EFFECTS OF PHONON–PHONON COUPLING ON PROPERTIES OF PYGMY RESONANCE IN $^{40–48}$Ca

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Starting from the Skyrme interaction SLy5, we study the effect of phonon–phonon coupling on the low-energy electric dipole response in $^{40–48}$Ca. This effect leads to the fragmentation of the E1 strength to a lower energy and improves the agreement with available experimental data.

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1. Introduction

Much experimental effort has been focused on measuring the low-energy spectrum of the $1^-$ states in nuclei across the nuclear landscape [1]. The observation of dipole states provides rich information on the various collective and single-particle nuclear excitation modes, in particular, the pygmy dipole resonance (PDR). The concentration of the electric dipole (E1) strength below or in the vicinity of the particle separation energy is commonly known as the PDR, because of its weak strength in comparison with the giant dipole resonance (GDR), which dominates the E1 strength in nuclei. Moreover, the total sum of the measured PDR energy-weighted sum rule (EWSR) is less than about 1–2% of the Thomas–Reiche–Kuhn (TRK) sum rule value for stable nuclei and less than 5–6% for unstable neutron-rich nuclei [1]. In analogy to the GDR, the PDR may be interpreted as a collective oscillation of the neutron skin with respect to an inert core of the nucleus (see Ref. [2] and references therein).

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The existence of the PDR mode near the neutron threshold has important astrophysical implications. For example, the occurrence of non-negligible low-lying E1 strength can influence the radiative neutron capture cross section by orders of magnitude and, consequently, also the rate of the astrophysical r-process nucleosynthesis [3].

The quasiparticle random phase approximation (QRPA) with a self-consistent mean-field derived from Skyrme energy density functionals (EDF) is one of the most successful methods for studying the low-energy dipole strength, see e.g., [2]. Such an approach can describe the properties of the low-lying states reasonably well by using existing Skyrme interactions. Due to the anharmonicity of vibrations, there is a coupling between one-phonon and more complex states [4]. The main difficulty is that the complexity of calculations beyond the standard QRPA increases rapidly with the size of the configuration space, so that one has to work within limited spaces. Using a finite-rank separable approximation (FRSA) [5, 6] for the residual interaction resulting from Skyrme forces, one can overcome this difficulty. The so-called FRSA was thus used to study the electric low-energy excitations and giant resonances within and beyond the QRPA [6–9]. In particular, we applied the FRSA approach to the PDR strength distribution [10, 11]. In the present report, we analyze the effects of the phonon–phonon coupling (PPC) on the E1 response for $^{40–48}$Ca, focusing on the emergence and the properties of the PDR. There are reliable experimental data for the properties of the PDR in $^{40,44,48}$Ca isotopes [12]. Thus, the isotopes in the Ca chain are ideal candidates for such an evolutionary PDR study.

2. Details of calculations and results

The FRSA approach has been discussed in detail in Refs. [5, 6] and it is presented here briefly for completeness. The Hartree–Fock–BCS (HF–BCS) calculations are performed by using the SLy5 [13] EDF in the particle–hole channel and a density-dependent zero-range interaction in the particle–particle channel. The parameters of the force SLy5 have been adjusted to reproduce nuclear matter properties, as well as nuclear charge radii, binding energies of doubly magic nuclei. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian in a harmonic oscillator basis. The strength of the zero-range volume force is taken equal to $\approx 270 \text{ MeV} \times \text{fm}^3$ in connection with the soft cutoff at 10 MeV above the Fermi energy as introduced in Ref. [6]. This value of the pairing strength is fitted to reproduce the experimental neutron pairing energies of $^{42,44,46}$Ca. The residual interaction in the particle-hole channel and in the particle–particle channel can be obtained as the second derivative of the energy density functional with respect to the particle density and the pair density, accordingly. By means of the standard procedure [14], we obtain the familiar QRPA equa-
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tions in the configuration space. The eigenvalues of the QRPA equations are found numerically as the roots of a relatively simple secular equation within the FRSA [5]. Since the FRSA enables us to use a large space, there is no need to introduce effective charges.

Taking into account the basic ideas of the quasiparticle–phonon model (QPM) [4], the Hamiltonian is then diagonalized in a space spanned by states composed of one and two QRPA phonons [7]

\[
\Psi_\nu(\lambda\mu) = \left( \sum_i R_i(\lambda\nu) Q^+_{\lambda i\mu i} + \sum_{\lambda_1\lambda_2} P_{\lambda_2\lambda_1}^{i_1i_2}(\lambda\nu) \left[ Q^+_{\lambda_1\mu_1i_1} Q^+_{\lambda_2\mu_2i_2} \right]_{\lambda\mu} \right) |0\rangle ,
\]

where \( Q^+_{\lambda\mu i} |0\rangle \) is the QRPA excitation having energy \( \omega_{\lambda i} \); \( \lambda \) denotes the total angular momentum and \( \mu \) is its z-projection in the laboratory system. The ground state is the QRPA phonon vacuum \( |0\rangle \). The unknown amplitudes \( R_i(\lambda\nu), P_{\lambda_2\lambda_1}^{i_1i_2}(\lambda\nu) \) and the excited state energies \( E_\nu \) are determined by solving the corresponding secular equation (cf. Refs. [7, 9]). We take into account all two-phonon terms that are constructed from the phonons with multipolarities \( \lambda^{\pi} = 1^-, 2^+, 3^-, 4^+, 5^- \) [10]. All dipole excitations with energies below 35 MeV and 15 most collective phonons of the other multipolarities are included in the wave function \( (1) \). In addition, we have checked that extending the configurational space plays a minor role in our calculations.

As the first step in the present analysis, we examine the PPC effects in the case of \(^{48}\)Ca since the E1 strength distribution for this nucleus is well-studied. The dipole spectrum is displayed in Fig. 1. The right part of the figure shows the photo-absorption cross section up to 26 MeV. The left panel shows the low-lying part of the corresponding spectrum below 11 MeV. Figure 1 (a) displays the experimental distribution [12, 15]. Results of RPA calculations are shown in Fig. 1 (b), and the RPA plus PPC results are presented in Fig. 1 (c). The cross section is computed by using a Lorentzian smearing with an averaging parameter \( \Delta = 1.0 \) MeV. The coupling between the one- and two-phonon states yields a noticeable redistribution of the GDR strength in comparison with the RPA results. In particular, the coupling increases the GDR width from 6.9 MeV to 7.3 MeV in the energy region \( E_x = 10\div26 \) MeV. Also, the PPC induces a 300 keV downward shift of the GDR energy (19.3 MeV for the RPA). The experimental GDR width and energy are 6.98 MeV and 19.5 MeV [15], respectively. The calculated characteristics of the GDR are in agreement with the observed values. The general shapes of the GDR obtained in the PPC are rather close to those observed in experiment. This demonstrates the improvement of the description within PPC in comparison with RPA. We conclude that the main mechanisms of the GDR formation in \(^{48}\)Ca are taken into account correctly and consistently in the PPC approach.
Let us now study the PPC effects on the low-energy E1 strength in more detail (cf. the left part of Fig. 1). In the case of the RPA, there is no $1^-$ state below 10 MeV. The RPA calculations predict the first dipole state around 10.5 MeV. In contrast to the case of the RPA, as can be seen from the left-hand side of panel (c) of Fig. 1, the inclusion of the two-phonon terms results in the formation of low-lying $1^-$ states in this energy region. The dominant contribution in the wave function of the $1^-$ states comes from the two-phonon configurations (> 60%). These states originate from the fragmentation of the RPA states above 10 MeV. The effects of the phonon–phonon coupling produce a sizable impact on the low-energy E1 strength of $^{48}$Ca. As a result, PPC calculations give a total dipole strength of 0.063 $e^2$fm$^2$, where the summation includes the dipole states below 10 MeV. The experimental value $\sum B(E1)$ is 0.0687±0.0075 $e^2$fm$^2$ in the same interval [12]. The relativistic quasiparticle time blocking approximation (RQTBA) calculations estimate the value of the summed $B(E1)$ as 0.1 $e^2$fm$^2$ [16]. The $^{48}$Ca($\gamma$, $\gamma'$) experiments give an integrated energy-weighted E1 strength of the PDR of about 0.33±0.04% of the TRK sum rule [12], while the calculations with the PPC effects lead to 0.28% (RQTBA results — 0.55% [16]). Finally, to illustrate the wide range of the effect of the two-phonon correlations on dipole
spectrum, we consider the dipole response of $^{40-48}\text{Ca}$ isotopes in the energy region below 10 MeV. Figure 2 shows the total dipole strength in the studied Ca isotopes. One can see that with the increase of the neutron number, the value $\sum B(E1)$ increases. Moreover, we have shown that it is necessary to include the phonon–phonon coupling to describe the PDR. As can be seen from Fig. 2, the $\sum B(E1)$ values are in good agreement with the experimental data [12], except for $^{44}\text{Ca}$. The theoretical value of the summed E1 strength in $^{44}\text{Ca}$ underestimates the experimental one by a factor of 2. In the case of RQTBA calculations [16], the calculated $\sum B(E1)$ value is also substantially less than the experimental value.

![Fig. 2. (Color online) Integrated dipole strength below 10 MeV in the Ca isotopic chain as a function of the neutron number calculated within the QRPA (open squares), the QRPA plus PPC (filled squares) and the RQTBA (filled stars); compared with the experimental data (filled diamonds) from Ref. [12].](image)

In summary, by starting from the Skyrme mean-field calculations, the properties of the electric dipole strength in Ca isotopes are studied by taking into account the coupling between one- and two-phonons terms in the wave functions of the excited states. The finite-rank separable approach for the QRPA calculations enables one to reduce considerably the dimensions of the matrices that must be inverted to perform nuclear structure calculations in very large configuration spaces. Neutron excess effects on the PDR excitation energies and transition strengths have been investigated for the even–even nuclei $^{40-48}\text{Ca}$. We have shown that in order to explain the properties of the PDR, one has to consider the phonon–phonon coupling in addition to the QRPA approach.
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