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STABILITY OF THE ELECTROWEAK VACUUM IN A SCALE-INVARIANT EXTENSION OF THE SM*

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We explore the possibility that scale symmetry is a quantum symmetry that is broken only spontaneously in flat space and apply this idea to the Standard Model (SM). The one-loop scalar potential is scale-invariant, since the loop calculations preserve the scale symmetry, with the DR subtraction scale generated spontaneously by the dilaton vacuum expectation value, $\langle\sigma\rangle$. At the quantum level, the Higgs mass is protected although the theory is non-renormalizable. It is argued that the instability of the effective potential in the Higgs sector that is driven by the quartic coupling running towards negative values becomes worse in the scale-invariant version, since the effective potential becomes unbounded from below.

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1. Introduction

In this letter we explore the proposal that scale symmetry is a quantum symmetry and study its implications for physics beyond SM. However, this symmetry is broken in the real world. We shall here consider only spontaneous breaking of this (quantum) symmetry[1]. One motivation of this study is that scale symmetry plays a role in the ultraviolet (UV) behaviour of the models. In particular, the SM with a classical Higgs mass parameter $m_\phi = 0$ has an increased symmetry: it is scale-invariant at the tree level; this was invoked [1] to protect m_ϕ naturally [2] from large quantum corrections, but a full quantum study is needed.

Consider a classically scale-invariant theory. One known issue when studying scale symmetry at the quantum level is that the regularization of the loop corrections introduces a dimensionful parameter (the subtraction

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[1] By quantum scale symmetry we mean that the full 1PI quantum action is scale-invariant.

scale μ) which breaks *explicitly* the scale symmetry, thus destroying the symmetry we actually want to investigate[2] and affecting the UV properties of the quantum theory. To avoid this, the UV regularization must preserve this symmetry. This is done by using a subtraction *function* $\mu(\sigma)$ which generates (dynamically) a subtraction scale $\mu(\langle\sigma\rangle)$ when the field σ acquires a v.e.v. $\langle\sigma\rangle$ after spontaneous scale symmetry breaking, see [3] and recent examples at one loop [4–8] and higher loops [9, 10]. Here, σ is the Goldstone mode (dilaton) of the spontaneously broken scale symmetry.

The model we consider is a scale-invariant SM, defined as SM with classical $m_\phi = 0$ and extended by the dilaton. The goal is to use this scale-invariant regularization to compute quantum corrections to the scalar potential. The quantum result is scale-invariant, so it can only have *spontaneous* scale symmetry breaking, with a flat direction for the dilaton (σ). This result should be compared to that in the "usual" dimensional regularization (DR) of $\mu =$ constant scale, which breaks explicitly the scale symmetry at the quantum level.

Let us consider first a simplified scale-invariant (classical) theory (*e.g.* [11–27]) of two real scalar fields ϕ (Higgs-like) and σ. The potential V is a homogeneous function

$$V(\phi,\sigma) = \sigma^4\, W(\phi/\sigma)\,, \qquad \text{where} \qquad W(\phi/\sigma) = V(\phi/\sigma, 1)\,. \tag{1}$$

We assume that $V(\phi,\sigma)$ has spontaneous scale symmetry breaking *i.e.* that σ acquires a non-zero v.e.v. $\langle\sigma\rangle \neq 0$. We thus search for such a solution and for the necessary condition for this spontaneous breaking to happen. With $\langle\sigma\rangle \neq 0$ it is then easy to see that the minimum conditions $V_\sigma = V_\phi = 0$ ($V_\alpha = \partial V/\partial\alpha$) are equivalent to

$$W(\rho) = W'(\rho) = 0\,, \qquad \rho \equiv \phi/\sigma\,. \tag{2}$$

These equations can have a common solution $\rho_0 \equiv \langle\phi\rangle/\langle\sigma\rangle$, if the couplings satisfy a particular condition (constraint), see below. Then a *flat direction* exists in the plane (ϕ,σ) with $\phi = \rho_0\,\sigma$. Indeed, if $(\langle\phi\rangle, \langle\sigma\rangle)$ is a ground state with $V = 0$, then so is $(t\langle\phi\rangle, t\langle\sigma\rangle)$, t real. Besides, the second derivatives matrix $V_{\alpha\beta}$ w.r.t. $\alpha,\beta = \phi,\sigma$ has $\det(V_{\alpha\beta}) \propto (4WW'' - 3W'^2) = 0$ on the ground state, so a massless state is indeed present corresponding to the flat direction. Finally, since ρ_0 is a root of both W and of its derivative W', then $W(\phi/\sigma) \propto (\phi/\sigma - \rho_0)^2$, while if V depends only on even powers of the scalar fields (our model below), then the general structure is

$$W(\phi/\sigma) \propto \left(\phi^2/\sigma^2 - \rho_0^2\right)^2\,. \tag{3}$$

[2] One could use a regularization that does not keep manifest scale symmetry and attempt to restore it "by hand" at the end, but this would miss scale-invariant operators if the theory is non-renormalizable, see later.

Note that the vanishing vacuum energy $V(\langle\phi\rangle, \langle\sigma\rangle) = 0$ follows from the (spontaneously broken) scale symmetry, see Eq. (2). A scale-invariant regularization of this theory leads to a scale-invariant quantum potential, which thus remains of the form of Eq. (1). Hence the above discussion around Eqs. (1), (2), and (3) *remains true at the quantum level*, including the possibility of spontaneous-only breaking of the scale symmetry.

One of the two minimum conditions in (2) fixes the ratio $\rho_0 = \langle\phi\rangle/\langle\sigma\rangle$ in terms of the (dimensionless) couplings of the theory. Thus, all v.e.v.s of such a theory, including $\langle\phi\rangle$ are proportional to $\langle\sigma\rangle \neq 0$ which is a parameter (unknown) of this theory. The second minimum condition, after eliminating ρ_0 between the two equations in (2), gives a relation among the couplings of the theory in the order of perturbation in which V is computed. This means one coupling, say λ_σ (the dilaton self-coupling), is defined in terms of the rest $\lambda_\sigma = f(\lambda_{j\neq\sigma}) +$ loops. This relation follows from demanding that V has a *flat direction* that is itself a consequence of our requiring that scale symmetry, which is a quantum symmetry, be broken spontaneously (in that order). The relation $\lambda_\sigma = f(\lambda_{j\neq\sigma}) +$ loops is corrected in a given order of perturbation theory by $\mathcal{O}(\lambda_j)$ relative to previous order; it is ultimately related to the vanishing of the vacuum energy $V(\langle\phi\rangle, \langle\sigma\rangle) \sim W(\rho_0) = 0$, see minimum conditions (2).

We stress that the above picture, that builds on previous studies [3–10], is very different from that obtained in the "traditional" DR scheme ($\mu =$ constant scale) that is often used in classically scale-invariant models *e.g.* [18–27]; in such a case, scale symmetry is broken *explicitly* by the (regularization of) quantum effects and then conditions (1), (2) are not true any more at quantum level and the flat direction is lifted by quantum corrections.

What about the hierarchy problem? In the absence of gravity (not included here), the Standard Model has no hierarchy problem. However, this situation is no longer true under the reasonable assumption that there is some "new physics" beyond SM, *e.g.* a large v.e.v. of a new scalar field that couples to Higgs, *etc.* In the model we consider, defined by the invariant version of the SM extended by the dilaton, we have "new physics" beyond the SM, represented by the v.e.v. $\langle\sigma\rangle$ that breaks spontaneously the scale symmetry. $\langle\sigma\rangle$ can be very large compared to $\langle\phi\rangle$ where the latter fixes the electroweak scale[3]. We simply enforce such a hierarchy by choosing a very weak coupling of the visible to the hidden sector of the dilaton [31]. Such a hierarchy is however stable under quantum corrections, so $m_\phi \sim \langle\phi\rangle \ll \langle\sigma\rangle$ without tuning at the quantum level [4, 8], and we verify this in our model at one-loop. This is expected to remain true to all orders in perturbation theory since scale symmetry is preserved by the regularization and is broken

[3] Such a hierarchy can be generated dynamically [28, 29] or as in [30].

only spontaneously. We thus have an example of a quantum stable hierarchy, with a vanishing vacuum energy at the loop level that follows from the demand of spontaneous-only broken quantum scale symmetry.

In the following, we apply these ideas to the scale-invariant version of the SM (with $m_\phi = 0$) extended by the dilaton. The Higgs and the dilaton have a potential dictated solely by the classical scale symmetry and thus can contain higher dimensional operators such as ϕ^6/σ^2, ϕ^8/σ^4, *etc.* (ϕ is the neutral Higgs boson). We then compute the one-loop potential with a scale-invariant regularization, so a flat direction is maintained at the quantum level. Even if the tree-level potential does not include the aforementioned higher dimensional operators (by tuning their couplings to zero), they are generated at one loop with finite coefficient [8] or as two loop or higher counterterms [9, 10] — this means the scale-invariant quantum theory is non-renormalizable. Further, the quantum consistency of the theory is shown by verifying the Callan–Symanzik equation of the potential in the presence of these non-polynomial operators, gauge and Yukawa interactions. We also compare the scale-invariant one-loop potential to its counterpart computed in the "usual" DR scheme that breaks scale symmetry explicitly ($\mu =$ constant), in the presence at tree level of these (non-polynomial) effective operators.

If scale symmetry is preserved by one-loop V, there is no dilatation anomaly which is a result of *explicit* scale symmetry breaking by quantum calculations with $\mu =$ constant. Contrary to common lore, the couplings still run with momentum [6–8] since the vanishing of the beta functions *is not a necessary* condition for scale invariance. Their one-loop running is identical to that in the theory with explicit scale symmetry breaking ($\mu =$ constant), but at two loop, they start to differ in theories with spontaneous *versus* explicit breaking [7, 10].

This analysis in flat space-time should be extended to include the effects of gravity which we ignored. Since Einstein gravity breaks scale symmetry, a natural setup to include such effects is then to consider the Weyl or Brans–Dicke–Jordan theory of gravity, see examples in [4, 28, 29, 32–38]. In such a setup it may still be possible to perform a scale-invariant regularization and then examine such a scale-invariant theory at quantum level.

2. SM with a scale-invariant one-loop potential

2.1. The tree-level scale-invariant potential

Consider the SM Lagrangian with tree-level Higgs mass $m_\phi = 0$, so it is scale-invariant. The Higgs sector is weakly coupled to the "hidden" sector of the dilaton σ with

$$\mathcal{L} = |D_\mu H|^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - V_0\,, \tag{4}$$

where

$$H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(\phi + i\,G^0\right) \end{pmatrix} \tag{5}$$

and

$$V_0 = \frac{\lambda_\phi}{3!}\left(H^\dagger H\right)^2 + \frac{\lambda_m}{2}\left(H^\dagger H\right)\sigma^2 + \frac{\lambda_\sigma}{4!}\sigma^4 + \frac{4\,\lambda_6}{3}\,\frac{\left(H^\dagger H\right)^3}{\sigma^2} + \cdots\,, \tag{6}$$

where the dots stand for higher powers of $H^\dagger H$. The neutral Higgs (ϕ) and dilaton part is

$$V(\phi,\sigma) = \frac{1}{4!}\,\lambda_\phi\,\phi^4 + \frac{1}{4}\,\lambda_m\,\phi^2\,\sigma^2 + \frac{1}{4!}\,\lambda_\sigma\sigma^4 + \frac{\lambda_6}{6}\,\frac{\phi^6}{\sigma^2} + \ldots \tag{7}$$

We take $\lambda_\phi, \lambda_\sigma > 0$ and $\lambda_m < 0$ and that the two sectors of ϕ and σ are weakly coupled, with $|\lambda_m| < \lambda_\phi$. Regarding the terms suppressed by powers of σ, they respect the (classical) scale symmetry of the action, so they can be present in the theory. They are well-defined since σ acquires spontaneously a v.e.v. $\langle\sigma\rangle \neq 0$ under conditions that we identify shortly (see (a) in Eqs. (9), (11) below). One can expand such terms about the ground state, into an infinite sum of familiar polynomial (effective) operators

$$\lambda_6\frac{\phi^6}{\sigma^2} = \lambda_6\frac{\phi^6}{\langle\sigma\rangle^2}\left(1 - 2\frac{\sigma'}{\langle\sigma\rangle} + 3\frac{\sigma'^2}{\langle\sigma\rangle^2} + \cdots\right), \quad \sigma = \langle\sigma\rangle + \sigma', \quad \sigma' : \text{fluctuation.} \tag{8}$$

However, we prefer to use the form in Eq. (7) since it keeps manifest the scale symmetry of $\mathcal{L}$. Finally, we keep $\lambda_6 \neq 0$ but set to 0 the coefficients of $(H^\dagger H)^4/\sigma^4$ and higher terms.

Consider first $\lambda_6 = 0$. We demanded spontaneous breaking of scale symmetry, so we seek the condition for which $\langle\sigma\rangle \neq 0$. The minimum of V exists if derivatives $V_\phi = V_\sigma = 0$, giving

$$\text{(a)}:\quad \lambda_\sigma = \frac{9\lambda_m^2}{\lambda_\phi}\,[1 + \text{loops}] \qquad \text{and} \qquad \text{(b)}:\quad \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = \frac{-3\lambda_m}{\lambda_\phi}[1 + \text{loops}]\,, \tag{9}$$

so also $\langle\phi\rangle \neq 0$; here, "loops" stands for loop corrections.

Let us then assume that λ_σ is indeed that of (a) up to "loop" effects that one can identify order by order in perturbation theory and that we ignore for the classical discussion here. If (a) is true, we have spontaneous breaking of scale symmetry and

$$V = \frac{1}{4!}\,\lambda_\phi \sigma^4 \left(\frac{\phi^2}{\sigma^2} + \frac{3\lambda_m}{\lambda_\phi}\right)^2 \tag{10}$$

with $V = 0$ at the minimum. A flat direction, corresponding to the Goldstone of scale symmetry (dilaton) exists in the plane (ϕ, σ). The neutral Higgs acquires a mass $m^2_{\tilde\phi} = (\lambda_\phi/3)(1 - 3\lambda_m/\lambda_\phi)\langle\phi\rangle^2$, while the EW Goldstone bosons are massless. Thus, spontaneous scale symmetry breaking triggers EW symmetry breaking, with a vacuum energy $V = 0$.

Consider now $\lambda_6 \neq 0$, with $\lambda_6 > 0$ for a well-defined V at large ϕ. Then Eqs. (9) become

$$\begin{aligned}
&\text{(a)}: \quad \lambda_\sigma = \rho_0^2\left[2\lambda_6\,\rho_0^4 - 3\,\lambda_m\right] + \text{loops}\,, \qquad \text{where} \\
&\text{(b)}: \quad \rho_0^2 \equiv \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = \frac{1}{12\lambda_6}\left[-\lambda_\phi + \left(\lambda_\phi^2 - 72\lambda_6\,\lambda_m\right)^{1/2}\right] + \text{loops}\,.
\end{aligned} \tag{11}$$

We assume from now on that λ_σ is indeed given by relation (a), up to small quantum corrections (ignored here), to ensure spontaneous scale symmetry breaking; this relation is "protected" by scale symmetry. The potential is then

$$V = \frac{\lambda_6}{6}\sigma^4 \left(\frac{\phi^2}{\sigma^2} - \rho_0^2\right)^2 \left(\frac{\phi^2}{\sigma^2} + \xi_0^2\right)\,, \tag{12}$$

in agreement with (3). Here, $\xi_0^2 = (\lambda_\phi + 2\,(\lambda_\phi^2 - 72\lambda_6\lambda_m)^{1/2})/(12\lambda_6) > 0$. If $\lambda_6 \to 0$, one recovers Eq. (10). The neutral Higgs mass can again be computed and recovers the above value for small λ_6; the dilaton is again massless, with the flat direction mildly changed by λ_6. To conclude, spontaneous scale symmetry breaking triggers EW symmetry breaking and ensures $V = 0$ on the ground state. We would like to know if this can remain true at quantum level.

The scale $\langle\sigma\rangle$ of "new physics" beyond SM should be larger than $\langle\phi\rangle \sim \mathcal{O}(100)$ GeV. In Weyl or Brans–Dicke–Jordan theory of gravity (not considered here) that can generalise this study, one actually expects $\langle\sigma\rangle \sim M_{\text{Planck}}$. So a hierarchy $\langle\phi\rangle \ll \langle\sigma\rangle$ may be generated dynamically [28, 29]. Here, we take a common view of a very weak coupling of the hidden (σ) to visible (ϕ) sector: $|\lambda_m| \ll \lambda_\phi$ [31]; then[4] from Eq. (11), $\lambda_\sigma \ll |\lambda_m|$. This classical "tuning" can ensure a hierarchy of scales $\langle\phi\rangle \ll \langle\sigma\rangle$ (λ_6 only brings sub-leading corrections, since the hierarchy is controlled by λ_m, the main coupling of the two sectors).

[4] This hierarchy is stable under renormalization group [31] due to a shift symmetry, $\sigma \to \sigma + \text{constant}$.

This concludes the picture of the classical potential with scale symmetry. At the quantum level, one question is whether the (quantum) scale symmetry, when spontaneously broken, maintains the hierarchy $m_{\tilde{\phi}}^2 \sim \langle\phi\rangle^2 \ll \langle\sigma\rangle^2$ without additional tuning of the couplings. If quantum corrections $\lambda_\phi^2\langle\sigma\rangle^2$ are generated, a tuning of the Higgs self-coupling λ_ϕ would be needed and this would re-introduce the hierarchy problem.

2.2. The one-loop scale-invariant potential

Let us compute the one-loop potential by preserving scale symmetry at quantum level and thus avoid its explicit breaking by the UV regularization. The method is described in [4, 6–10]. To do this, note that we already have a v.e.v. $\langle\sigma\rangle$ that can act as a subtraction scale. The starting point is in $d = 4 - 2\epsilon$ dimensions where the tree level potential is modified into

$$\tilde{V} = \mu(\sigma)^{2\epsilon}\, V\,, \qquad \mu(\sigma) = z\,\sigma^{1/(1-\epsilon)}\,, \tag{13}$$

$\tilde{V}$ is thus scale-invariant in $d = 4 - 2\epsilon$. The function $\mu(\sigma)$ generates a subtraction scale $\mu(\langle\sigma\rangle)$ when σ acquires a v.e.v. spontaneously. The definition of $\mu(\sigma)$ follows on dimensional grounds, with z an arbitrary dimensionless subtraction parameter [7]. If we set $\mu(\sigma) = \text{constant}$, one immediately recovers the "traditional" DR scheme that breaks explicitly the scale symmetry in $d = 4-2\epsilon$. We thus have two possible analytical continuations to $d = 4-2\epsilon$ of the classical scale-invariant theory in $d = 4$: one is scale-invariant (Eq. (13)), the other is not ($\mu = \text{constant}$), and they lead to distinct quantum theories (of different symmetry) [8, 10], as discussed below. The one-loop potential in $d = 4 - 2\epsilon$ is then [8, 10]

$$V_1 = \tilde{V} - \frac{i}{2}\int \frac{\mathrm{d}^d p}{(2\pi)^d}\,\mathrm{Tr}\ln\left[p^2 - \tilde{V}_{ij} + i\varepsilon\right]. \tag{14}$$

This is computed in the Landau gauge. The field-dependent squared masses are eigenvalues of the matrix of second derivatives denoted $\tilde{V}_{ij}$ where subscripts i, j stand for: the EW Goldstone scalars G^0, $\Re(G^+)$, $\Im(G^+)$, neutral Higgs ϕ and dilaton σ. Unlike the EW Goldstone modes or fermions and gauge bosons, the field-dependent masses of ϕ and σ acquire a correction $\propto\epsilon$ relative to their values induced by V alone, from derivatives of $\mu(\sigma)$

$$\begin{aligned}
m_t^2 &= \frac{\mu(\sigma)^{2\epsilon}}{2}\,h_t^2\phi^2\,, \quad m_W^2 = \frac{\mu(\sigma)^{2\epsilon}}{4}\,g_2^2\,\phi^2\,, \quad m_Z^2 = \frac{\mu(\sigma)^{2\epsilon}}{4}\,\left(g_1^2 + g_2^2\right)\,\phi^2,\\
m_G^2 &= \frac{\mu(\sigma)^{2\epsilon}}{6}\,\left[\lambda_\phi\,\phi^2 + 3\lambda_m\sigma^2 + 6\,\lambda_6\,\frac{\phi^4}{\sigma^2}\right],\\
M_k^2 &= \mu(\sigma)^{2\epsilon}\,\left[m_k^2 + \epsilon\,\delta_k\right]\,, \qquad k = \phi, \sigma\,,
\end{aligned} \tag{15}$$

where m_t (h_t) is the field-dependent top mass (Yukawa coupling), $m_{W,Z}$ denote the gauge boson masses and m_G denote the three EW Goldstone field-dependent masses. Finally, M_k^2 are eigenvalues of $\tilde{V}_{\alpha\beta}$, while m_k^2 are eigenvalues of the 2×2 sub-matrix $V_{\alpha\beta}$ of V_{ij} with $V_{\alpha\beta}=\partial^2 V/\partial\alpha\,\partial\beta$, $\alpha,\beta=\phi,\sigma$. Then, one finds at one loop ($\kappa=(4\pi)^2$)

$$V_1=\mu(\sigma)^{2\epsilon}\times\left\{V-\frac{1}{4\kappa}\left[\sum_{j=\phi,\sigma;G,W,Z,t} n_j\, m_j^4(\phi,\sigma)\left(\frac{1}{\epsilon}-\ln\frac{m_j^2(\phi,\sigma)}{c_j\,\mu^2(\sigma)}\right)+\frac{4\,(V_{\alpha\beta}\,N_{\beta\alpha})}{\mu^2(\sigma)}\right]\right\} \tag{16}$$

with summation over $\alpha,\beta=\phi,\sigma$ and $N_{\alpha\beta}=\mu(\mu_\alpha V_\beta+\mu_\beta V_\alpha)-\mu_\alpha\mu_\beta)\,V$ and $\mu_\alpha=\partial\mu/\partial\alpha$. Besides, $n_j=\{3,1,6,3,-12\}$ for $j=\{G,S,W,Z,t\}$, with $S=\phi,\sigma$; $c_j=4\pi\,\mathrm{e}^{3/2-\gamma_E}$ if $j=\phi,\sigma,t,G$ and $c_j=4\pi\,\mathrm{e}^{5/6-\gamma_E}$ if $j=W,Z$. The one-loop term $(V_{\alpha\beta}\,N_{\beta\alpha})$ is a new correction, absent in the case of $\mu=$ constant (*i.e.* explicit scale symmetry breaking by the regularization).

The SM one-loop potential U_1 becomes

$$U_1=V+V^{(1)}+V^{(1,n)}\,, \tag{17}$$

where

$$V^{(1)} = \frac{1}{4\kappa}\sum_{j=\phi,\sigma;G,t,W,Z} n_j\, m_j^4(\phi,\sigma)\,\ln\frac{m_j^2(\phi,\sigma)}{c_j\,(z\,\sigma)^2}\,, \tag{18}$$

$$\begin{aligned} V^{(1,n)} = \frac{1}{48\kappa}\Bigg[&\left(-16\lambda_m\lambda_\phi-18\lambda_m^2+\lambda_\phi\lambda_\sigma\right)\phi^4-\lambda_m\left(48\lambda_m+25\lambda_\sigma\right)\phi^2\sigma^2\\ &-7\lambda_\sigma^2\sigma^4+\left(\lambda_\phi\lambda_m+6\lambda_6\lambda_\sigma\right)\frac{\phi^6}{\sigma^2}+8\,\lambda_6\left(4\lambda_\phi-2\lambda_m\right)\frac{\phi^8}{\sigma^4}\\ &+\lambda_6\left(192\lambda_6+2\lambda_\phi\right)\frac{\phi^{10}}{\sigma^6}+40\lambda_6^2\frac{\phi^{12}}{\sigma^8}\Bigg]\,, \end{aligned} \tag{19}$$

and the U_1 is manifestly scale-invariant.

There is also a finite one-loop contribution $V^{(1,n)}$ due to "evanescent" corrections ($\propto\epsilon$) to the field-dependent masses of ϕ and σ (Eq. (15)), induced by *derivatives* of $\mu\sim\sigma$. Therefore, $V^{(1,n)}$ is not present in the other case of $\mu=$ constant when the regularization breaks the scale symmetry; thus, $V^{(1,n)}$ can distinguish between these two cases at one loop[5]. Further, in the classical decoupling limit of the hidden sector from the SM, $\lambda_m\to 0$ and $\lambda_6\to 0$, then $V^{(1,n)}$ vanishes.

[5] These two cases are different quantum theories (have different symmetry).

$V^{(1,n)}$ also contains terms non-polynomial in fields like $\lambda_m \lambda_\phi \phi^6/\sigma^2$ that remain present even if we set $\lambda_6 = 0$.

At two loop, such non-polynomial operators, including higher order ϕ^8/σ^4, *etc.*, emerge as two-loop counterterms [10] even if we set $\lambda_6 = 0$[6].

Although we do not show it, one can immediately Taylor expand both $V^{(1)}$ and $V^{(1,n)}$ about the non-zero v.e.v. of σ, with $\sigma = \langle\sigma\rangle + \sigma'$ and eventually of ϕ too, $\phi = \langle\phi\rangle + \phi'$. One then obtains a representation with only polynomial operators in the field fluctuations (ϕ', σ').

2.3. Absence of dilatation anomaly

Let us analyze the situation of the dilatation current D^μ and its divergence [6, 7]. For a set of fields ϕ_j (ϕ, σ, *etc.*),

$$D^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu \phi_j)}\,(x^\nu \partial_\nu \phi_j + d_\phi) - x^\mu\,\mathcal{L}\,,$$
$$\partial_\mu D^\mu = (d_\phi + 1)\,(\partial_\mu\phi_j)\,\frac{\partial\mathcal{L}}{\partial(\partial_\mu \phi_j)} + d_\phi\,\phi_j\,\frac{\partial\mathcal{L}}{\partial\phi_j} - d\mathcal{L}\,, \tag{20}$$

with d_ϕ the mass dimension of ϕ, $d_\phi = (d-2)/2$ for a scalar in d dimensions. For standard kinetic terms and using the equations of motion, we find for a potential $\mathcal{V}$ in d dimensions

$$\partial_\mu D^\mu = d\,\mathcal{V} - \frac{d-2}{2}\phi_j\,\frac{\partial\mathcal{V}}{\partial\phi_j}\,. \tag{21}$$

Consider now that $\mathcal{V}$ is scale-invariant at both classical and quantum level as in our case. Therefore, for a dimensionless parameter ρ, $\mathcal{V}$ has the property $\mathcal{V}(\rho\,\phi_j) = \rho^{2d/(d-2)}\,\mathcal{V}(\phi_j)$ in $d = 4-2\epsilon$ dimensions (homogeneous function). Differentiating this equation with respect to ρ and then taking $\rho \to 1$ gives $2d/(d-2)\,\mathcal{V} = \phi_j\,\partial\mathcal{V}/\partial\phi_j$ so the r.h.s. of Eq. (21) vanishes. Therefore, $\partial_\mu D^\mu = 0$ at both classical and quantum level, so there is no anomalous breaking of the quantum scale symmetry. Nevertheless, the couplings still "run" and have non-zero beta functions with their corresponding poles in $\mathcal{L}$.

To understand this better, let us also see what happens if $\mathcal{V}$ is not scale-invariant in $d = 4-2\epsilon$. This happens when $\mathcal{V} = \mu^{2\epsilon} V(\phi_j)$ which is the case of the "traditional" DR scheme with the explicit scale symmetry breaking, with μ a fixed scale (not a function of the fields) and V the potential, scale-invariant in $d = 4$ (assuming no mass terms). Then $V(\rho\,\phi_j) = \rho^4\,V(\phi_j)$, but

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[6] The two-loop beta functions of such terms are non-zero even if $\lambda_6 = 0$, so setting these to zero (at some scale) will not remove them since they are again generated at a different scale [10].

$\mathcal{V}$ is no longer scale-invariant in $d = 4 - 2\epsilon$. Then from Eq. (21),

$$\partial_\mu D^\mu = d\,\mu^{2\epsilon}\,V - 2(d-2)\,\mu^{2\epsilon}\,V(\phi_j) = 2\epsilon\,\mu^{2\epsilon}\,V = 2\epsilon\,\mu^{2\epsilon}\lambda_j\frac{\partial V}{\partial\lambda_j}\,. \tag{22}$$

While at the classical level the r.h.s. vanishes when $\epsilon \to 0$, at the quantum level the quartic couplings λ_j in V acquire a pole $\beta_{\lambda_j}/\epsilon$ which cancels the ϵ in front, to give a finite non-zero r.h.s. $\partial_\mu D^\mu \propto \beta_{\lambda_j}(\partial\mathcal{L}/\partial\lambda_j)$. This is the familiar scale anomaly breaking of the conservation of this current in the "traditional" DR scheme[7,8].

In conclusion, it is scale invariance of the action in $d = 4-2\epsilon$ that ensures that no scale anomaly is present. This invariance in $d = 4-2\epsilon$ is lost in the "usual" DR regularization with explicit breaking (μ = constant). Thus, the vanishing of the beta function is not a necessary condition for the theory to be scale-invariant; one must also specify how the quantum theory was regularized, with or without respecting its scale symmetry. In other words the non-vanishing of the beta function does not mean the theory cannot be scale-invariant.

3. Additional comments

It is interesting to note that the chiral symmetry breaking at low energies can also be viewed as the spontaneous breaking of the scale invariance due to strong interactions. Parametrizing the renormalization scale as $\mu = \mathrm{e}^t\,\sigma$ and taking into account the 1-loop running of the strong coupling constant, $\frac{\mathrm{d}g_\mathrm{s}(t)}{\mathrm{d}t} = -\frac{b_0 g_\mathrm{s}^3}{2}$, one finds the renormalization group invariant scale expressed as

$$\Lambda s_\mathrm{s} = \langle\sigma\rangle \mathrm{e}^{-\frac{1}{b_0\,g_\mathrm{s}^2(t)}}\,. \tag{23}$$

This implies that the contributions to the quark masses that are due to strong interactions can be parametrized by scale-invariant terms in the effective Lagrangian

$$\delta L_\mathrm{s} = -\sigma\,\mathrm{e}^{-\frac{1}{b_0\,g_\mathrm{s}^2(t)}}\,\bar{q}q\,. \tag{24}$$

The consistency of the boundary values for the running couplings with high scale physics that may fix the value of $\langle\sigma\rangle$ should be investigated. This discussion requires that one extends this quantum calculation to the case of curved space time while respecting this symmetry. The appropriate setup is in the context of Weyl gravity or Brans–Dicke–Jordan theory of gravity. For investigations along this direction, see [28, 29, 32–42].

[7] Even if at classical level it was conserved.

[8] If V contained mass terms, $\partial_\mu D^\mu$ also contains a "classical" breaking of scale symmetry term, $m^2\phi^2$.

4. Quantum instability

The careful study of the renormalization group improved SM potential has revealed new extrema located at field strengths larger than 10^{11} GeV. The upshot depends critically on the precise value of the measured Higgs mass and on the measured value of the top quark Yukawa coupling. In particular, one finds that for the central value of the top mass and for the central value of the measured Higgs mass, the physical electroweak symmetry breaking minimum becomes metastable with respect to the tunnelling from the physical EWSB minimum to a deeper minimum located at super-Planckian values of the Higgs field strength. The lifetime of the metastable SM Universe is much larger than the presently estimated age of the Universe, however the instability border in the space of parameters $M_{\rm top}$–$M_{\rm Higgs}$ looks uncomfortably close, which suggests that the result is rather sensitive to modifications that can be brought in by the BSM extensions. The question of stability in the presence of non-renormalizable operators was raised in [10]. This question has been studied further in [43]. In particular, a map of the vacua lifetime in SM with new non-renormalizable scalar couplings was presented. Moreover, the RGE improvement of the new couplings has been included and shown to have a significant impact on the resulting lifetime. The results were also obtained by a fully numerical approach instead of the analytical approximations previously used in the literature.

Now, it is an interesting question how the stability issue gets modified in the context of the quantum scale-invariant extension of the SM[9]. First of all, there appear non-renormalizable operators suppressed by the powers of the expectation value of the dilaton, the $\langle\sigma\rangle$. The suppression scale needs to be assumed to be pretty high to avoid significant modifications of the Standard Model predictions, since all couplings of the additional field σ which are not assumed numerically small to arrange for the hierarchy of the EW scale are suppressed by $\langle\sigma\rangle$. Besides, the new, with respect to the SM, corrections to the couplings of the non-renormalizable operators are proportional to powers of the small couplings λ_σ, λ_m. Hence, the influence of the non-renormalizable operators in the stability of the EW vacuum is the same in the pure SM and in the quantum scale-invariant SM. However, the interesting new effect does appear and it is related to the exact scale invariance of the potential. The result of this invariance to all orders in perturbation theory is the general structure of the scalar potential which takes the form of

$$V_{\rm eff}(\phi,\sigma) = M^4 W(\theta)\,, \tag{25}$$

where $M^2 = \phi^2 + \sigma^2$ and $\tan(\theta) = \frac{\phi}{\sigma}$. This structure makes it obvious that the running of the Higgs quartic coupling $\lambda_{\rm eff}$ towards negative values

[9] This section is based on the work with Paweł Olszewski, paper in preparation [44].

implies that the function $W(\theta)$ becomes negative for $\theta > \theta_0$ for certain value θ_0 corresponding to the vanishing effective potential. However, if such a finite θ_0 does exist, this implies an incurable instability, since by making M suitably large for $\theta > \theta_0$, one can make the effective potential as negative as one wishes. The upshot is that to stabilise the electroweak vacuum in the simplest quantum scale-invariant extension of the SM, one has to prevent the effective Higgs quartic coupling from becoming negative. Precise numerical calculations confirm the validity of the reasoning sketched above. However, one should notice that the coupling to gravity should be carefully taken into account. When one couples the flat space action to Weyl gravity or to Brans–Dicke gravity [39–42, 45], the picture may become modified.

5. Conclusion

We explored the possibility that scale symmetry is a quantum symmetry of the SM that is broken only spontaneously. Following previous developments on this idea, we considered the case of the classically scale-invariant version of the SM which has vanishing tree-level mass for the Higgs (ϕ) and is extended by the dilaton σ (the Goldstone mode of scale symmetry). The v.e.v. $\langle\sigma\rangle \neq 0$ breaks scale symmetry spontaneously and generates dynamically a subtraction scale $\mu \sim \langle\sigma\rangle$. The classical scalar potential is dictated by scale symmetry and may contain non-polynomial effective operators such as ϕ^6/σ^2, *etc.*; these may be expanded into a sum of infinitely many polynomial operators in fields (suppressed by $\langle\sigma\rangle$).

The one-loop computation of the potential respected the scale symmetry of the classical Lagrangian. As a result, a scale-invariant one-loop potential for the Higgs and dilaton is obtained. The quantum potential has corrections from gauge and Yukawa interactions and also from the higher dimensional, non-polynomial operators. The latter were included in the classical Lagrangian and their couplings are one-loop renormalized, with beta functions that we computed from the potential (and which are otherwise difficult to compute by other means). Tuning the couplings of these non-polynomial operators to zero at the tree-level will not allow to avoid their presence at the quantum level; they re-emerge at the loop level with a finite one-loop coefficient and as two-loop counterterms, due to the non-renormalizability of theories with quantum scale invariance.

It has been argued that the instability of the effective potential in the Higgs sector that is driven by the quartic coupling running towards negative values becomes worse in the scale-invariant version, since the effective potential becomes unbounded from below. This presentation has been based on [10, 43, 44, 46].

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