

NUCLEAR ASTROPHYSICS IN THE MULTIMESSENGER ERA: A PARTNERSHIP MADE IN HEAVEN*

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On August 17, 2017 the LIGO–Virgo Collaboration detected, for the first time, gravitational waves from the binary merger of two neutron stars (GW170817). Unlike the merger of two black holes, the associated electromagnetic radiation was also detected by a host of telescopes operating over a wide range of frequencies — opening a brand new era of multimessenger astronomy. This historical detection is providing fundamental new insights into the astrophysical site for the r-process and on the nature of dense matter. In this contribution, we examine the impact of GW170817 on the equation of state of neutron rich matter, particularly on the density dependence of the symmetry energy. Limits on the tidal polarizability extracted from GW170817 seem to suggest that the symmetry energy is soft, thereby excluding models that predict overly large stellar radii.

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1. Introduction

Almost a century ago in 1915, Albert Einstein published his landmark paper on “*The Field Equations of Gravitation*” [1]. Shortly after, Einstein predicted the existence of gravitational waves — ripples in space-time that travel at the speed of light [2, 3]. The stretching and squeezing of spacetime induces a periodic increase and decrease of the distance between objects that could, in principle, be detected by sophisticated laser interferometers. In analogy to electromagnetic waves that are created by accelerating charges, gravitational waves are created by time variations of massive objects with a non-uniform mass distribution, such as an intrinsic mass quadrupole moment.

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It would take nearly six decades to confirm, albeit indirectly, the existence of gravitational radiation. Back in 1974, using the Arecibo telescope in Puerto Rico, Hulse and Taylor discovered the first binary pulsar (PSR B1913+16) [4], a remarkable achievement for which they were awarded the Nobel Prize in Physics in 1993. Since initially discovered, the orbit of the binary neutron star system has been slowly and steadily shrinking for over 30 years in a manner precisely predicted by the general theory of relativity. At the observed rate of energy loss due to the emission of gravitational radiation, the Hulse–Taylor binary pulsar will merge in about 300 million years.

Fittingly, it was near the centennial celebration of the birth of general relativity that the LIGO–Virgo scientific collaboration reported the first direct detection of gravitational waves from a binary black hole merger [5]. On September 14, 2015, shortly after the advanced interferometers were turned on, a gravitational-wave signal corresponding to a binary black hole merger was detected at both detectors; Hanford in Washington State and Livingston in the state of Louisiana. Using theoretical waveforms predicted by general relativity, individual black holes with masses of about 36 and 29 solar masses merged to produce a final black hole with a mass of $62 M_{\odot}$. This implies that about 3 solar masses were radiated in the form of gravitational waves, or about ten billion times the amount of energy radiate by our own sun in one year. As impressive, a peak gravitational strain of 10^{-21} was measured, suggesting that the 4 km arms of both interferometers were stretched and squeezed by a few femtometers. This dramatic discovery opened the new and exciting era of gravitational-wave astronomy.

Soon after, the first detection of gravitational waves from a binary neutron star merger (GW170817) at a distance of about 40 Mpc opened the brand new era of multimessenger astronomy [6]. About two seconds after the arrival of the gravitational-wave signal, the Fermi Gamma-ray Space Telescope identified a short duration γ -ray burst in association with the neutron star merger [7]. Within eleven hours of the initial detection, ground- and spaced-based telescopes operating at a variety of wavelengths identified the associated *kilonova* — the electromagnetic transient powered by the radioactive decay of the heavy elements synthesized in the rapid neutron-capture process (r-process). Distinct features of the kilonova light curve — such as its fast rise, decay, and rapid color evolution from blue to red — are consistent with the large opacity typical of the lanthanides, spanning atomic number from 57 to 71. Such characteristic features of the optical spectrum have revealed that about 0.05 solar masses (or nearly 10,000 earth masses!) of r-process elements were synthesized in this single event [8–11]. The gravitational wave detection confirmed the long-held belief of the association of short gamma-ray bursts to neutron star mergers. Further, GW170817

established that binary neutron star mergers play a critical site in the production of heavy elements in the cosmos. Finally, constraints on the tidal polarizability (or deformability) of the binary system are starting to provide fundamental new insights into the nature of dense matter. Thus, in one clean sweep, GW170817 is providing compelling answers to two of the “Eleven science questions for the next century” [12] identified by the National Academies Committee on the Physics of the Universe: “What are the new states of matter at exceedingly high density and temperature?” and “How were the elements from iron to uranium made?” Colloquially, one can say that GW170817 created gravitational waves, light, and gold.

The first direct detection of the binary neutron star merger GW170817 is already providing valuable clues into the enigmatic nature of dense matter [6]. In particular, fundamental properties of the equation of state are encoded on the tidal polarizability, an intrinsic neutron-star property that describes its tendency to develop a mass quadrupole as a response to the tidal field induced by its companion [13, 14]. When the separation between the two neutron stars is large relative to their intrinsic size, the gravitational wave profile is practically indistinguishable from that of a binary black hole. Yet, as the neutron stars approach each other, tidal distortions become progressively more important. Tidal distortions modify the phase of the gravitational wave from its point-mass nature and increase the efficiency of gravitational wave emission, thereby precipitating the merger. How early during the inspiral phase do tidal effects become important is highly sensitive to the stellar compactness. That is, for a neutron star of a given mass, a star with a large radius — and thus a lower average density — is easier to tidally distort than a star with a smaller radius and, therefore, more compact. Essentially, the tidal polarizability probes the “fluffiness” of the neutron star. Among the many critical results inferred from GW170817 were relatively small tidal polarizabilities that “disfavor equations of state that predict less compact stars” [6]. For example, for a $1.4 M_{\odot}$ neutron star, limits on the tidal polarizability translate into an associated stellar radius of $R_{\star}^{1.4} \lesssim 13.76$ km [15, 16].

The main goal of this contribution is to examine how the first detection of gravitational waves from GW170817 improves our knowledge of the equation of state (EOS) of dense matter. Measurements of the tidal polarizability of the two stars have resulted in valuable limits on the stellar radius of neutron stars, a quantity that has been traditionally difficult to determine. Indeed, the determination of stellar radii by photometric means has been plagued by large systematic uncertainties, often revealing discrepancies as large as 5–6 km [17–19]. It appears, however, that the situation has improved considerably through a better understanding of systematic uncertainties, important theoretical developments, and the implementation

of robust statistical methods [20–27]. Nevertheless, that the detection of gravitational waves from a binary neutron star merger offers a compelling complement to photometric techniques is a welcome alternative.

We have organized the paper as follows. In Sec. 2, we sketch some of the important details that are needed to compute the tidal polarizability of a neutron star. We then proceed to Sec. 3 to display results that highlight the impact of GW170817 on the underlying equation of state. Finally, we conclude in Sec. 4.

2. Sensitivity of the tidal polarizability to the equation of state

2.1. Tolman–Oppenheimer–Volkoff equations

The structure of neutron stars is encapsulated in the TOV equations, named after Tolman, Oppenheimer, and Volkoff [28, 29]. The TOV equations represent the generalization of Newtonian gravity to the domain of general relativity. Remarkably, the only input required for their solution is the equation of state of cold (zero temperature) matter in chemical (or “beta”) equilibrium. Indeed, for static, spherically symmetric stars in hydrostatic equilibrium, the TOV equations may be written as a pair of first order differential equations. That is,

$$\frac{dP(r)}{dr} = -\frac{G(\mathcal{E}(r) + P(r)) \left(M(r) + 4\pi r^3 \frac{P(r)}{c^2} \right)}{c^2 r^2 \left(1 - \frac{2GM(r)}{c^2 r} \right)}, \quad (1a)$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \frac{\mathcal{E}(r)}{c^2}, \quad (1b)$$

where $M(r)$, $\mathcal{E}(r)$, and $P(r)$ are the mass, energy density, and pressure profiles, respectively. Given boundary conditions in terms of a central pressure $P(0) = P_c$ and enclosed mass at the origin $M(0) = 0$, the TOV equations may be solved using any suitable numerical solver. Note that the stellar radius R and mass M are determined from the following two conditions: $P(R) = 0$ and $M = M(R)$. Also note that the solution of the problem requires a relation connecting the pressure to the energy density, namely, an equation of state.

2.2. Composition and equation of state

Although it is fairly well-understood how the number of electrons determines the chemistry of the atom and how chemistry is responsible for binding atoms into molecules and molecules into both traditional and fascinating new materials, one would like to understand *how does matter organize*

itself at densities significantly higher than those found in everyday materials; say, from 10^4 – 10^{15} g/cm³. In this units, the equilibrium (or saturation) density of atomic nuclei equals 2.48×10^{14} g/cm³. This density, corresponding to about 0.15 nucleons per cubic fermi, is found in the interior of nuclei. At these enormous densities, it is the pressure rather than the temperature that is responsible for squeezing electrons out of the atoms. As depicted in Fig. 1, neutron stars contain a non-uniform crust above a uniform liquid core that is comprised of a uniform assembly of neutrons, protons, electrons, and muons in chemical equilibrium. Given that the densities in the stellar core may exceed that of normal nuclei by up to an order of magnitude, both electrons and muons contribute to neutralize the positive charge carried by the protons. Although the highest densities attained in massive neutron stars is presently unknown, for soft equations of state — namely, those with a pressure that rises slowly with density — the highest density may be such as to favor the formation of new and exotic states of matter [30–34]. However, at densities below saturation density, other novel phases of matter emerge under these extreme conditions. As the density falls below about 1/2 to 1/3 of saturation density, the average separation between nucleons increases to such an extent that it becomes energetically favorable for the system to segregate into regions of normal density (nuclear clusters) and

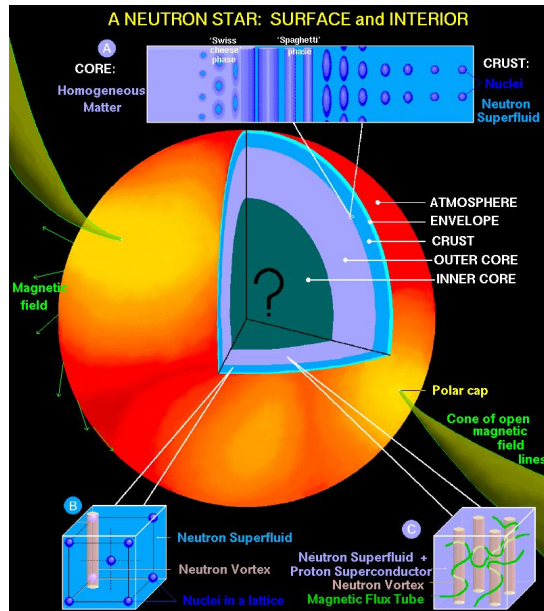


Fig. 1. An accurate rendition of the structure and phases of a neutron star, courtesy of Dany Page. Of great relevance to the neutron star is the composition of both the stellar crust and core, as well as their contribution to the equation of state.

regions of low density (a dilute, likely superfluid, neutron vapor). Such a clustering instability signals the transition from the uniform liquid core to the non-uniform crust. The crust itself is divided into an outer and an inner region. Structurally, the outer crust is comprised of a Coulomb lattice of neutron-rich nuclei embedded in a uniform electron gas [35–40]. Given that the electronic density increases rapidly with density, it becomes energetically favorable for electrons to capture into protons, resulting in nuclear clusters that are progressively more neutron rich. Eventually, the neutron excess becomes too large for the nuclear clusters to absorb any more neutrons, marking the transition to the inner crust — a region of the star characterized by a Coulomb crystal of neutron rich nuclei embedded in a uniform Fermi gas of electrons and a dilute vapor of likely superfluid neutrons. Even deeper in the crust, distance scales that were well-separated in both the crystalline phase — where the long-range Coulomb interaction dominates — and in the uniform phase — where the short-range strong interaction dominates — become comparable, giving rise to a universal phenomenon known as *Coulomb frustration*. Coulomb frustration is characterized by a myriad of complex structures radically different in topology yet extremely close in energy — collectively referred to as *nuclear pasta* [41, 42]; see Fig. 1. The fascinating and subtle pasta dynamics has been captured using either semi-classical numerical simulations [43–51] or quantum-mechanical approaches in a mean-field approximation [52–56]. Yet, despite the undeniable progress in understanding the nuclear-pasta phase, no theoretical framework exists at present that can simultaneously incorporate quantum-mechanical effects and complex dynamical correlations. As a result, a reliable equation of state for the inner crust is still missing. In the past, we have adopted a simple polytropic interpolation formula [57] to estimate the equation of state in the inner crust [58] and we will continue do so in this contribution.

The following prescription is adopted for the neutron-star matter equation of state. First, the EOS for the outer crust follows the seminal work of Baym, Pethick, and Sutherland (BPS) [35], slightly modified to incorporate the accurate mass formula of Duflo and Zuker [60]. The boundary of the outer crust is determined by demanding that the chemical potential be equal to the bare neutron mass. Second, given the complexity of the inner crust, we adopt a polytropic equation of state that interpolates between the outer crust and the liquid core [58]. In particular, the crust–core boundary is determined from an RPA analysis that signals the instability of the uniform ground state to cluster formation. Finally, The EOS in the uniform liquid core is derived from a relativistic model that is accurate in the description of both the properties of finite nuclei and neutron stars [61]. As an example of such an equation of state, we display in Fig. 2 the predictions from the recently calibrated relativistic density functional “FSUGarnet” [59].

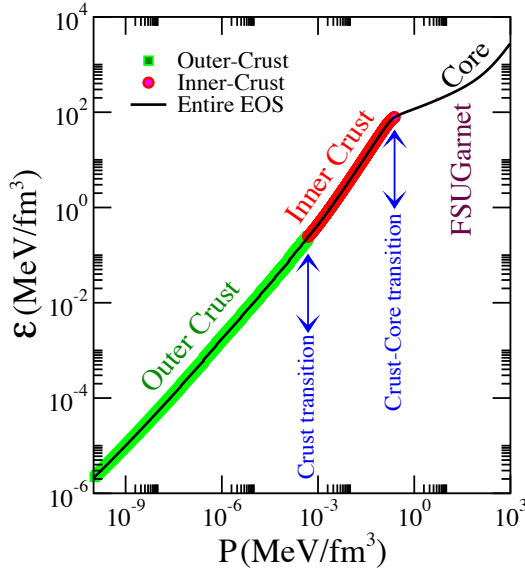


Fig. 2. Neutron-star-matter equation of state as predicted by the relativistic density functional “FSUGarnet” [59]. The composition and equation of state in the non-uniform crust is described in the text. Note that the features that are mostly sensitive to the choice of density functional are the crust–core transition pressure and the EOS in the entire uniform liquid core.

2.3. Tidal polarizability

Finally, we focus on the tidal polarizability. In the linear regime, *i.e.*, in the limit of weak tidal fields, the ratio of the induced mass quadrupole to the external tidal field defines the tidal polarizability. The tidal polarizability is the gravitational analog to the electric polarizability of a drop of water. A polar molecule such as water develops an electric dipole moment in response to an external electric field. The magnitude of the response is encoded in the dielectric constant — an intrinsic property of the material. In the same manner, the response of a neutron star to an external tidal field is encoded in the tidal polarizability. Moreover, a time-dependent mass quadrupole emits gravitational radiation in analogy to the electromagnetic radiation generated by a time-dependent electric dipole moment. Particularly useful is the *dimensionless* tidal polarizability Λ that is defined as [6]

$$\Lambda = \frac{2}{3}k_2 \left(\frac{c^2 R}{GM} \right)^5 = \frac{64}{3}k_2 \left(\frac{R}{R_s} \right)^5, \quad (2)$$

where k_2 is the second Love number [62, 63], M and R are the mass and radius of the neutron star, and $R_s \equiv 2GM/c^2$ the associated Schwarzschild

radius. Evidently, Λ is a very sensitive quantity of the compactness parameter $\xi \equiv R_s/R$ [64–69]. In turn, the second Love number k_2 depends on both ξ and y_R — a dimensionless parameter that is sensitive to the entire equation of state [64, 65]. The parameter y_R is associated to the non-spherical component of the gravitational potential at the surface of the star. In the limit of axial symmetry, the leading non-spherical component of the gravitational potential is proportional to the product $H(r)Y_{20}(\theta, \varphi)$, where $Y_{20}(\theta, \varphi)$ is the “quadrupole” spherical harmonic [64]. In turn, $H(r)$ encodes the dynamical changes to the gravitational potential and satisfies a linear, homogeneous, second order differential equation that may be solved in conjunction with the corresponding TOV equations [64, 65, 67, 68]. The value of y_R is obtained from the logarithmic derivative of $H(r)$ evaluated at the surface of the star and, hence, depends on the entire equation of state. Once y_R is known, the second Love number k_2 can be computed and from it — and the compactness ξ — the tidal polarizability Λ .

3. Results

Having defined the entire formalism, we are now in a position to display results for the tidal polarizabilities inferred from GW170817. In Fig. 3, we show tidal polarizabilities Λ_1 and Λ_2 associated with the high-mass M_1 and low-mass M_2 components of GW170817 using a collection of ten relativistic models that provide an accurate description of the properties of finite nuclei and neutron stars [15]. The various models differ in their choice of the density dependence of the symmetry energy, particularly the slope of the symmetry energy at saturation — a quantity commonly denoted by L that is proportional to the pressure of pure neutron matter at saturation. Moreover, the neutron skin thickness of ^{208}Pb is a laboratory observable that has been shown to be strongly correlated to L [70–73]. Thus, the various models in the figure are labeled using the neutron skin thickness of ^{208}Pb as a proxy for L . The combination of neutron star masses displayed in the figure are constrained by maintaining the very well measured “chirp” mass fixed at [6]

$$\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}} = 1.188 M_\odot. \quad (3)$$

In turn, the solid circles are used to indicate predictions for a binary system having masses of $M_1 = 1.4 M_\odot$ and $M_2 = 1.33 M_\odot$, respectively, corresponding to a chirp mass as in Eq. (3). Finally, we show the 90% probability contour extracted from the low-spin scenario assumed in Fig. 5 of the discovery paper [6]. Models to the right of the contour predict a symmetry energy that is too stiff and, as a consequence, stellar radii that are too large, to be consistent with the LIGO-Virgo analysis.

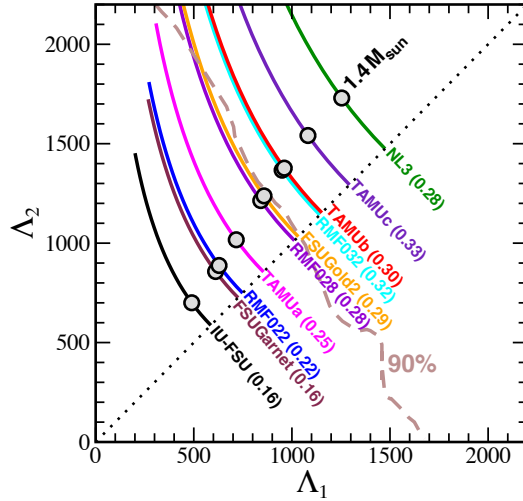


Fig. 3. Tidal polarizabilities Λ_1 and Λ_2 associated with the high-mass M_1 and low-mass M_2 components of the binary neutron star system GW170817 as predicted by a set of ten distinct relativistic mean-field models [15]. Models to the right side of the 90% probability contour extracted from Ref. [6] are ruled out. The solid circles represent model predictions for a binary system having masses of $M_1 = 1.4 M_\odot$ and $M_2 = 1.33 M_\odot$, respectively.

We conclude by displaying in Fig. 4 the mass *vs.* radius relation. For the models of the kind described here, the maximum stellar mass is largely controlled by the high-density component of the EOS of symmetric matter with equal numbers of neutrons and protons. In contrast, stellar radii — as well as tidal polarizabilities — are sensitive to the density dependence of the symmetry energy. All ten models used in this contribution generate an EOS that is sufficiently stiff to support an $M_\star \approx 2M_\odot$ neutron star [74, 75]. However, by incorporating the new constraints on the tidal polarizability of an $M_\star = 1.4 M_\odot$ neutron star, we deduced an upper limit on the stellar radii of 13.76 km [15]. Note that a revised analysis of the original GW170817 data that now assumes the same EOS for the two stars seems to suggest even more restrictive bounds on the tidal polarizability [76]. Interestingly enough, by combining electromagnetic and gravitational wave information in this new era of multimessenger astronomy, additional constraints have been obtained on both the maximum stellar mass ($2.17 M_\odot$) [77] and the minimum radius of a 1.6 solar mass neutron star (10.7 km) [78].

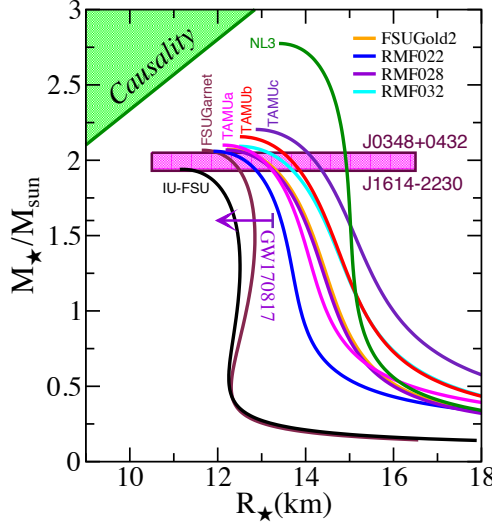


Fig. 4. Mass *vs.* radius relation predicted by the same ten models used in Fig. 3. Mass constraints obtained from electromagnetic observations of two neutron stars are indicated with a combined uncertainty bar [74, 75]. In contrast, the arrow incorporate constraints on stellar radii obtained exclusively from GW170817 and exclude many of the otherwise acceptable equations of state [15]. The excluded causality region was adopted from Fig.2 of Ref. [79].

4. Conclusions

Neutron stars provide a powerful intellectual bridge between Nuclear Physics and Astrophysics. This synergy will strengthen even further with the recent detection of gravitational waves from the merger of two neutron stars. In this contribution, we explored the fascinating structure of neutron stars, their connection to nuclear physics through the underlying equation of state, and the new limits imposed on the EOS from tidal distortions. The connection between the two fields is strong because of the sensitivity of the tidal polarizability to the stellar radius, which, in turn, probes the symmetry energy at about twice nuclear matter saturation density. In particular, limits on the tidal polarizability of a $1.4 M_{\odot}$ neutron star translated into an upper limit of $R_{\star}^{1.4} \lesssim 13.76$ km for the associated stellar radius. The multimessenger era is in its infancy, yet it is remarkable that the very first observation of a neutron star merger is already providing a treasure trove of insights into the nature of dense matter. The third observing run by the LIGO–Virgo Collaboration is scheduled to start in 2019 and with it the expectation of many more detections of neutron star mergers. The future of multimessenger astronomy is very bright indeed!

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