THE DARK SIDE OF A BOSE GAS* **

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We consider the possibility that a single scalar extension of the Standard Model can be used to account for the presence of dark matter. We consider such an extension where the dark sector has a global U(1) symmetry, in which case dark matter can exhibit Bose–Einstein condensation, even when relativistic. We show that a condensate indeed forms at sufficiently early times for all masses, but that consistency and observational constraints imply that the condensate persists at present only for masses in the 10^{-12} eV region. We also briefly discuss constraints derived from relic abundance and direct detection limits.

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1. Introduction

The latest cosmic census indicates that about 23% of the matter of the universe is composed of a new type of non-luminous (dark) matter (DM). All information about the properties of DM is derived form its gravitational effects [1] that provide few constraints on the details of the DM Lagrangian, allowing for a vast number of DM models. Some of the simplest are based on the assumption that the dark and Standard Model (SM) sectors interact via the exchange of one or more mediators, which are neutral with respect to all SM and DM internal symmetries (it is usual to assume that the dark sector has some internal, stabilizing symmetry that ensures the stability of DM, and explains the presence of a relic abundance).

The simplest DM model assumes a dark sector populated only by a scalar particle χ that couples to the SM via the so-called Higgs portal $|\chi|^2|\phi|^2$, where ϕ denotes the SM scalar isodoublet. Many models assume χ is a real field, but here we assume that it is complex, and that the dark sector stabilizing symmetry is a (global) dark U(1); as a consequence, the dark

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sector carries a conserved charge. We use this model as a simple system that can be used to study possible collective effects of bosonic DM (a significant amount of work in this direction already exists [2]).

2. The model

This Lagrangian that we use is

$$\mathcal{L} = |\partial\chi|^2 - m_{\rm be}{}^2|\chi|^2 - \tfrac{1}{2}\lambda_{\rm be}|\chi|^4 - \epsilon|\chi|^2|\phi|^2 + \mathcal{L}_{\rm sm}\,, \tag{1}$$

where ϵ is the portal coupling constrained by $-\epsilon < \sqrt{\lambda_{\rm be}\lambda_{\rm sm}}$ (where $\mathcal{L}_{\rm sm} \supset -\lambda_{\rm sm}|\phi|^4/2$ is the isodoublet quartic coupling). We will consider a wide range for $m_{\rm be}$: from 10^{-20} eV to 1 TeV, ignoring naturality concerns.

In the early universe, this extension of the SM can be described as two statistical systems: DM$_\chi$ and the SM. The thermodynamics of DM$_\chi$ is determined by $\lambda_{\rm be}$, and we assume large enough so that the dark sector is in equilibrium with a well-defined temperature T_χ; similarly, we assume the SM is in equilibrium with a temperature T (T and T_χ are time-dependent). The SM and DM$_\chi$ will be in equilibrium depending on the strength of the portal coupling ϵ and the Hubble rate $\mathbb{H}$. Associated with the DM$_\chi$ conserved charge there will be, in general, a non-zero chemical potential μ, so this is a simple example of asymmetric DM.

Concerning the geometry of the universe, we will assume that it is flat, homogeneous and isotropic.

3. Cosmology

To lowest order the occupation numbers for the DM particles and antiparticles are given by ($x = m_{\rm be}/T\,,\ \varpi = \mu/m_{\rm be}$)

$$n_{\rm be}^{\pm} = \left(e^{(E\mp\mu)/T} - 1\right)^{-1} = \left(e^{x\left(\sqrt{u^2+1}\mp\varpi\right)} - 1\right)^{-1}\,, \tag{2}$$

where $|\mu| \le m_{\rm be}$ (to lowest order in $\lambda_{\rm be}$) with the equality corresponding to the presence of a Bose–Einstein condensate (BEc).

The relevant thermodynamic quantities are the conserved charge $q_{\rm be}$, the entropy and energy densities ($s_{\rm be}$ and $\rho_{\rm be}$ respectively); explicitly,

$$q_{\rm be} = q_{\rm be}^{(c)} + m_{\rm be}{}^3\nu_{\rm be}\,; \qquad \nu_{\rm be} = \frac{1}{2\pi^2}\int_0^\infty {\rm d}u\, u^2\left(n_{\rm be}^+ - n_{\rm be}^-\right)\,;$$

$$s_{\rm be} = m_{\rm be}{}^3\sigma_{\rm be}\,; \qquad \sigma_{\rm be} = \frac{1}{2\pi^2}\int_0^\infty {\rm d}u\, u^2 \sum_{n=n_{\rm be}^{\pm}}\left[(1+n)\ln(1+n) - n\ln n\right]\,;$$

$$\rho_{\rm be} = m_{\rm be} q_{\rm be}^{(c)} + {m_{\rm be}}^4 r_{\rm be}\,; \qquad \frac{1}{2\pi^2}\int\limits_0^\infty {\rm d}u\, u^2 \sqrt{u^2+1}\left(n_{\rm be}^+ + n_{\rm be}^-\right)\,, \quad (3)$$

where $q_{\rm be}^{(c)}$ is the charge in the BEc, if present, and ${m_{\rm be}}^3 \nu_{\rm be}$ the charge in the excited states; $m_{\rm be} q_{\rm be}^{(c)}$ is the energy in the condensate. When DM$_\chi$ and the SM are in equilibrium, the homogeneity and isotropy of the universe ensures that $q_{\rm be}/s_{\rm tot}$ is conserved, where $s_{\rm tot} = s_{\rm be} + s_{\rm sm}$ is the total entropy density. When the two sectors are not in equilibrium $q_{\rm be}/s_{\rm be}$ and $q_{\rm be}/s_{\rm sm}$ are separately conserved.

4. The Bose–Einstein condensate

A BEc is present if

$$\frac{q_{\rm be}}{s_{\rm tot}} > \left.\frac{{m_{\rm be}}^3 \nu_{\rm be}}{s_{\rm tot}}\right|_{\varpi=1} = Y^{(e)}(\varpi = 1)\,. \quad (4)$$

For large T, $\nu_{\rm be} \simeq T^2$ and $s_{\rm tot} \sim T^3$, so $Y^{(c)} \to 0$: a BEc will always appear at sufficiently early times (Fig. 1). For WIMP-like masses, $m_{\rm be} > O$ (GeV), the transition temperature is

$$T_{\rm BEc} \simeq {m_{\rm be}}^2 \frac{1.9~{\rm eV}^{-1}}{g_{\star{\rm s}}(T_{\rm BEc}) + 2}\,, \quad (5)$$

where $g_{\star{\rm s}}$ is the number of relativistic degrees of freedom contributing to the entropy; for this mass range, the BEc carries a small fraction of the energy ($\sim O(100~{\rm eV})/m_{\rm be}$).

One can also consider the possibility that a BEc was present when the SM and DM$_\chi$ decoupled. We assume (to be justified later) that the Bose gas was non-relativistic at the time. In this case,

$$\underbrace{\frac{q_{\rm be}}{s_{\rm sm}} \simeq \frac{1}{m_{\rm be}} \frac{\rho_{\rm DM}}{s_{\rm sm}} = \frac{0.4~{\rm eV}}{m_{\rm be}}}_{\text{relic abundance constraint}}, \quad \underbrace{\frac{q_{\rm be}(T_{\rm d})}{(m_{\rm be} T_{\rm d})^{3/2}} > \frac{\zeta_{3/2}}{(2\pi)^{3/2}}}_{\text{presence of a BEc}} \;\Rightarrow\; m_{\rm be} < 1.3~{\rm keV}\,. \quad (6)$$

Similarly, $m_{\rm be} < 88$ eV for a BEc to be now present; but for these light masses, there are additional constraints that require $m_{\rm be}$ to be much smaller.

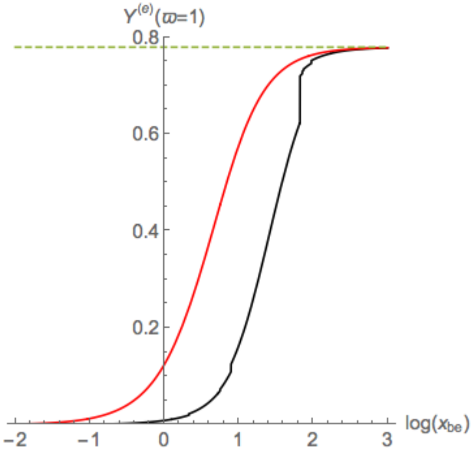

Fig. 1. Charge content of the excited states as a function of temperature for $m_{\rm be} = 10^{-12}$ eV and 10 GeV (left and right curves respectively).

5. Relic abundance

The first relation in Eq. (6) implies, for a non-relativistic gas,

$$\frac{0.4\ \mathrm{eV}}{m_{\rm be}} s_{\rm sm}(T_{\rm d}) \simeq 2(m_{\rm be}T_{\rm d}/2\pi)^{3/2}\cosh(\mu/T_{\rm d})\, e^{-m_{\rm be}/T_{\rm d}}\,, \tag{7}$$

so that the relic abundance constraint can be satisfied by an appropriate choice of μ. The decoupling temperature can then be obtained by considering the equation of energy transfer ($\vartheta = T_{\rm be} - T_{\rm sm}$) [3]

$$\dot{\vartheta} + 4\mathbb{H}\vartheta = -\mathit{\Gamma}\vartheta\,, \qquad \mathit{\Gamma} = \left(\frac{1}{c_{\rm be}} + \frac{1}{c_{\rm sm}}\right)\frac{\epsilon^2 G}{T}\,, \tag{8}$$

where $c_{\rm be}$, $c_{\rm sm}$ denote the heat capacities, $\mathbb{H}$ the Hubble rate, and

$$\boldsymbol{G} = \int_0^{\beta} \mathrm{d}s \int_0^{\infty} \mathrm{d}t \int \mathrm{d}^3\boldsymbol{x} \left\langle \mathcal{O}_{\rm be}(-is,\boldsymbol{x})\dot{\mathcal{O}}_{\rm be}(t,\boldsymbol{0})\right\rangle \left\langle \mathcal{O}_{\rm sm}(-is,\boldsymbol{x})\dot{\mathcal{O}}_{\rm sm}(t,\boldsymbol{0})\right\rangle\,,$$

where $\mathcal{O}_{\rm sm} = |\phi|^2\,,\ \mathcal{O}_{\rm be} = |\chi|^2$; $\boldsymbol{G}$ can be obtained using standard techniques of finite-temperature field theory [4]. The decoupling temperature $T_{\rm d}$ is obtained from the condition $\mathit{\Gamma} = \mathbb{H}$; the results are presented in Fig. 2 and justify our treating the gas non-relativistically.

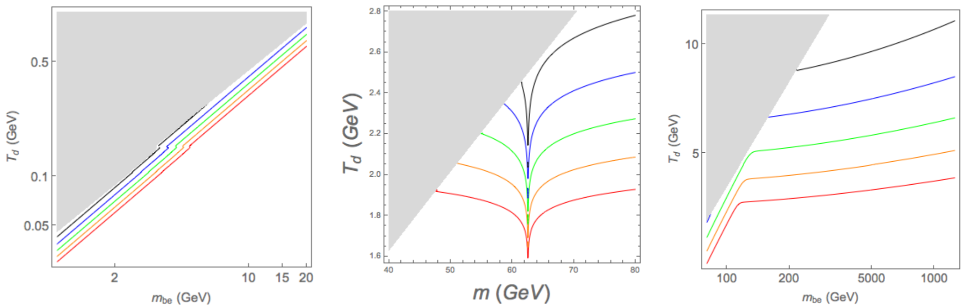

Fig. 2. $T_{\rm d}$ as a function of $m_{\rm be}$ for ϵ = 0.001, 0.01, 0.1, 1, 10 (lower to upper curves). Gray areas are excluded by the relic abundance constraint.

6. Direct detection

We imagine a SM particle with momentum $\boldsymbol{p}$ scattering off the DM$_\chi$ which is initially in a state X and ends in a state Y. Taking a thermal average over X and summing over all possible Y, we find that the cross section is given by (derived also using standard techniques of finite-temperature field theory [4])

$$\begin{aligned}\sigma &= \frac{1}{2q_{\rm be}|\boldsymbol{p}|}\left(\frac{1}{\mathcal{V}}\int' \frac{{\rm d}^3\boldsymbol{q}}{2E_{\boldsymbol{q}}\,(2\pi)^3}, \langle W_{\rm i\to f}\rangle_\beta\right) \\ &= \left[\frac{m_{\rm be}m_N}{m_{\rm be}+m_N}\frac{\epsilon v g_{\rm N-H}}{\sqrt{8\pi}\,m_{\rm be}m_{\rm H}^2}\right]^2\left[\frac{\exp\left(-u^2\right)}{\sqrt{\pi}\,u}+\left(1+\frac{1}{2u^2}\right){\rm Erf}(u)\right], \quad (9)\end{aligned}$$

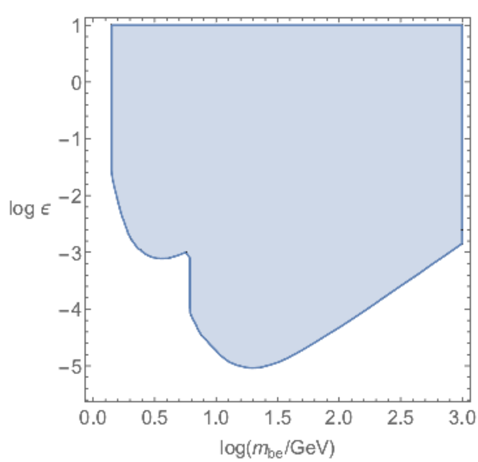

Fig. 3. Region excluded (shaded) by the LUX and CDMSLite limits.

where $u = |\boldsymbol{p}|\sqrt{x/2}/m_{\rm H}$ and the prime indicates $q \sim p$ is excluded. In obtaining the above, we used standard techniques in the finite-temperature field theory to obtain the expression for the average transition probability $\langle W_{\rm i\to f}\rangle_\beta$. The limits on the model parameters can be derived from these expressions and are presented in Fig. 3.

7. BEc now: tiny masses

The Bose gas must have been non-relativistic at the beginning of the large-scale structure formation era (corresponding to $z \sim 3400$ [5]); conservation of entropy $a^3 s_{\rm be} =$ constant (where a is the metric scale parameter) then requires the gas to be currently extremely non-relativistic: $x_{\rm now} > 3.5 \times 10^7$. In this case, the region in the temperature-mass plane for which a BEc is present is given in Fig. 4.

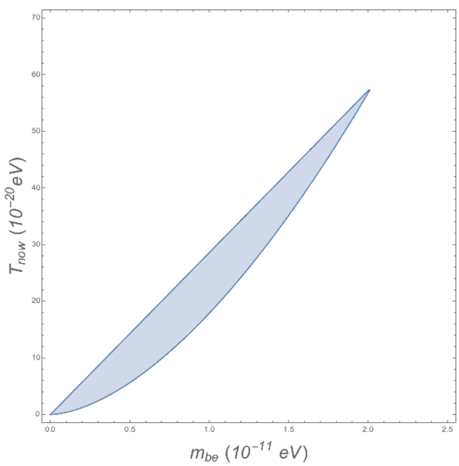

Fig. 4. Mass-temperature region for the DM_χ where a BEc is present.

Constraints derived from BBN are weaker: since $s_{\rm be}/s_{\rm sm}$ is conserved, we find that $x_{\rm BBN} \simeq 6.6 \times 10^{-9}\sqrt{x_{\rm now}}$; expressing then the DM_χ contribution to the energy density in terms of an effective number of neutrino species ΔN_ν [3], we have

$$\rho_{\rm be}|_{\rm BBN} = \frac{3}{\pi^2}\frac{7}{4}\left(\frac{4}{11}\right)^{4/3} T_\gamma^4 \Delta N_\nu\,, \quad T_\gamma \simeq 0.06\,\mathrm{MeV} \;\Rightarrow\; \Delta N_\nu \lesssim 7.2 \times 10^{-5}\,. \tag{10}$$

For these small masses, the DM_χ and SM never equilibrate: in this region of parameter space, the gas is axion-like (except that a BEc can occur even if the gas is non-relativistic).

8. Comments

We discussed various aspects of DM_χ, including the possibility that it exhibits collective behavior in the form of a BEc. Indirect detection effects for this system are the same as for a regular scalar DM. The chemical potential is used to meet the relic abundance requirement [6], so this does not restrict the model parameters; direct detection limits [7] provide the most stringent constraints. For WIMP-like masses, a BEc is only present at very early times, when the BEc carries a very small fraction of the total energy. For very small masses, a condensate could be present at present, but detecting this type of DM would be very difficult; in this case, the BEc can also address issues in galactic dynamics [8] such as the core *versus* cusp [10] and the "too big to fail" [11] problems.

REFERENCES

[1] G. Bertone, D. Hooper, J. Silk, *Phys. Rep.* **405**, 279 (2005) [arXiv:hep-ph/0404175].

[2] T. Matos, L.A. Urena-Lopez, *Phys. Rev. D* **63**, 063506 (2001) [arXiv:astro-ph/0006024]; B. Kain, H.Y. Ling, *Phys. Rev. D* **85**, 023527 (2012) [arXiv:1112.4169 [hep-ph]]; L.A. Urena-Lopez, *J. Cosmol. Astropart. Phys.* **0901**, 014 (2009) [arXiv:0806.3093 [gr-qc]]; A.P. Lundgren, M. Bondarescu, R. Bondarescu, J. Balakrishna, *Astrophys. J.* **715**, L35 (2010) [arXiv:1001.0051 [astro-ph.CO]]; D.J.E. Marsh, P.G. Ferreira, *Phys. Rev. D* **82**, 103528 (2010) [arXiv:1009.3501 [hep-ph]]; I. Rodríguez-Montoya, J. Magaña, T. Matos, A. Pérez-Lorenzana, *Astrophys. J.* **721**, 1509 (2010) [arXiv:0908.0054 [astro-ph.CO]]; I. Rodríguez-Montoya *et al.*, *Phys. Rev. D* **87**, 025009 (2013) [arXiv:1110.2751 [astro-ph.CO]]; A. Suárez, T. Matos, *Mon. Not. R. Astron. Soc.* **416**, 87 (2011) [arXiv:1101.4039 [gr-qc]]; T. Harko, *Phys. Rev. D* **83**, 123515 (2011) [arXiv:1105.5189 [gr-qc]]; T. Harko, *Mon. Not. R. Astron. Soc.* **413**, 3095 (2011) [arXiv:1101.3655 [gr-qc]]; T. Matos, A. Vázquez-González, J. Magaña, *Mon. Not. R. Astron. Soc.* **393**, 1359 (2009) [arXiv:0806.0683 [astro-ph]]; P.H. Chavanis, *Astron. Astrophys.* **537**, A127 (2012) [arXiv:1103.2698 [astro-ph.CO]]; H. Velten, E. Wamba, *Phys. Lett. B* **709**, 1 (2012) [arXiv:1111.2032 [astro-ph.CO]]; A. Aguirre, A. Diez-Tejedor, *J. Cosmol. Astropart. Phys.* **1604**, 019 (2016) [arXiv:1502.07354 [astro-ph.CO]]; J. Magaña, T. Matos, A. Suárez, F.J. Sánchez-Salcedo, *J. Cosmol. Astropart. Phys.* **1210**, 003 (2012) [arXiv:1204.5255 [astro-ph.CO]].

[3] B. Grzadkowski, J. Wudka, *Phys. Rev. D* **80**, 103518 (2009) [arXiv:0809.0977 [hep-ph]].

[4] M.L. Bellac, *Thermal Field Theory*, Cambridge University Press, 2000, ISBN-10: 0521654777, ISBN-13: 978-0521654777.

[5] E.W. Kolb, M.S. Turner, *The Early Universe*, Westview Press, 1994, ASIN: B019NE1EQC.

[6] P.A.R. Ade *et al.* [Planck Collaboration], *Astron. Astrophys.* **594**, A13 (2016) [arXiv:1502.01589 [astro-ph.CO]].

[7] B. Sadoulet, *NATO Sci. Ser. C* **511**, 517 (1998); P. Beltrame [LUX Collaboration], DOI:10.3204/DESY-PROC-2015-02/beltrame_paolo; E. Aprile *et al.* [XENON Collaboration], arXiv:1705.06655 [astro-ph.CO]; B. Serfass [CDMS and SuperCDMS collaborations], *J. Low. Temp. Phys.* **167**, 1119 (2012).

[8] A. Arbey, J. Lesgourgues, P. Salati, *Phys. Rev. D* **64**, 123528 (2001) [arXiv:astro-ph/0105564]; W. Hu, R. Barkana, A. Gruzinov, *Phys. Rev. Lett.* **85**, 1158 (2000) [arXiv:astro-ph/0003365]; A. Suárez, V.H. Robles, T. Matos, *Astrophys. Space Sci. Proc.* **38**, 107 (2014) [arXiv:1302.0903 [astro-ph.CO]]; S.J. Sin, *Phys. Rev. D* **50**, 3650 (1994) [arXiv:hep-ph/9205208]; J.w. Lee, I.g. Koh, *Phys. Rev. D* **53**, 2236 (1996) [arXiv:hep-ph/9507385]; J. Goodman, *New Astron.* **5**, 103 (2000) [arXiv:astro-ph/0003018]; T. Rindler-Daller, P.R. Shapiro, *Mon. Not. R. Astron. Soc.* **422**, 135 (2012) [arXiv:1106.1256 [astro-ph.CO]].

[9] T. Rindler-Daller, P.R. Shapiro, *Mod. Phys. Lett. A* **29**, 1430002 (2014) [arXiv:1312.1734 [astro-ph.CO]]; C.G. Boehmer, T. Harko, *J. Cosmol. Aastropart. Phys.* **0706**, 025 (2007) [arXiv:0705.4158 [astro-ph]]; T. Harko, E.J.M. Madarassy, *J. Cosmol. Aastropart. Phys.* **1201**, 020 (2012) [arXiv:1110.2829 [astro-ph.GA]].

[10] W.J.G. de Blok, *Adv. Astron.* **2010**, 789293 (2010) [arXiv:0910.3538 [astro-ph.CO]].

[11] M. Boylan-Kolchin, J.S. Bullock, M. Kaplinghat, *Mon. Not. R. Astron. Soc.* **415**, L40 (2011) [arXiv:1103.0007 [astro-ph.CO]].