THE pd → η³He REACTION AND η³He BOUND STATE?
THE B⁺B⁺ρ SYSTEM*

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(Received August 10, 2017)

We analyze the data on cross sections and asymmetries for the pd(dp) → η³He reaction close to threshold and look for bound states of the η³He system. Fitting these data in terms of an η³He optical potential, we find a local Breit–Wigner form of the η³He amplitude T below threshold with a clear peak in |T|², which corresponds to an η³He binding of about 0.3 MeV and a width of about 3 MeV. However, this corresponds to a pole in the complex plane above threshold. We also discuss a state found for BB⁺ρ with J = 3.

DOI:10.5506/APhysPolB.48.1793

1. Introduction

The pd → η³He reaction has been used in the past to investigate the possible existence of an η³He bound state. The data on the pd(dp) → η³He total cross section show a sharp rise from threshold before becoming stable at an excess energy of about Q = 1 MeV [1, 2]. These data have been analyzed before in Refs. [1, 3]. In Ref. [1] only an s-wave amplitude for η³He was considered, while in Ref. [3] the s-wave and p-wave interference data were

* Presented at the 2nd Jagiellonian Symposium on Fundamental and Applied Subatomic Physics, Kraków, Poland, June 3–11, 2017.
considered in order to further constrain the $\eta^3$He amplitude. This analysis suggested a pole with a binding energy of around 0.3 MeV and with a very small width.

Here, we report on the work of [4], where a different analysis is performed with also different conclusions.

### 2. Formalism

Let us depict diagrammatically the $pd \rightarrow \eta^3$He process. This is done in Fig. 1.

![Diagram 1](image1)

Fig. 1. The process $pd \rightarrow \eta^3$He considering explicitly the $\eta^3$He rescattering. The square box in the first diagram indicates the full transition amplitude, while the circle in the second diagram stands for the bare transition amplitude prior to the $\eta^3$He final state interaction. It contains all diagrams that do not have $\eta^3$He as an intermediate state. The oval stands for the $\eta^3$He optical potential.

The $\eta^3$He scattering amplitude is given by the diagrams depicted in Fig. 2, and formally by

$$T = V + VGT,$$

where $V$ is the $\eta^3$He optical potential, which contains an imaginary part to account for the inelastic channels.

![Diagram 2](image2)

Fig. 2. Diagrammatic representation of the $\eta^3$He scattering matrix.

In many-body theory, it is known that at low densities, the optical potential is given by

$$V(\vec{r}) = 3t_{\eta N} \hat{\rho}(\vec{r}),$$

(2)
where \( t_{\eta N} \) is the forward \( \eta N \) amplitude and \( \tilde{\rho}(\vec{r}) \) is the \(^3\)He density normalized to unity. However, we use it only to provide the range of \( \eta \)-nucleus interaction, since the \( \eta \) can interact with all the nucleons in the nucleus distributed according to \( \rho(\vec{r}) \).

In momentum space, the potential is given by

\[
V\left(\vec{p}_\eta, \vec{p}'_\eta\right) = 3t_{\eta N} \int d^3 \vec{r} \tilde{\rho}(\vec{r}) e^{i(\vec{p}_\eta - \vec{p}'_\eta) \cdot \vec{r}}
\]

\[
= 3t_{\eta N} F\left(\vec{p}_\eta - \vec{p}'_\eta\right),
\]

where \( F(\vec{q}) \) is the \(^3\)He form factor with \( F(\vec{0}) = 1 \). A good approximation to this form factor at small momentum transfers is given by a Gaussian,

\[
F(\vec{q}) = e^{-\beta^2|\vec{q}|^2},
\]

where \( \beta^2 = \langle r^2 \rangle / 6 \), which gives \( \beta^2 = 13.7 \text{ GeV}^{-2} \).

After integrating over the angle between \( \vec{p}'_\eta \) and \( \vec{p}_\eta \), the s-wave projection of the optical potential becomes

\[
V\left(\vec{p}_\eta, \vec{p}'_\eta\right) = 3t_{\eta N} \frac{1}{2} \int_{-1}^{1} d \cos \theta e^{-\beta^2(|\vec{p}_\eta|^2 + |\vec{p}'_\eta|^2 - 2|\vec{p}_\eta||\vec{p}'_\eta| \cos \theta)}
\]

\[
= 3t_{\eta N} e^{-\beta^2|\vec{p}_\eta|^2} e^{-\beta^2|\vec{p}'_\eta|^2} \left[ 1 + \frac{1}{6} \left( 2\beta^2 |\vec{p}_\eta| |\vec{p}'_\eta| \right)^2 + \ldots \right].
\]

The term \( 2\beta^2|\vec{p}_\eta||\vec{p}'_\eta|/6 \) is negligible in the region, where \( e^{-\beta^2|\vec{p}_\eta|^2} e^{-\beta^2|\vec{p}'_\eta|^2} \) is sizeable and can be neglected, and this leads to a potential that is separable in the variables \( \vec{p}_\eta \) and \( \vec{p}'_\eta \), which makes the solution of Eq. (1) trivial.

Thus, we find, substituting \( 3t_{\eta N} \) by \( \tilde{V} \)

\[
\tilde{T}\left(\vec{p}_\eta, \vec{p}'_\eta\right) = \tilde{T} e^{-\beta^2|\vec{p}_\eta|^2} e^{-\beta^2|\vec{p}'_\eta|^2}.
\]

The Bethe–Salpeter equation becomes then algebraic

\[
\tilde{T} = \tilde{V} + \tilde{V} G \tilde{T},
\]

with

\[
G = \frac{M_{^3\text{He}}}{16\pi^3} \int \frac{d^3 \vec{q}}{\omega_{\eta}(\vec{q}) E_{^3\text{He}}(\vec{q})} \frac{e^{-2\beta^2|\vec{q}|^2}}{\sqrt{s - \omega_{\eta}(\vec{q}) - E_{^3\text{He}}(\vec{q}) + i\epsilon}}.
\]
3. Production amplitude in the s-wave

Following the formalism of Ref. [3], we write for the $pd \rightarrow \eta^3\text{He}$ transition depicted as a circle in Fig. 1

$$V_P = A\vec{\epsilon} \cdot \vec{p} + iB(\vec{\epsilon} \times \vec{\sigma}) \cdot \vec{p},$$

(9)

where $\vec{\epsilon}$ is the polarization of the deuteron, $\vec{\sigma}$ denotes the Pauli matrix standing for the spin of the proton, and $\vec{p}$ is the momentum in the initial state. The cross section then becomes

$$\sigma = \frac{m_p M_{^3\text{He}}}{12\pi s} \left| A' \right|^2 + 2 \left| B' \right|^2 |\vec{p}_\eta| |\vec{p}| e^{-2\beta^2 |\vec{p}_\eta|^2},$$

(10)

with

$$A' = \frac{A}{1 - VG}; \quad B' = \frac{B}{1 - VG}$$

(11)

which takes into account rescattering of the $\eta^3\text{He}$ in the denominators of $A'$ and $B'$.

In [4], we also took into account $p$-waves and s-wave–p-wave interference calculating the asymmetry parameter “$\alpha$”. Then we make a fit to the production cross section and the asymmetry parameter, and we find the results depicted in Figs. 3 and 4.

![Fig. 3. The fitted $dp \rightarrow \eta^3\text{He}$ total cross sections compared with experimental data [1, 2].](image-url)

With the optical potential obtained from the fit, we calculate the amplitude and find the results shown in Fig. 5. This corresponds to a local Breit–Wigner amplitude with binding and width given by

$$B_E = (0.30 \pm 0.10 \pm 0.08) \text{ MeV},$$

(12)

$$\Gamma = (3.0 \pm 0.5 \pm 0.7) \text{ MeV},$$

(13)
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Fig. 4. The fit in the model to the asymmetry parameter $\alpha$ as a function of the center-of-mass $\eta$ momentum $p_\eta$ compared with the experimental data [1, 2].

where the first errors are statistical and the second systematic. Yet, it is worth to note that if we look for poles, the pole appears at $Q = (1.5 - i0.7)$ MeV, i.e., in the unbound region.

Yet, since from the experimental point of view what matters is the amplitude in the real axis, the results obtained indicate that in some reaction one could see a structure below the threshold. The problem is always how big this signal would be compared to background from other sources, the general problem that has prevented so far the identification of such states.

In the discussion session some issues were clarified. In [4], it was mentioned that the theoretical calculations for light systems of [5] predict $B_E$ of around 1 MeV or less and $\Gamma = 15$ MeV for $\eta^3He$, which seems too big. Gal

Fig. 5. Real and imaginary parts of the $\eta^3He \to \eta^3He$ amplitude $T$ as a function of the excess energy $Q$. 
stated that, indeed, there was a mistake in this calculation which has been corrected in a recent paper [6]. On the other hand, if one associates $\tilde{V}$ with $3t\eta_N$, one can obtain an $\eta N$ scattering length

$$a'_{\eta N} = \left[ -(0.48 \pm 0.05) - i(0.18 \pm 0.02) \right] \text{ fm.} \quad (14)$$

Yet, Gal pointed out that in the version of the optical potential of [7] (see also [8]), the $t\rho$ potential should be multiplied by a factor $(A - 1)/A$, where $A$ is the mass number. Actually, it is more subtle since a new $t'$ matrix is defined, obtained from this optical potential with the Lippmann–Schwinger equation, but then the real $t$ should be obtained from $t'$ by multiplying by the inverse factor $A/(A - 1)$. Without entering into further details, if we ignore these subtle details one might think that following this approach we would get the $a'_{\eta N}$ scattering $3/2$ the result of Eq. (14), which would be perfectly acceptable. However, we should also remember that the optical potential can have two body contributions which would go beyond the $t\rho$ approximation. Note that we parametrized $\tilde{V}\rho$ to the data and did not make the assumption that it should be $t\rho$. On the other hand, there is a magnitude that comes straight from our calculations and this is the $\eta^3\text{He}$ scattering length for which we obtain

$$a_{\eta^3\text{He}} = \left[ (2.23 \pm 1.29) - i(4.89 \pm 0.57) \right] \text{ fm.} \quad (15)$$

Note that we are able to give the sign of $\eta^3\text{He}$, where in most analyses only the modulus of the real part could be provided.

4. States of $\rho\text{B}^*\bar{\text{B}}^*$ with $J = 3$

We briefly discuss here the results obtained for the system with a $\rho$ meson and the beauty vector mesons $B^*$ and $\bar{B}^*$ within the Fixed Center approximation to the Faddeev equations in [9]. Systems of this type have been studied before, and the most remarkable one is the case of multirho states with their spins aligned [10], which are identifiable up to an $f_6$ state. Similarly, there are also $K^*$ multirho states which are identifiable up to a $K^*_5$ [11]. In the present case, we assume the $B^*\bar{B}^*$ system forming a cluster, and in terms of the two-body $\rho B^*$ unitarized scattering amplitudes in the local Hidden Gauge approach [12], we find a new $I(J^{PC}) = 1(3^{--})$ state. The mass of the new state corresponds to a two-particle invariant mass of the $\rho B^*$ system close to the resonant energy of the $B_2^*(5747)$, indicating that the role of this $J = 2$ resonance is important in the dynamical generation of the new state. We refer the reader to [9] for details and quote here just the final result, which is the prediction of a state of mass $10987 \pm 40$ MeV and width $40 \pm 15$ MeV.
5. Conclusions

We have presented some information concerning the analysis of the \(pd \rightarrow \eta^3\)He reaction and the issue of the \(\eta^3\)He bound state. The novel approach used here allowed us to get an insight into the \(\eta^3\)He scattering matrix which, we found, behaves as a Breit–Wigner very close to threshold in the bound region. However, as a pole it is lightly unbound. While this latter finding agrees with some calculations, we reported on the new information of the behaviour of the amplitude below threshold.

The \(B^*\bar{B}^*\rho\) with their spins aligned was investigated here in analogy to multirho and \(K^*\) multirho states studied before. We found that there should be a resonance made out of these components with an energy around 11000 MeV with the total spin equal three.

REFERENCES