# OPERATOR FORM OF THE THREE-NUCLEON SCATTERING AMPLITUDE*

Kacper Topolnicki, Jacek Golak, Roman Skibiński
Henryk Witała, Yuriy Volkotrub

M. Smoluchowski Institute of Physics, Jagiellonian University
Łojasiewicza 11, 30-348 Kraków, Poland

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We present a general form of the three-nucleon scattering amplitude. Our result is an operator form in which the scattering amplitude is written as a linear combination of scalar functions and operators acting on spin states. Using this form greatly reduces the numerical complexity of the so-called, *three dimensional* treatment of the Faddeev equations and can potentially lead to more accurate calculations of scattering observables at higher energies.

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## 1. Introduction

A general operator form of the three-nucleon ($3N$) scattering amplitude has potential applications in calculations that employ the so-called *three dimensional* (3D) formalism to calculate observables in the nucleon–deuteron scattering process. Solutions of the relevant equations using first order terms in the nucleon–nucleon transition operator were obtained in [1] and demonstrated that for certain kinematical configurations, the precision of the 3D calculations is better than the traditional partial wave approach. This observation motivates the development of a full 3D calculation.

In our approach, we use the Faddeev equation

$$\check{T} = \check{t}\check{P} + \check{t}\check{G}_0\check{P}\check{T}\,, \tag{1}$$

where $\check{T}$ is the $3N$ transition operator, $\check{t}$ is the two-nucleon transition operator, $\check{G}_0$ is the free propagator, and $\check{P}$ is a permutation operator composed from particle transpositions $\check{P} = \check{P}_{12}\check{P}_{23} + \check{P}_{13}\check{P}_{23}$. Observables for the $3N$

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elastic scattering and breakup processes are constructed from two types of matrix elements containing the same initial state $\mid \phi\rangle$ with a deuteron and a free nucleon with the relative momentum $\boldsymbol{q}_0$. In elastic scattering, the final state $\langle\phi' \mid$ corresponds to a deuteron and a free nucleon with the relative momentum $\boldsymbol{q}_0'$ and in the breakup, the final state $\langle\phi_0 \mid$ describes three free particles. For the latter, observables are constructed from

$$A^{\rm breakup} = \langle\phi_0 \mid \left(\check{1} + \check{P}\right) \check{T} \mid \phi\rangle \,, \tag{2}$$

while for the elastic channel from

$$A^{\rm elastic} = \langle\phi' \mid \check{P}\check{G}_0^{-1} + \check{P}\check{T} \mid \phi\rangle \,. \tag{3}$$

Looking at (2) and (3), it can be concluded that, in order to describe both processes at a given energy, we only need to solve equation (1) for the state $\check{T} \mid \phi\rangle$.

In the 3D approach, quantum mechanical operators are represented without using angular momentum decomposition. Instead, the *three-dimensional* momentum degrees of freedom of the nucleons are used. In practice, this means that we will be interested in matrix elements of the scattering amplitude $\langle\boldsymbol{p}\boldsymbol{q} \mid \check{T} \mid \phi\rangle$, where $\boldsymbol{p}$ and $\boldsymbol{q}$ are *three-dimensional* Jacobi momentum vectors. Considerations of numerical complexity lead to the conclusion that spatial rotation symmetry must be taken into account by employing the newly developed general form of the scattering amplitude [2] in order to create a practical numerical realization. Since there are 8 possible spin and 8 possible isospin states for the $3N$ system, a naïve numerical representation of $\langle\boldsymbol{p}\boldsymbol{q} \mid \check{T} \mid \phi\rangle$ requires knowledge of $8\times 8 = 64$ complex numbers for every $\boldsymbol{p}$ and $\boldsymbol{q}$. If each component of the momentum vectors is discretized over a grid of 32 points, then the numerical representation of the scattering amplitude would require $\approx 10^{15}$ complex numbers. This is clearly unfeasable. However, by utilizing the general form of the $3N$ scattering amplitude [2], this large number can be reduced to $\approx 10^{11}$. More details on the numerical complexity of the problem and the general form of the $3N$ scattering amplitude can be found in [2]. In the following, we briefly discuss the operator form of $\check{T} \mid \phi\rangle$ following the considerations in [2].

## 2. Rotation invariance

Considering the $\check{T} \mid \phi\rangle$ state in more detail, it is easy to work out that when projected onto a final momentum eigenstate $\langle\boldsymbol{p}\boldsymbol{q} \mid$, with $\boldsymbol{p}$, $\boldsymbol{q}$ being Jacobi momenta, it has the following general form:

$$\langle\boldsymbol{p}\boldsymbol{q} \mid \check{T} \mid \phi\rangle = \left[\check{X}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)\right]^{\rm is} \mid s\rangle \,. \tag{4}$$

In (4), $\left[\check{X}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)\right]^{\rm is}$ is an isospin–spin operator that can depend on the final Jacobi momenta $\boldsymbol{p}$, $\boldsymbol{q}$ and on the free nucleon momentum $\boldsymbol{q}_0$. Further, $|\, s\rangle$ is a pure $3N$ isospin–spin state. Square brackets are used to denote the matrix representation for operators in the spin (s) or isospin–spin (is) space.

Symmetry with respect to spatial and isospin rotations results in $\check{X}$ having a rotational symmetry. This allows us to use the algorithm from [3] to find the general operator form of $\check{X}$. First, we split this operator into isospin and spin components $\check{X} = \check{X}^{\rm isospin} \otimes \check{X}^{\rm spin}$. Symmetry of $\check{X}$ with respect to isospin rotations is achieved by using scalar combinations of the three single nucleon isospin operators acting in the spaces of particles 1, 2 and 3. Symmetry with respect to spatial rotations is more complicated since the spin part, $\check{X}^{\rm spin}$, can depend on three momenta. The algorithm from [3] takes, as input, the building blocks of the operator $\check{X}^{\rm spin}$, in this case, the Jacobi momenta $\boldsymbol{p}$, $\boldsymbol{q}$ in the final state, the free nucleon momentum $\boldsymbol{q}_0$ and the vectors of spin operators $\check{\boldsymbol{\sigma}}(1)$, $\check{\boldsymbol{\sigma}}(2)$, $\check{\boldsymbol{\sigma}}(3)$ acting in the spaces of particles 1, 2 and 3. The resulting general form of $\check{X}^{\rm spin}$ that is invariant under spatial rotations reads

$$\left[\check{X}^{\rm spin}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)\right]^{\rm s} = \sum_{r=1}^{64} x_r(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)\left[\check{O}_r(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)\right]^{\rm is}, \tag{5}$$

where $\check{X}^{\rm spin}$ is defined by the scalar functions $x_r$ and the operators $\check{O}_r$ are [2]

$$\begin{aligned}
&\hat{O}_1(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = 1\,, && \hat{O}_{21}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{q}_0\times\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_2(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\,, && \hat{O}_{22}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{q}_0\times\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_3(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(2)\,, && \hat{O}_{23}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \hat{\boldsymbol{\sigma}}(1)\times\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_4(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(3)\,, && \hat{O}_{24}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\;\boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(2)\,,\\
&\hat{O}_5(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(1)\,, && \hat{O}_{25}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\;\boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_6(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\,, && \hat{O}_{26}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\;\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\,,\\
&\hat{O}_7(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(3)\,, && \hat{O}_{27}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\;\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_8(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(1)\,, && \hat{O}_{28}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\;\boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(2)\,,\\
&\hat{O}_9(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(2)\,, && \hat{O}_{29}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\;\boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_{10}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(3)\,, && \hat{O}_{30}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\;\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_{11}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(2)\,, && \hat{O}_{31}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\;\boldsymbol{p}\times\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_{12}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(3)\,, && \hat{O}_{32}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\;\boldsymbol{q}\times\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_{13}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,, && \hat{O}_{33}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\;\boldsymbol{q}_0\times\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_{14}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\times\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(2)\,, && \hat{O}_{34}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\;\hat{\boldsymbol{\sigma}}(1)\times\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_{15}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\times\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(3)\,, && \hat{O}_{35}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(2)\;\boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_{16}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\times\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,, && \hat{O}_{36}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(2)\;\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_{17}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{q}\times\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(2)\,, && \hat{O}_{37}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(2)\;\boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_{18}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{q}\times\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(3)\,, && \hat{O}_{38}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(2)\;\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_{19}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{q}\times\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,, && \hat{O}_{39}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(2)\;\boldsymbol{p}\times\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(3)\,,\\
&\hat{O}_{20}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{q}_0\times\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(2)\,, && \hat{O}_{40}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) = \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(2)\;\boldsymbol{q}\times\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(3)\,,
\end{aligned}$$

$$\hat{O}_{41}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{q}_0\times\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{42}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\,,$$
$$\hat{O}_{43}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{44}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(2)\,,$$
$$\hat{O}_{45}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{46}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(1)\ \hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{47}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{p}\times\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{48}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{q}\times\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{49}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{q}_0\times\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{50}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(1)\ \hat{\boldsymbol{\sigma}}(1)\times\hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{51}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{52}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{53}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\ \hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{54}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{p}\times\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{55}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{q}\times\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{56}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{q}_0\times\hat{\boldsymbol{\sigma}}(1)\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{57}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(1)\ \hat{\boldsymbol{\sigma}}(2)\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{58}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{59}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{60}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{61}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{62}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{p}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{63}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(3)\,,$$
$$\hat{O}_{64}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)=\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(1)\ \boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}(2)\ \boldsymbol{q}_0\cdot\hat{\boldsymbol{\sigma}}(3)\,.$$

Using (5) in [2], we proposed to write the scattering amplitude $\langle \boldsymbol{pq} \mid \check{T} \mid \phi\rangle$ in the general form

$$\left[\langle \boldsymbol{p},\boldsymbol{q} \mid \check{T} \mid \phi\rangle\right]^{\mathrm{is}} = \sum_{\gamma}\sum_{r=1}^{64} \tau_r^{\gamma}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) \mid \gamma\rangle \otimes \left(\left[\check{O}_r(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0)^{\mathrm{s}}\right] \mid s\rangle\right)\,, \tag{6}$$

where $\mid \gamma\rangle$ is one of the eight possible isospin states of the $3N$ system, $\mid s\rangle$ is a pure $3N$ spin state, and the amplitude is defined by the scalar functions $\tau_r^{\gamma}$. We also discussed the arguments of these scalar functions and suggested, following [4], to use

$$\tau_r^{\gamma}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{q}_0) \equiv \tau_r^{\gamma}\left(\boldsymbol{p}^2,\boldsymbol{q}^2,\boldsymbol{q}_0^2,\widehat{\boldsymbol{q}_0\times\boldsymbol{q}}\cdot\widehat{\boldsymbol{q}_0\times\boldsymbol{p}},\boldsymbol{q}\cdot\boldsymbol{q}_0,\boldsymbol{q}_0\cdot\boldsymbol{p}\right)\,. \tag{7}$$

Since $\tau_r^{\gamma}$ are scalar functions, they depend on only six real arguments which leads to a significant reduction of numerical work in comparison to the naïve representation of $\langle \boldsymbol{pq} \mid \check{T} \mid \phi\rangle$ mentioned in the introduction.

The operator form (6) can be plugged into the Faddeev equation (1). Next, the spin dependecies can be removed and the Faddeev equation can be rewritten as a set of coupled linear equations for the scalar functions $\tau_r^{\gamma}$ that define the scattering amplitude. Details on this procedure are given in [2]. Solving the resulting linear equations requires the carefull numerical treatment of the so-called moving singularities resulting from the singular behaviour of the free propagator and two-nucleon ($2N$) transition operator at the deuteron binding energy. More details on the moving singularities and their treatment can be found in [5].

## 3. Summary and outlook

In [2], we showed that in order to construct a practical, 3D realization of calculations for nucleon–deuteron scattering, the general operator form of the scattering amplitude (6) should be used. The discussion in this paper assumes that only two nucleon interactions are present. Additionally, we do not consider any relativistic corrections to the scattering amplitude.

Unfortunately, in order to have a good description of experimental data, $3N$ forces should be included in the calculations [8–13]. Furthermore, we expect the direct 3D treatment of the momentum degrees of freedom to have the largest benefits over traditional calculations that use angular momentum decomposition at high energies, where many partial waves need to be taken into account in order to achieve convergence. Both of these issues determine the direction of our future work.

Some steps towards including the $3N$ force were already taken in [6] where we developed the general operator form of the non-local $3N$ potential; the operator form of the local $3N$ force is available in [7]. Our ambition is also to include relativistic corrections into the calculations, thus significantly extending the energy range of our calculations. This is still a distant goal, however, we believe the general form of the $3N$ scattering amplitude can easily be extended to facilitate relativistic corrections. We expect that this can be achieved by including the dependence on the total momentum of the $3N$ system in the scattering amplitude.

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