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# SINGLE PION PRODUCTION INDUCED BY $\nu$-NUCLEON INTERACTIONS[*]

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This article presents a short review of the single pion production (SPP) in the neutrino–nucleon scattering. The attention is focused on the discussion of the main difficulties in modeling the SPP processes. New physical observables, which may constrain the theoretical models, are proposed.

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## 1. Motivation

Last years a big effort has been made to investigate the basic properties of neutrinos. However, a further progress in studies of the neutrino oscillation phenomenon, the CP violation in lepton sector requires headway in the knowledge of the neutrino–nucleon ($\nu N$) and the neutrino–nucleus ($\nu A$) scattering cross sections [1]. Indeed, a lack of accuracy in the predictions of the $\nu N$ and the $\nu A$ scattering cross sections results in a large systematic uncertainty for the measurement of the neutrino oscillation parameters and the CP-violation phase [2].

In this article, the single pion production (SPP) in the $\nu N$ scattering is considered. For the sake of simplicity, our attention is concentrated on the charged current (CC) interactions of the muon neutrinos with the nucleons, namely,

$$\nu_\mu + N \to \mu^- + N' + \pi \,, \qquad (1)$$

where $N$ and $N'$ denotes incoming and outgoing nucleon respectively.

The first theoretical works dedicated to the SPP were published in sixties [3, 4]. The new interest in the topic has been initiated by the development of the long and short baseline oscillation experiments [5, 6] with accelerator

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source of neutrinos. Indeed, the knowledge of the SPP processes is necessary for performing neutrino oscillation analyses. On the other hand, investigation of the pion production induced by interactions of the neutrinos with the nucleons gives opportunity to study weak excitations of the nucleon to the resonance states.

## 2. Basic properties

The pions occurred in process (1) can be produced through the resonant and nonresonant mechanisms. In the first case, the nucleon, $N$, is excited to resonant state, $N^*$, which subsequently decays to $\pi N$ system. In the other, there is no intermediate resonance state. In principle, to describe the $N \to N^*$ transition amplitude, one should take into consideration the resonances from the first, second and third resonance regions. In this paper, we consider the scattering processes in which the neutrino has the energy $E \sim 1$ GeV, which is a typical kinematic range of the neutrino oscillation experiments with the accelerator source. Hence, to describe the resonant contribution to the SPP, it is enough to include only

$$W^+N \to \mathit{\Delta}(1232) \to \pi N' \tag{2}$$

transition. This property is illustrated in Fig. 1 where the SPP cross sections for various CC processes and for neutrino energy $E = 1$ GeV and $E = 3$ GeV are plotted. The most difficult task in modeling the SPP is to construct the consistent theoretical model for the resonant and the nonresonant contributions.

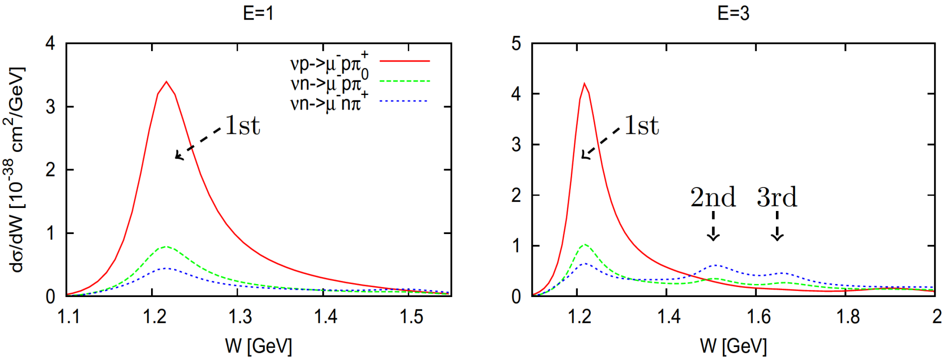

Fig. 1. Differential cross section ($W$ is the hadronic invariant mass) for the SPP induced by the $\nu N$ interaction, calculated for neutrino energy 1 GeV (left figure) and 3 GeV (right figure). Results obtained within the Rein–Sehgal model [7] updated in [8, 9]. The first, second, and third resonance regions are indicated by the arrows.

The $\Delta(1232)$ is 3/2 spin particle. Therefore, it can be described by the Rarita–Schwinger field $\Psi_\mu$ [15]. Then the amplitude for $N \to \Delta(1232)$ excitation is given by the weak current which has a vector-axial ($J^\mu = J^\mu_{\rm V} + J^\mu_{\rm A}$) structure. While the form factors of the vector part are obtained from the analysis of the electroproduction and photoproduction data [13], the axial form factors are extracted from the neutrino–nucleon scattering data.

The axial part of the transition matrix element for (2) reads

$$\langle \Delta(p+q) \mid J^\mu_{\rm A} \mid N(p)\rangle = \Psi_\nu(p+q)\left(\frac{C_3^{\rm A}}{M}\left(g^{\nu\mu}\not{q} - q^\nu\gamma^\mu\right)\right.$$
$$\left.\frac{C_4^{\rm A}}{M^2}\left(g^{\nu\mu} q\cdot(p+q) - q^\nu(p^\mu+q^\mu)\right) + g^{\nu\mu}C_5^{\rm A} + \frac{C_6^{\rm A}}{M^2}q^\nu q^\mu\right)\gamma_5 u(p)\,, \quad (3)$$

where $M$ is the nucleon mass, $g_{\mu\nu}$ is the metric in Minkowski space and $u(p)$ is the nucleon field in the momentum space.

The axial form factors in (3) are obtained from the analysis of data collected in two bubble chamber experiments: ANL [17] and BNL [18]. Some time ago, it was believed that the SPP data of the ANL and the BNL experiments are inconsistent. The problem has been studied by several groups. It has been shown that it is possible to get a consistent (statistically) model which reproduces [11, 12, 19] the ANL and BNL data[1].

However, the experimental data are not enough informative to obtain four independent fits of all axial form factors [21]. Hence, some simplifications are made [22]. Namely, the axial current (3) is expressed by only $C_5^{\rm A}(Q^2)$, where it is assumed that $C_3^{\rm A} = 0$, and $C_4^{\rm A}$ as well as $C_6^{\rm A}$ are proportional to $C_5^{\rm A}$. Unfortunately, the extraction of the $C_5^{\rm A}$ from the data is quite model-dependent. It is shown in Fig. 2 where we plot fits of $C_5^{\rm A}$ obtained from the analysis of the scattering data within various theoretical models. The differences between fits are caused by:

*(i)* details of the statistical model, see [11];

*(ii)* different treatment of the $NW^+\Delta$ vertex;

*(iii)* details of the model for the $\Delta(1232)$ propagation, see the discussion in [14];

*(iv)* treatment of the nonresonant contribution.

Obviously points *(ii)*–*(iv)* are correlated — the particular choice of the model for resonant production affects the construction of the nonresonant terms.

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[1] The consistency was demonstrated only for the $\nu_\mu p \to \mu^-\pi^+ p$ channel data. Showing the consistency of the measurements for other charged current channels is problematic [13, 14, 20].

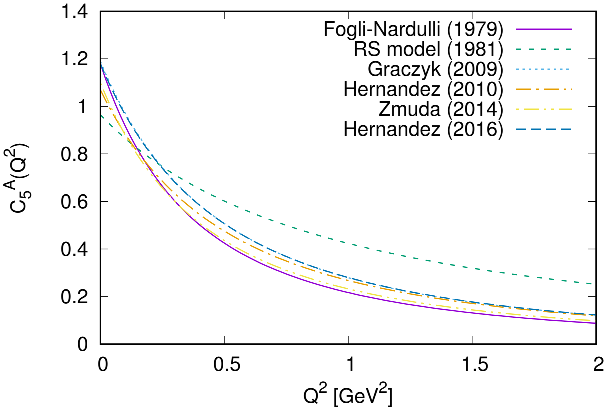

Fig. 2. $C_5^{\mathrm{A}}$ axial form factor for weak $N \to \Delta(1232)$ transition. The fits are taken from: Fogli–Nardulli (1979) [10], RS model (1981) [7], Graczyk (2009) [11], Hernandez (2010) [12], Żmuda (2014) [13], Hernandez (2016) [14].

## 3. Resonant versus nonresonant

There are many phenomenological approaches dedicated to the SPP in neutrino–nucleon interactions [23–26] (for more complete list, see [2, 27]) but in the experimental analyses still the Rein–Sehgal (RS) model [7] is in the usage. It is implemented in almost all Monte Carlo (MC) neutrino event generators except NuWro [28].

The RS approach is based on the relativistic quark model formulated by Feynman *et al.* [30]. The resonance contribution is given by transition amplitudes for weak excitation of the nucleon to 18 different resonance states belonging to the first, second and third resonance regions. However, the RS model, in its original form, describes only effectively the nonresonant contribution. Moreover, the resonant transition amplitudes have oversimplified form [8] and do not describe the properties of $N \to N^*$ transition in details. Certainly, in modern experimental analyses of the scattering data, the RS model should be replaced by more consistent description of the SPP processes.

As it has been mentioned, the main problem is to combine properly the nonresonant and resonant contributions. A choice of the resonant dynamics: parametrization of the $W^+N\Delta$ vertex, a form of the propagator of $\Delta(1232)$ *etc.* reflects in the description of nonresonant contribution [14]. The nonresonant amplitudes can be modeled by some number of diagrams [10, 31] motivated by chiral symmetry [29]. The degrees of freedom of the model are nucleons, resonances and pions.

If one concentrates the attention only on the SPP total and single differential cross sections, some important differences between models are not visible. They are simply integrated out. It is illustrated in Fig. 3 where we show how small modifications of the nonresonant part affect the cross section. We plot predictions of two models: Fogli–Nardulli (FN) [10] and Nieves *et al.* (HNV) [29]. In the first one, the nonresonant and resonant contributions are described by four diagrams. The model is simple but it re-

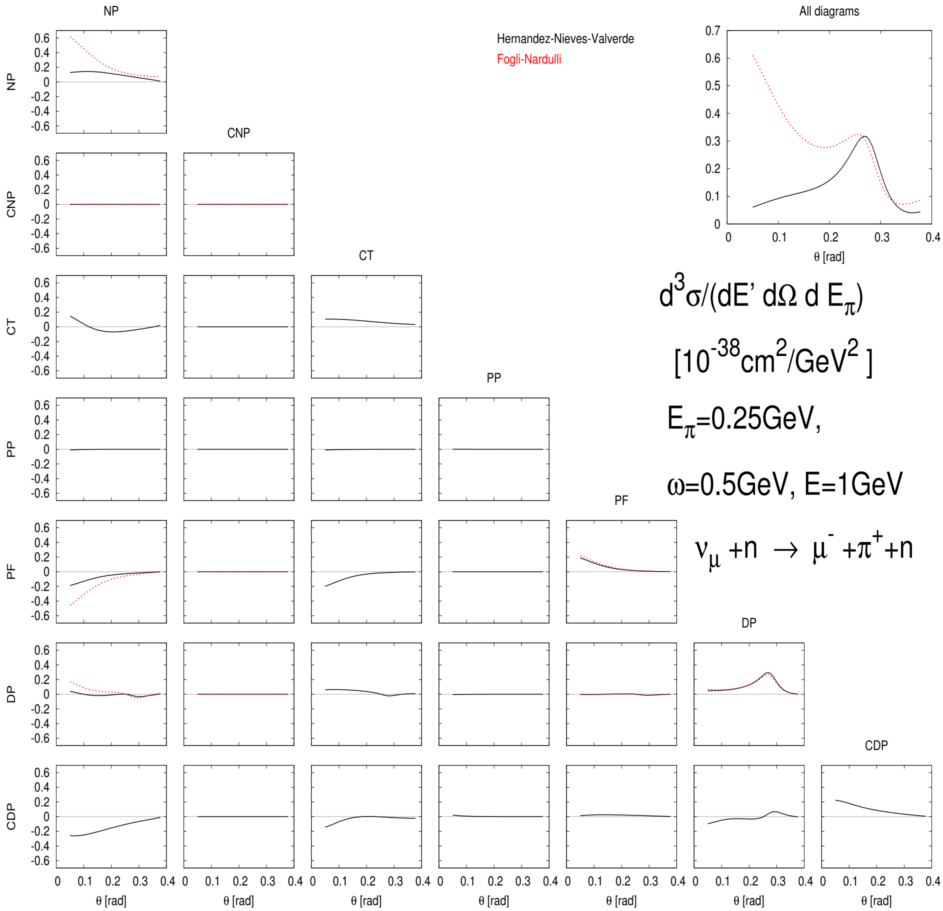

Fig. 3. (Color online) Separation of the differential cross section into various components. The solid/black and dashed/red lines represent the predictions of the HNV [29] and FN [10] models respectively. To denote various diagrams contributions, we used the same abbreviations as in Ref. [16]. The contribution from the $|\mathcal{A}_i|^2$ are on the diagonal, below the diagonal the interference terms, $2\Re(\mathcal{A}_i\mathcal{A}_j^*)$ ($i$ — column, $j$ — row), are shown. The energy transfer is denoted by $\omega$, while $E_\pi$ is the pion energy, $\theta$ angle is in $\pi$ units.

produces the total cross section and $\mathrm{d}\sigma/\mathrm{d}Q^2$ data with reasonable accuracy. In the HNV model, the number of diagrams is larger and equal to seven — all contributions allowed by the gauge invariance are included. This model also reproduces well the experimental measurements. However, if one compares triple differential cross sections calculated at some kinematics, the interesting differences are visible. Namely, at low scattering angle, the cross section of the FN model rapidly diverges in contrast to the predictions of the HNV model. It is the result of the lack of the crossed $\Delta$-pole term in the FN model.

It is rather obvious that for more critical and detailed studies of the SPP, one needs to have new precise measurements of the interaction of the neutrinos with the free nucleon target. In particular, proposal of new observables which contain nontrivial information about the resonance and nonresonance contributions is wanted. We believe that the measurement of the polarizations of the final particles in (1) may deliver such new information.

In our latest work [32], we propose to investigate the polarization transfer observables like a polarization of the charged lepton. Figure 4 presents the predictions of the normal component of the polarization of the $\mu^-$ lepton and its degree of polarization. The first quantity vanishes if only $\Delta$-pole resonant contribution is taken into account. Indeed, the normal component of the polarization is given by the interferences between various amplitudes. The degree of polarization of the muon is also sensitive on the details of the SPP model. Hence, it seems that investigation of the polarization transfer observables may bring valuable information about features of the SPP processes.

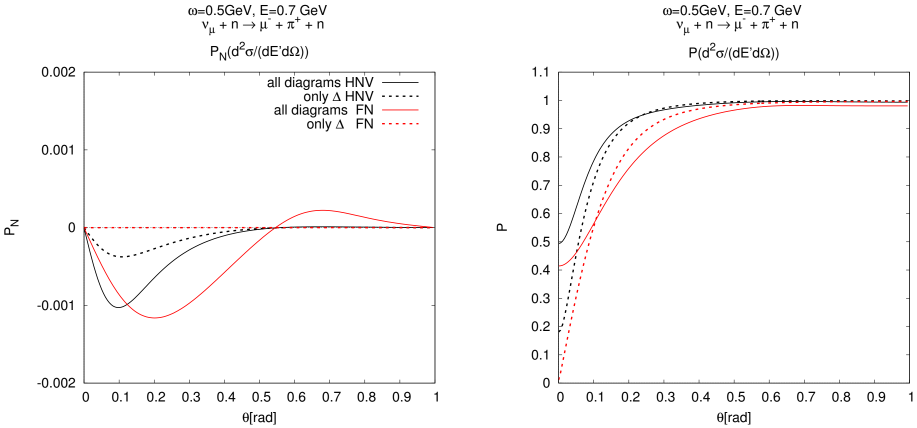

Fig. 4. The angular dependence ($\theta$ is the scattering angle) of the normal component of the polarization (left figure) and the degree of polarization (right figure) of the $\mu^-$. The predictions are obtained within HNV [29] and FN [10] models. By $\omega$ the energy transfer is denoted, $\theta$ angle is in $\pi$ units.

## 4. Summary

We have shortly reviewed the single pion production induced by neutrino scattering off the nucleons. The main difficulties have been sketched. Eventually, we have proposed the physical observables which contain new information about interference of the resonant and nonresonant contributions.

The calculations have been carried out in Wrocław Centre for Networking and Supercomputing (`http://www.wcss.wroc.pl`), grant No. 268. A part of the algebraic calculations presented in this talk has been performed using FORM language [33].

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