

ELASTIC NUCLEON–DEUTERON SCATTERING WITH THE NUCLEON–NUCLEON OPE-GAUSSIAN FORCE AT $E = 65$ MeV — INTRODUCTORY STUDIES*

YURIY VOLKOTRUB, ROMAN SKIBIŃSKI, JACEK GOLAK
KACPER TOPOLNICKI, HENRYK WITAŁA

The M. Smoluchowski Institute of Physics, Jagiellonian University
Łojasiewicza 11, 30-348 Kraków, Poland

(Received August 10, 2017)

We study the elastic nucleon–deuteron (Nd) scattering process at the incoming nucleon laboratory energy $E = 65$ MeV working within the formalism of Faddeev equations. We employ, for the first time in the elastic Nd scattering, the OPE-Gaussian nucleon–nucleon (NN) potential and confirm its high quality by comparing our predictions for the differential cross section with results based on the AV18 potential as well as with available data. We also estimate the theoretical uncertainty of this observable originating from uncertainties of the OPE-Gaussian model parameters. We find the relative uncertainties to be smaller than 0.8% for the differential cross section. The correlations between various parameters of the OPE-Gaussian are also shown.

DOI:10.5506/APhysPolB.48.1995

1. Introduction

Various models of the NN potential have been derived up to now. They usually combine the approach based on the meson-exchange picture and/or on the chiral Lagrangian with more phenomenological paths. All modern models are constructed in such a way that the NN phase shifts and the deuteron properties are described with high precision. The AV18 model [1] — a semi-phenomenological force whose long-range part, given by the one-pion exchange (OPE), is supplemented by a purely phenomenological short-range term — can be a good example. That short-range part is given by a set of operators (like *e.g.* 1 , $\vec{L} \cdot \vec{S}$, L^2 , S_{12} , *etc.*) accompanied by radial functions, *e.g.* of the Saxon–Woods type.

* Presented at the 2nd Jagiellonian Symposium on Fundamental and Applied Subatomic Physics, Kraków, Poland, June 3–11, 2017.

A reliable estimation of theoretical uncertainties is becoming an increasingly important task in nuclear physics. These uncertainties can be partially related to uncertainties of the NN interaction, which in turn depend on the finite accuracy of the experimental data. In the case of predictions based on the chiral forces, also the so-called truncation errors have to be taken into account. Such errors have been studied *e.g.* in [2] and in [3], where a general prescription for estimating such errors has been proposed. It is also possible to estimate uncertainties related with a given numerical scheme, and here many computer science methods can be used [4]. The propagation of theoretical uncertainties from the NN force to many-body observables is an open question.

The newly developed OPE-Gaussian potential [5] offers a unique opportunity to study this issue. This is because of the intense attention paid by the authors of [5] to the determination of statistically well-defined uncertainties of the potential parameters. The OPE-Gaussian force, similarly to the AV18 interaction, can be decomposed to the long-range and short-range components

$$V(\vec{r}) = V_{\text{short}}(\vec{r})\theta(r_c - r) + V_{\text{long}}(\vec{r})\theta(r - r_c), \quad (1)$$

where $r_c = 3$ fm and the V_{long} part contains the OPE part and electromagnetic corrections for the proton–proton interaction. The short-range component is built from $\hat{O}_1, \dots, \hat{O}_{18}$ operators, from which 16 are the same as in the AV18 model, see [6] for details. It has a form of

$$V_{\text{short}}(\vec{r}) = \sum_{n=1}^{18} \hat{O}_n \left[\sum_{i=1}^4 V_{i,n} F_{i,n}(r) \right], \quad (2)$$

where $F_{i,n}(r)$ are radial Gaussian functions which depend on a free parameter a_0 defining their widths and strength parameters $V_{i,n}$ which are fixed from the NN data. Note that in Ref. [5], the above given sum contains 21 operators but in the final expression three of them are dropped.

To study Nd scattering, we use the formalism of Faddeev equations which belongs to standard techniques used to investigate three-nucleon ($3N$) reactions. Our approach is described in detail *e.g.* in [7, 8]. In the present work, we neglect the $3N$ interaction and apply only the two-body force, which enters the Faddeev equation via the t-matrix operator. That operator and the free $3N$ propagator are used to obtain the complete transition amplitude, from which any observable for this process can be calculated. Our numerical solution is obtained using $3N$ partial waves, and we take into account all states with the two-body angular momentum j up to $j = 5$ and the three-body total angular momentum J up to $J = \frac{25}{2}$.

2. The correlation matrix

The authors of Ref. [5] carefully analyzed the NN data and constructed the “ 3σ self-consistent” database. This allowed them, by means of a fitting procedure, to obtain not only the central values of the potential parameters but also their correlation coefficients. More precisely, the fitting procedure was applied to the potential parameters on the partial wave basis. From these parameters, the central values and the covariance matrix for the $V_{i,n}$ and a_0 parameters can be calculated. In Fig. 6 of [5], the genuine correlation matrix for potential parameters on the partial wave basis is shown. We have been equipped by authors of Ref. [5] with a sample of 50 sets of potential parameters what allowed us to estimate the correlation matrix directly for $V_{i,n}$ and a_0 parameters (avoiding any partial wave decomposition). This matrix is shown in Fig. 1. The tick labels denote the operators numbered as in Eqs. (A4) and (A5) of [5] and there are four strength parameters for the Gaussian functions $V_{1\dots 4,n}$ for each operator \hat{O}_n . This creates the 4×4 submatrices shown in Fig. 1. We observe that, beside the obvious strong correlations between $V_{i,n}$ for a given operator \hat{O}_n , there are also strong correlations between various operators *e.g.* \hat{O}_1 (the unit operator) and \hat{O}_{15} (T_{12}). The interested reader can obtain more precise values of the correlations parameters directly from Fig. 1.

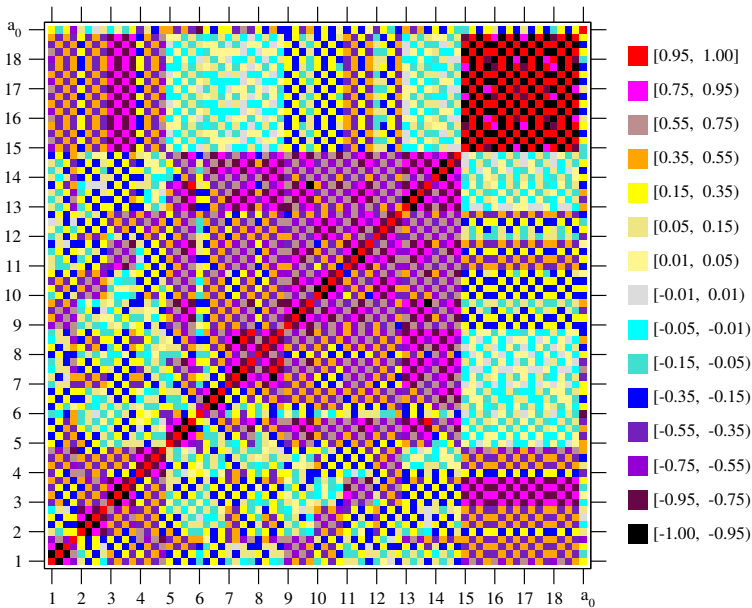


Fig. 1. The correlation matrix for the $V_{i,n}$ and a_0 parameters of the OPE-Gaussian model. For tick labels, see the text.

3. The differential cross section

In Fig. 2, we show predictions for the differential cross section for elastic Nd scattering at the incoming nucleon laboratory energy $E = 65$ MeV. The dark gray/red band comprising 50 curves based on different sets of potential parameters is hardly visible since it overlaps with the black curve representing predictions obtained with the central values of the parameters. The width of the band remains smaller than 1.6% of the magnitude of the cross section at a given scattering angle. The OPE-Gaussian predictions are close to the ones obtained with the AV18 force (dashed/blue curve) and to the data. Thus, we conclude that basing on the differential cross section only, it is impossible to judge if the new force delivers a better data description than the other models. A study of polarization observables using the OPE-Gaussian force is in progress.

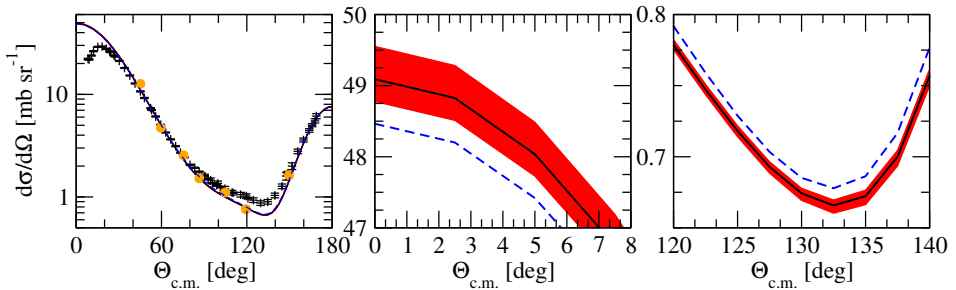


Fig. 2. (Color online) The differential cross section for elastic Nd scattering at incoming nucleon laboratory energy $E = 65$ MeV. Graphs show the same cross section but at various ranges of the scattering angle $\Theta_{\text{c.m.}}$. For the description of the band and curves, see the text. Data are from [9] (pluses) and [10] (circles).

We thank Dr. E. Ruiz Arriola and Dr. R. Navarro Pérez for sending us sets of parameters for the OPE-Gaussian model and for the valuable discussions. This work was supported by the National Science Center, Poland (NCN) under grants Nos. 2016/22/M/ST2/00173 and DEC-2016/21/D/ST2/01120. The numerical calculations were performed on the supercomputer cluster of the JSC, Jülich, Germany.

REFERENCES

- [1] R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, *Phys. Rev. C* **51**, 38 (1995).
- [2] R.J. Furnstahl, N. Klco, D.R. Phillips, S. Wesolowski, *Phys. Rev. C* **92**, 024005 (2015).

- [3] S. Binder *et al.*, *Phys. Rev. C* **93**, 044002 (2016).
- [4] M.L. Overton, *Numerical Computing with IEEE Floating Point Arithmetic*, SIAM, Philadelphia 2001.
- [5] R. Navarro Pérez, J.E. Amaro, E. Ruiz Arriola, *Phys. Rev. C* **89**, 064006 (2014).
- [6] R. Navarro Pérez, J.E. Amaro, E. Ruiz Arriola, *Phys. Rev. C* **88**, 064002 (2013) [*Erratum ibid.* **91**, 029901 (2015)].
- [7] W. Glöckle *et al.*, *Phys. Rep.* **274**, 107 (1996).
- [8] H. Witała *et al.*, *Phys. Rev. C* **63**, 024007 (2001).
- [9] S. Shimizu *et al.*, *Phys. Rev. C* **52**, 1193 (1995).
- [10] H. Rühl *et al.*, *Nucl. Phys. A* **524**, 377 (1991).