authors have different opinions on the underlying mechanism. The difficulty is that the higher-order effects are more important than for other pQCD processes. In the present short review, we have considered only so-called colour-singlet mechanisms that, in contrast to colour-octet mechanisms, can be calculated from first principles. To include higher order effects, we have used the $k_t$-factorization method and modern unintegrated gluon distributions. In this presentation, we have focused on the description of recent LHCb data for forwardly produced $J/\psi$ mesons. Both direct and feed-down contributions were included. The rather forward production, especially for $\chi_c$ feed-down, is sensitive to small values of longitudinal momentum fraction. We have obtained a good description of lower energy rapidity distributions and gradually worse and worse description with increasing collisions energy when using the KMR UGDFs. Whether this is a signal of nonlinear effects (saturation) it is, in our opinion, still an open issue. We have shown that one can avoid the conflict with the LHCb experimental data using UGDFs that include nonlinear effects.

Finally, we have addressed shortly the simultaneous production of two $J/\psi$ mesons at intermediate transverse momenta. We have made calculation relevant for recent ATLAS data. Several mechanisms have been discussed. Several distributions have been discussed. The leading (box) contribution was calculated in collinear approach. The double parton scattering was calculated using a data-driven approach using a simple parametrization of inclusive $J/\psi$ data. It seems impossible to describe the new data with the two mechanisms. We have also presented results including the two-gluon exchange mechanism between two quark–antiquark fluctuations of gluons which turned out to be important at large rapidity difference between the two $J/\psi$ mesons. Some plans for the future have been presented.
REFERENCES