SPECTROSCOPY OF Ω_{cc}, Ω_{bb} AND Ω_{bc} BARYONS IN HYPERCENTRAL CONSTITUENT QUARK MODEL VIA ANSATZ METHOD

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In this paper, we exhibited the ground and excited state masses of doubly heavy Ω baryons. For this purpose, we have analytically solved the six-dimensional radial Schrödinger equation for three identical particles with the hypercentral potential by using the Ansatz method. The hypercentral potential is considered as a combination of the hypercoulomb, linear confining, and the harmonic oscillator terms which has a two-body character and turns out to be exactly hypercentral. We also incorporated the first order correction and the spin-dependent part to the confinement potential. Our calculations have been performed for the radial excited states as well as orbital excited states masses of Ω_{cc}, Ω_{bb} and Ω_{bc} baryons. The obtained masses are compared with other theoretical predictions, which could be a useful tool for the interpretation of experimentally unknown doubly heavy baryons spectrum.

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1. Introduction

Baryons are strongly interacting fermions made up of three quarks. Singly heavy baryons have one heavy quark and two light quarks. Doubly heavy baryons, with two heavy quarks and one light quark, are expected to exist in QCD and their masses have been predicted in the quark model. Ω baryons have a light strange quark with two heavy quarks (c and b) [1]. In the last years, experimental and theoretical outcomes have been used in studying the heavy baryons. A lot of new experimental results have been reported by different collaborations including CLEO, Belle, BaBar and LHCb [2, 3] on ground and many new excited states of heavy flavour baryons. The Particle Data Group (PDG) listed 20 known charm baryons [4]. The experimental evidence comes for Ξ_{cc}^{+} (containing two charm quarks) with a mass of $\sim 3\,520$ MeV$/c^2$ by the SELEX experiment, and LHCb has determined the five excited states of Ω_c baryon and the ground state of Ξ_{cc}^{+} baryon [4–7]. Recently, various phenomenological approaches have been used to study the doubly heavy baryons including relativistic quark model [8], the chiral unitary model [9], the extended local hidden gauge approach [10], the relativistic fluxtube (RFT) model [11], the Hamiltonian model [12], Regge phenomenology [13], QCD sum rule [14, 15], *etc.*

The nonrelativistic constituent quark models (CQM) among them have also yielded good results studying the baryons static properties [16, 17], such as the baryon spectrum [1, 8, 12, 18–20], the magnetic moments, the photocouplings [21, 22], the electromagnetic form factors [23] and the strong decay amplitudes. The spectrum of baryon is usually well-described. However, the various models are quite different. It is necessary to note that the study of hadron spectroscopy is not enough to distinguish the different forms of quark dynamics. To do so, we need to study in a consistent way all the physical observables of interest. This systematic study of baryon properties is better done using a general framework. Here, a hypercentral approach to quark dynamics can be applied [24, 25]. As proposed by lattice QCD calculations [26, 27], the model comprises a hypercentral quark interaction involving a linear plus Coulomb-like term. The hCQM scheme can be used for baryons, which is an average two-body potential for the three-quark system over the hyperangle and performs quite well. Our study here is also based on the hypercentral constituent quark model (hCQM). Since the solution of the hyper-radial Schrödinger equation with the Cornell potential cannot be obtained analytically [23], therefore, we added the harmonic oscillator terms which has a two-body character and turns out to be exactly hypercentral [2, 4, 18]. In fact, we have used a modified version of the original model with only hypercoulomb and linear confinement in the hyper-radious [2]. We also added the first order correction and the spin-dependent part to the potential, and calculations for doubly heavy Ω baryons masses have been performed by solving six-dimensional hyper-radial Schrödinger equations by using the Ansatz method. We have obtained the mass spectra of radial excited states up to $5S$ and orbital excited states for $1P$–$5P$, $1D$–$4D$ and $1F$–$2F$ states.

This paper is organized as follows: after the introduction of doubly heavy baryons, the hCQM and the interaction potentials between three quarks in baryons are explained in Sec. 2. In Sec. 3, we present the quasi-exact analytical solution of the radial Schrödinger equation for our proposed potential. In Sec. 4, our masses spectra results are given compared with other predictions. Our concluding remarks are given in Sec. 5.

2. The hypercentral model and hypercentral potential

In this paper, the hypercentral Constitute Quark Model (hCQM) has been used to generate the mass spectrum of doubly heavy baryons. The brief description of hCQM model is as follows.

By considering baryon as a three-body system, in the center-of-mass frame ($R_{\rm cm} = 0$), the internal quark motion is described by the Jacobi coordinates, ρ and λ [28], defined as:

$$\vec{\rho} = \frac{1}{\sqrt{2}}\left(\vec{r}_1 - \vec{r}_2\right) , \qquad \vec{\lambda} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 - (m_1+m_2)\,\vec{r}_3}{\sqrt{m_1^2 + m_2^2 + (m_1+m_2)^2}} . \tag{1}$$

The respective reduced masses are given by

$$m_\rho = \frac{2m_1m_2}{m_1+m_2} , \qquad m_\lambda = \frac{2m_3\left(m_1^2+m_2^2+m_1m_2\right)}{(m_1+m_2)\,(m_1+m_2+m_3)} . \tag{2}$$

Here, m_1, m_2 and m_3 are the constituent quark masses.

The angles of the hyperspherical coordinates are given by $\Omega_\rho = (\theta_\rho, \phi_\rho)$ and $\Omega_\lambda = (\theta_\lambda, \phi_\lambda)$. In order to describe the three-quark dynamics, we define hyper-radius x and hyperangle ξ in terms of the absolute values ρ and λ of the Jacobi coordinates [29] as

$$x = \sqrt{\rho^2+\lambda^2} , \qquad \xi = \arctan\left(\frac{\rho}{\lambda}\right) . \tag{3}$$

In the present paper, the confining three-body potential is regarded as a combination of three hypercentral interacting potentials. First, the six-dimensional hypercoulomb potential [30, 31] which is attractive for small separations

$$V_{\rm hyc}(x) = \frac{\tau}{x} , \tag{4}$$

while at large separations, a hyperlinear term gives rise to quark confinement [1, 2]

$$V_{\rm com} = \beta x . \tag{5}$$

β corresponds to the string tension of the confinement [32]. Third, the six-dimensional harmonic oscillator potential, which has a two-body character, and turns out to be exactly hypercentral since [33]

$$V_{\rm ho} = \sum_{i<j}^{i=3} \frac{1}{2}k(r_i - r_j)^2 = \frac{3}{2}kx^2 = px^2 . \tag{6}$$

The first order correction $V^{(1)}(x)$ can be written as [33–35]

$$V^{(1)}(x) = -C_{\mathrm{F}}C_{\mathrm{A}}\frac{\alpha_{\mathrm{s}}^2}{4x^2}\,. \tag{7}$$

The parameters $C_{\mathrm{F}} = \frac{2}{3}$ and $C_{\mathrm{A}} = 3$ are the Casimir charges of the fundamental and adjoint representation. The hypercoulomb strength $\tau = -\frac{2}{3}\alpha_{\mathrm{s}}$, $\frac{2}{3}$ is the color factor for the baryon. α_{s} is the strong running coupling constant, which is written as

$$\alpha_{\mathrm{s}} = \frac{\alpha_{\mathrm{s}}(\mu_0)}{1+\left(\frac{33-2n_{\mathrm{f}}}{12\pi}\right)\alpha_{\mathrm{s}}(\mu_0)\ln\left(\frac{m_1+m_2+m_3}{\mu_0}\right)}\,. \tag{8}$$

The spin-dependent part $V_{\mathrm{SD}}(x)$ is given as

$$V_{\mathrm{SD}}(x) = V_{\mathrm{SS}}(x)\left(\vec{S}_\rho\cdot\vec{S}_\lambda\right) + V_{\gamma\mathrm{S}}(x)\left(\vec{\gamma}\cdot\vec{S}\right) + V_{\mathrm{T}}(x)\left[S^2 - \frac{3\left(\vec{S}\cdot\vec{x}\right)\left(\vec{S}\cdot\vec{x}\right)}{x^2}\right]. \tag{9}$$

The spin-dependent potential, $V_{\mathrm{SD}}(x)$ contains three types of the interaction terms [36] including the spin–spin term $V_{\mathrm{SS}}(x)$, the spin–orbit term $V_{\gamma\mathrm{S}}(x)$ and tensor term $V_{\mathrm{T}}(x)$ described in Ref. [37]. Here, $S = S_\rho + S_\lambda$, where S_ρ and S_λ are the spin vectors associated with the ρ and λ variables, respectively. The coefficient of these spin-dependent terms can be written in terms of the vector, $V_{\mathrm{V}}(x) = \frac{\tau}{x}$ and scalar, $V_{\mathrm{S}}(x) = \beta x + px^2$ parts of the static potential as [29]

$$V_{\gamma\mathrm{S}} = \frac{1}{2m_\rho m_\lambda x}\left(3\frac{\mathrm{d}V_{\mathrm{V}}}{\mathrm{d}x} - \frac{\mathrm{d}V_{\mathrm{S}}}{\mathrm{d}x}\right), \tag{10}$$

$$V_{\mathrm{T}}(x) = \frac{1}{6m_\rho m_\lambda}\left(\frac{3\mathrm{d}^2V_{\mathrm{V}}}{\mathrm{d}^2x} - \frac{1}{x}\frac{\mathrm{d}V_{\mathrm{V}}}{\mathrm{d}x}\right), \tag{11}$$

$$V_{\mathrm{SS}}(x) = \frac{1}{3m_\rho m_\lambda}\nabla^2 V_{\mathrm{V}}\,. \tag{12}$$

In our model, the hypercentral interaction potential is assumed as follows [38]:

$$V(x) = V^{(0)}(x) + \left(\frac{1}{m_\rho} + \frac{1}{m_\lambda}\right)V^{(1)}(x) + V_{\mathrm{SD}}(x)\,, \tag{13}$$

where $V^{(0)}(x)$ is given by

$$V^{(0)}(x) = V_{\mathrm{hyc}}(x) + V_{\mathrm{con}}(x) + V_{\mathrm{ho}}(x)\,, \tag{14}$$

$$V^{(0)}(x) = \frac{\tau}{x} + \beta x + px^2\,. \tag{15}$$

In our purposed hypercentral potential, we have six parameters: m_1, m_2, m_3, τ, β and p. The baryon masses are determined by the sum of the model quark masses plus kinetic energy, potential energy and the spin-dependent interaction as [39]

$$M_B = \sum_{i=1}^{3} m_i + \langle H \rangle \,. \tag{16}$$

First, we will solve the Schrödinger equation via the quasi-exact analytical Ansatz approach and obtain the corresponding eigenvalues.

3. Quasi-exact analytical solution of the six-dimensional radial Schrödinger equation via Ansatz approach

The Hamiltonian of the three-body baryonic system in the hypercentral constituent quark model is expressed as [40]

$$H = \frac{P_\rho^2}{2m} + \frac{P_\lambda^2}{2m} + V(x)\,, \tag{17}$$

and the hyper-radial wave function $\psi_{\nu\gamma}(x)$ is determined by the hypercentral Schrödinger equation. The hyper-radial Schrödinger equation corresponding to the above Hamiltonian can be written as [1, 2, 38]

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{5}{x}\frac{\mathrm{d}}{\mathrm{d}x} - \frac{\gamma(\gamma+4)}{x^2}\right)\psi_{\nu\gamma}(x) = -2m[E - V(x)]\psi_{\nu\gamma}(x)\,, \tag{18}$$

where γ is the grand angular quantum number given by $\gamma = 2n + l_\rho + l_\lambda$, $n = 0, 1, \ldots$; l_ρ and l_λ are the angular momenta associated with the $\vec{\rho}$ and $\vec{\lambda}$ variables. ν denotes the number of nodes of the space three-quark wave function [41]. In Eq. (18), m is the reduced mass [42] which is defined as

$$m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda}\,. \tag{19}$$

The transformation [1, 2]

$$\psi_{\nu\gamma}(x) = x^{-\frac{5}{2}}\phi_{\nu\gamma} \tag{20}$$

reduces Eq. (18) to the form of

$$\phi''_{\nu\gamma}(x) + \left[\varepsilon - r_1 x^2 - r_2 x - \frac{r_3}{x} - \frac{r_4}{x^2} - \frac{r_5}{x^3} + \frac{r_6}{x^5} + r_7 - \frac{(2\gamma+3)(2\gamma+5)}{4x^2}\right] \times\phi_{\nu\gamma}(x) = 0\,. \tag{21}$$

The hyper-radial wave function $\phi_{\nu\gamma}(x)$ is a solution of the reduced Schrödinger equation for each of the three identical particles with the mass m and interacting potential (13), where

$$\begin{aligned}
\varepsilon &= 2mE\,, \qquad r_1 = 2mp\,, \qquad r_2 = 2m\beta\,, \qquad r_3 = 2m\tau\,, \\
r_4 &= 2m\left(\frac{1}{m_\rho}+\frac{1}{m_\lambda}\right)\left(-C_{\mathrm{F}}C_{\mathrm{A}}\frac{\alpha_{\mathrm{s}}^2}{4}\right)\,, \\
r_5 &= 2m\left[\frac{2\tau}{3m_\rho m_\lambda}(S_\rho\cdot S_\lambda)-\frac{3\tau}{2m_\rho m_\lambda}(\vec{\gamma}\cdot\vec{s}\,)+\frac{7\tau}{6m_\rho m_\lambda}s^2\right]\,, \\
r_6 &= 2m\frac{21\tau}{6m_\rho m_\lambda}\,(\vec{s}\cdot\vec{x}\,)\,(\vec{s}\cdot\vec{x}\,)\,, \\
r_7 &= 2m\left(\frac{(\beta+2p)}{2m_\rho m_\lambda}\,(\vec{\gamma}\cdot\vec{s}\,)\right)\,.
\end{aligned} \tag{22}$$

We assume the following form for the wave function:

$$\phi_{\nu\gamma} = h(x)e^{g(x)}\,. \tag{23}$$

Now, we make use of the Ansatz for the $h(x)$ and $g(x)$ [43–45]

$$\begin{aligned}
h(x) &= \Pi(x-\alpha_i^\nu)\,, \qquad \nu = 1,2,\dots\,, \\
h(x) &= 1\,, \qquad \nu = 0\,, \\
g(x) &= a\ln x + qx^2 + cx + \frac{d}{x}\,,
\end{aligned} \tag{24}$$

where a, q, c and d are positive constants. From Eq. (23), we obtain

$$\phi''(x) = \left[g''(x)+g'^2(x)+\left(\frac{h''(x)+2h'(x)g'(x)}{h(x)}\right)\right]\phi(x)\,. \tag{25}$$

Comparing Eqs. (21) and (25), it can be found that

$$\begin{aligned}
&\left[r_1x^2+r_2x+\frac{r_3}{x}+\frac{r_4}{x^2}+\frac{r_5}{x^3}-\frac{r_6}{x^5}-r_7+\frac{(2\gamma+3)(2\gamma+5)}{4x^2}-\varepsilon\right] \\
&= \left[g''(x)+g'^2(x)+\frac{h''(x)+2h'(x)g'(x)}{h(x)}\right]\,.
\end{aligned} \tag{26}$$

By substituting Eq. (24) into Eq. (26), we obtain the following equation:

$$\begin{aligned}
&-\varepsilon+r_1x^2+r_2x+\frac{r_3}{x}+\frac{r_4}{x^2}+\frac{r_5}{x^3}-\frac{r_6}{x^5}-r_7+\frac{(2\gamma+3)(2\gamma+5)}{4x^2} \\
&= 4q^2x^2+4cqx+\frac{(2ac-4dq)}{x}+\frac{(a^2-a-2cd)}{x^2}+\frac{2d(1-a)}{x^3}+\frac{d^2}{x^4} \\
&\quad +\left(c^2+2q+4ac\right)\,.
\end{aligned} \tag{27}$$

By equating the corresponding powers of x on both sides of Eq. (27), we can obtain

$$\begin{aligned} a &= \frac{2\tau}{\beta}\sqrt{\frac{mp}{2}}\,, \qquad c=\frac{m\beta}{2}\sqrt{\frac{2}{mp}}\,, \qquad q=\sqrt{\frac{mp}{2}}\,, \\ \varepsilon &= -\left[\frac{m\beta^2}{2p}+2\sqrt{\frac{mp}{2}}+\frac{4mp\tau}{\beta}+2m\left(\frac{(\beta+2p)}{2m_\rho m_\lambda}\left(\vec{\gamma}\cdot\vec{s}\,\right)\right)\right]. \end{aligned} \tag{28}$$

Since $p=\frac{m\omega^2}{2}$, we have

$$a=\frac{2m\omega}{2\beta}\,, \qquad c=\frac{\beta}{\omega}\,, \qquad q=\frac{m\omega}{2}\,. \tag{29}$$

The energy eigenvalues for the mode $\nu=0$ and grand angular momentum γ from Eqs. (22) and (28) are given as follows:

$$E=-\left[\frac{\beta^2}{2m\omega}+\frac{\omega}{2}+\frac{m\omega^2\tau}{\beta}+\left(\frac{(\beta+m\omega^2)}{2m_\rho m_\lambda}\right)\left(\vec{\gamma}\cdot\vec{s}\,\right)\right]. \tag{30}$$

Finally, for calculating the best doubly heavy baryons masses (Ω_{cc}, Ω_{bb}, Ω_{bc}) predictions, the values of m_s, m_c, m_b, α_s, ω and β (which are listed in Table I) are selected using genetic algorithm. The cost function of genetic algorithms is the minimum difference between our calculated baryon mass and the reported baryons mass of other works.

TABLE I

The quark mass (in GeV) and the fitted values of the parameters used in our calculations.

m_s	m_c	m_b	α_s	C_F	C_A	β	ω
0.565	1.345	4.902	0.340	$\frac{2}{3}$	3	0.01	0.142 fm^{-1}

4. Results and discussions: mass spectrum

The ground and excited states of doubly heavy Ω baryons are unknown to us experimentally. Hence, we have obtained the ground and excited state masses of Ω_{cc}, Ω_{bb} and Ω_{bc} (Tables II, III, IV, V and VI). These mass spectra are estimated by using the hypercentral potential Eq. (13) in the hypercentral constituent quark model. We begin with the ground state $1S$ and the masses are computed for both parities $J^P=\frac{1}{2}^+$ and $J^P=\frac{3}{2}^+$. Our predicted ground state masses of doubly heavy Ω baryons are compared with other predictions in Table II. Our calculations for the ground state masses of Ω_{cc}^{+}, Ω_{bb}^{-} and Ω_{bc}^{0} are different from other predictions in the vicinity of ≈ 100 MeV, ≈ 500 MeV and ≈ 400 MeV, respectively.

TABLE II

The ground state masses of Ω_{cc}^{+}, Ω_{bb}^{-} and Ω_{bc}^{0} are listed with other theoretical predictions (in GeV).

Baryons	Ω_{cc}^{+}		Ω_{bb}^{-}		Ω_{bc}^{0}	
J^P	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$
Our calc.	3.662	3.677	10.870	10.866	7.329	7.339
Ref. [38]	3.650	3.810	10.446	10.467	7.136	7.187
Ref. [46]	3.719	3.746	10.442	10.432	6.999	7.024
Ref. [47]	3.778	3.872	10.359	10.389	7.088	7.130
Ref. [48]	3.648	3.770	10.271	10.289	6.994	7.017
Refs. [49, 50]	3.710	3.760	10.320	10.380		
Refs. [51, 52]	3.730	3.780	9.970	10.50	6.750	7.300
Ref. [53]	3.832	3.883	10.447	10.467		
Ref. [54]	3.815	3.876	10.454	10.486	7.136	7.187
Ref. [55]	3.697	3.769	10.293	10.321		
Ref. [56]	3.747	3.819				
Ref. [57]	3.713	3.785				
Ref. [58]	3.738	3.822	10.273	10.308	6.999	7.059
Ref. [59]	3.740	3.779				
Ref. [60]	3.654	3.724				
Refs. [61, 62]	3.650	3.809	10.320	10.430		
Ref. [63]	3.702	3.783	10.260	10.297	6.986	7.046
Ref. [64]	3.667	3.758	10.397	10.495	7.103	7.200
Ref. [65]	3.710	3.800	10.208	10.244	6.999	7.063
Ref. [66]	3.740	3.820	10.370	10.400	7.045	7.120

The radial excited state masses for these three doubly heavy baryons are computed from $2S$–$5S$ and are compared with Refs. [38, 47, 48, 53–55] in Table III.

We can observe that our $2S$ and $3S$ states show a smaller difference in MeV, see Refs. [38, 47], than other references, and for $4S$ and $5S$ states, have a difference in the range of ≈ 100 MeV comparing with Ref. [38] for Ω_{cc}^{+}. Our calculations for $2S$ and $3S$ states of Ω_{bb}^{-} are close to Refs. [38, 48] and for $4S$ and $5S$ states, have a difference in the range of ≈ 150 MeV comparing with Ref. [38]. In the case of Ω_{bc}, we can observe that our $2S$, $3S$, $4S$ and $5S$ state masses with $J^P = \frac{1}{2}^+$ are 5 MeV, 158 MeV, 243 MeV and 267 MeV, while 12 MeV, 156 MeV, 236 MeV and 261 MeV (with $J^P = \frac{3}{2}^+$) are lower than the results of Ref. [38], respectively.

TABLE III

The masses of radial excited states for Ω_{cc}, Ω_{bb} and Ω_{bc} (in GeV).

Baryons	State	J^P	Our calc.	[38]	[53]	[54]	[55]	[48]	[47]
Ω_{cc}	$2S$	$\frac{1}{2}^+$	3.964	4.041	4.227	4.180	4.112	4.268	4.075
		$\frac{3}{2}^+$	3.979	4.096	4.263	4.188	4.160	4.334	4.174
	$3S$	$\frac{1}{2}^+$	4.177	4.338	4.295			4.714	4.321
		$\frac{3}{2}^+$	4.193	4.365	4.265			4.776	
	$4S$	$\frac{1}{2}^+$	4.452	4.598					
		$\frac{3}{2}^+$	4.467	4.614					
	$5S$	$\frac{1}{2}^+$	4.787	4.836					
		$\frac{3}{2}^+$	4.802	4.845					
Ω_{bb}	$2S$	$\frac{1}{2}^+$	10.969	10.736	10.707	10.693	10.604	10.830	10.610
		$\frac{3}{2}^+$	10.964	10.743	10.723	10.721	10.622	10.839	10.645
	$3S$	$\frac{1}{2}^+$	11.036	10.983	10.744			11.240	10.806
		$\frac{3}{2}^+$	11.032	10.986	10.730			11.247	10.843
	$4S$	$\frac{1}{2}^+$	11.123	11.205	10.994				
		$\frac{3}{2}^+$	11.119	11.207	11.031				
	$5S$	$\frac{1}{2}^+$	11.230	11.411					
		$\frac{3}{2}^+$	11.225	11.412					
Ω_{bc}	$2S$	$\frac{1}{2}^+$	7.475	7.480				7.559	
		$\frac{3}{2}^+$	7.485	7.497				7.571	
	$3S$	$\frac{1}{2}^+$	7.609	7.767				7.976	
		$\frac{3}{2}^+$	7.619	7.775				7.985	
	$4S$	$\frac{1}{2}^+$	7.782	8.023					
		$\frac{3}{2}^+$	7.792	8.028					
	$5S$	$\frac{1}{2}^+$	7.993	8.260					
		$\frac{3}{2}^+$	8.002	8.263					

To calculate the orbital excited state masses ($1P$–$5P$, $1D$–$4D$, $1F$–$2F$), we have considered all possible isospin splitting and all combinations of total spin S and total angular momentum J. Our outcomes and the comparison of masses with other approaches are also tabulated in Tables IV, V and VI.

TABLE IV

The masses of orbital excited states of Ω_{cc} (in GeV).

State	Our calc.	[38]	[53]	[54]	[57]	[47]	[61]	Other
$(1^2P_{1/2})$	3.965	3.989	4.086	4.046	4.061	4.002		4.009 [55]
$(1^2P_{3/2})$	3.956	3.972	4.086	4.052	4.132	4.102	3.910	
$(1^4P_{1/2})$	3.987	3.998						
$(1^4P_{3/2})$	3.978	3.981						3.960 [52]
$(1^4P_{5/2})$	3.926	3.958	4.220	4.152			4.058	
$(2^2P_{1/2})$	4.194	4.273	4.199	4.135		4.251		4.101 [55]
$(2^2P_{3/2})$	4.155	4.259	4.201	4.140		4.345		
$(2^4P_{1/2})$	4.215	4.280						
$(2^4P_{3/2})$	4.176	4.266						
$(2^4P_{5/2})$	4.154	4.247						
$(3^2P_{1/2})$	4.453	4.529						
$(3^2P_{3/2})$	4.444	4.517						
$(3^4P_{1/2})$	4.475	4.536						
$(3^4P_{3/2})$	4.466	4.523						
$(3^4P_{5/2})$	4.414	4.506						
$(4^2P_{1/2})$	4.803	4.767						
$(4^2P_{3/2})$	4.764	4.755						
$(4^4P_{1/2})$	4.825	4.772						
$(4^4P_{3/2})$	4.788	4.761						
$(4^4P_{5/2})$	4.763	4.745						
$(5^2P_{1/2})$	5.183	4.989						
$(5^2P_{3/2})$	5.175	4.978						
$(5^4P_{1/2})$	5.206	4.994						
$(5^4P_{3/2})$	5.198	4.984						
$(5^4P_{5/2})$	5.145	4.969						
$(1^4D_{1/2})$	4.215	4.186						
$(1^2D_{3/2})$	4.156	4.162						
$(1^4D_{3/2})$	4.193	4.170						
$(1^2D_{5/2})$	4.116	4.141	4.264	4.202			4.153	
$(1^4D_{5/2})$	4.155	4.149						
$(1^4D_{7/2})$	4.086	4.122					4.294	
$(2^4D_{1/2})$	4.490	4.446						
$(2^2D_{3/2})$	4.429	4.425						
$(2^4D_{3/2})$	4.468	4.432						
$(2^2D_{5/2})$	4.391	4.407						
$(2^4D_{5/2})$	4.430	4.414	4.299	4.232				
$(2^4D_{7/2})$	4.360	4.391						

TABLE IV

Continued.

State	Our calc.	[38]	[53]	[54]	[57]	[47]	[61]	Other
$(3^4D_{1/2})$	4.823	4.642						
$(3^2D_{3/2})$	4.764	4.625						
$(3^4D_{3/2})$	4.802	4.631						
$(3^2D_{5/2})$	4.726	4.611						
$(3^4D_{5/2})$	4.763	4.616	4.410					
$(3^4D_{7/2})$	4.695	4.598						
$(4^4D_{1/2})$	5.221	4.911						
$(4^2D_{3/2})$	5.160	4.894						
$(4^4D_{3/2})$	5.198	4.900						
$(4^2D_{5/2})$	5.123	4.879						
$(4^4D_{5/2})$	5.161	4.885						
$(4^4D_{7/2})$	5.092	4.866						
$(1^4F_{3/2})$	4.467	4.348						
$(1^2F_{5/2})$	4.391	4.321						
$(1^4F_{5/2})$	4.429	4.328						
$(1^4F_{7/2})$	4.360	4.303						
$(1^2F_{7/2})$	4.353	4.296					4.383	
$(1^4F_{9/2})$	4.292	4.274					4.516	
$(2^4F_{3/2})$	4.802	4.593						
$(2^2F_{5/2})$	4.726	4.569						
$(2^4F_{5/2})$	4.764	4.575						
$(2^4F_{7/2})$	4.695	4.553						
$(2^2F_{7/2})$	4.688	4.547						
$(2^4F_{9/2})$	4.627	4.527						

Our obtained orbital excited masses for Ω_{cc} show differences with Ref. [38], $1P$ state $J^P = \frac{1}{2}^-$ shows 25 MeV, $J^P = \frac{3}{2}^-$ shows 16 MeV and $J^P = \frac{5}{2}^-$ shows 32 MeV, $2P$ state $J^P = \frac{1}{2}^-$ shows 6 MeV (with [53]), $J^P = \frac{3}{2}^-$ shows 15 MeV (with [54]) difference. Our results for $3P$ states masses are lower than in Ref. [38] in the range of ≈ 70 MeV. For $4P$ state $J^P = \frac{1}{2}^-$ shows 35 MeV, state $J^P = \frac{3}{2}^-$ shows 9 MeV and $J^P = \frac{5}{2}^-$ shows 19 MeV difference with Ref. [38]. Our $5P$ states masses are higher than those in Ref. [38] in the range of ≈ 190 MeV.

Our outcome for $1D$ state $J^P = \frac{3}{2}^+$ shows 7 MeV, $J^P = \frac{5}{2}^+$ shows 24 MeV, $J^P = \frac{7}{2}^+$ shows 36 MeV, $2D$ state $J^P = \frac{3}{2}^+$ shows 4 MeV, $J^P = \frac{5}{2}^+$ shows 16 MeV and $J^P = \frac{7}{2}^+$ shows 36 MeV (with [38]) difference. For the $3D$–$4D$ states, the difference is 97 and 226 MeV for $J^P = \frac{7}{2}^+$ with Ref. [38].

TABLE V

The masses of orbital excited states of Ω_{bb} (in GeV).

State	Our calc.	[38]	[53]	[54]	[47]	[62]	Other
$(1^2P_{1/2})$	10.968	10.646	10.607	10.616	10.532		10.519 [55]
$(1^2P_{3/2})$	10.957	10.641	10.608	10.619	10.566	10.593 ± 58	10.520 [49, 50]
$(1^4P_{1/2})$	10.976	10.648					
$(1^4P_{3/2})$	10.963	10.643					10.513 [52]
$(1^4P_{5/2})$	10.956	10.637	10.808	10.766	10.798	10.700 ± 60	
$(2^2P_{1/2})$	11.031	10.897	10.796	10.763	10.738		10.683 [55]
$(2^2P_{3/2})$	11.029	10.893	10.797	10.765	10.775		
$(2^4P_{1/2})$	11.039	10.899			10.924		
$(2^4P_{3/2})$	11.035	10.898			10.961		
$(2^4P_{5/2})$	11.019	10.888	11.028				
$(3^2P_{1/2})$	11.124	11.123	10.803		11.083		
$(3^2P_{3/2})$	11.111	11.120	10.805				
$(3^4P_{1/2})$	11.131	11.125					
$(3^4P_{3/2})$	11.119	11.122					
$(3^4P_{5/2})$	11.112	11.177	11.059				
$(4^2P_{1/2})$	11.225	11.332					
$(4^2P_{3/2})$	11.223	11.339					
$(4^4P_{1/2})$	11.232	11.334					
$(4^4P_{3/2})$	11.230	11.331					
$(4^4P_{5/2})$	11.213	11.322					
$(5^2P_{1/2})$	11.356	11.528					
$(5^2P_{3/2})$	11.343	11.525					
$(5^4P_{1/2})$	11.363	11.530					
$(5^4P_{3/2})$	11.351	11.527					
$(5^4P_{5/2})$	11.344	11.523					
$(1^4D_{1/2})$	11.038	10.804					
$(1^2D_{3/2})$	11.032	10.797					
$(1^4D_{3/2})$	11.033	10.800					
$(1^2D_{5/2})$	11.017	10.792	10.729	10.720		10.858 ± 77	
$(1^4D_{5/2})$	11.019	10.794					
$(1^4D_{7/2})$	11.007	10.786				10.964 ± 80	
$(2^4D_{1/2})$	11.126	11.036					
$(2^2D_{3/2})$	11.116	11.030					
$(2^4D_{3/2})$	11.119	11.032					
$(2^2D_{5/2})$	11.104	11.025	10.744	10.734			
$(2^4D_{5/2})$	11.106	11.027					
$(2^4D_{7/2})$	11.094	11.021					

TABLE V

Continued.

State	Our calc.	[38]	[53]	[54]	[47]	[62]	Other
$(3^4D_{1/2})$	11.232	11.249					
$(3^2D_{3/2})$	11.223	11.244					
$(3^4D_{3/2})$	11.225	11.246					
$(3^2D_{5/2})$	11.210	11.240	10.937				
$(3^4D_{5/2})$	11.213	11.241					
$(3^4D_{7/2})$	11.201	11.236					
$(4^4D_{1/2})$	11.358	11.448					
$(4^2D_{3/2})$	11.348	11.444					
$(4^4D_{3/2})$	11.351	11.445					
$(4^2D_{5/2})$	11.336	11.440					
$(4^4D_{5/2})$	11.339	11.441					
$(4^4D_{7/2})$	11.327	11.437					
$(1^4F_{3/2})$	11.118	10.943					
$(1^2F_{5/2})$	11.105	10.936					
$(1^4F_{5/2})$	11.107	10.938					
$(1^4F_{7/2})$	11.094	10.932					
$(1^2F_{7/2})$	11.082	10.930				11.118 ± 96	
$(1^4F_{9/2})$	11.073	10.924				11.221 ± 99	
$(2^4F_{3/2})$	11.226	11.162					
$(2^2F_{5/2})$	11.210	11.155					
$(2^4F_{5/2})$	11.214	11.157					
$(2^4F_{7/2})$	11.202	11.151					
$(2^2F_{7/2})$	11.189	11.149					
$(2^4F_{9/2})$	11.179	11.144					

Our result for $1F$ state $J^P = \frac{7}{2}^-$ is 30 MeV lower than in Ref. [61] and for the $1F$–$2F$ states $J^P = \frac{9}{2}^-$ are 18 and 100 MeV higher than in Ref. [38]. For the ground and excited states of doubly heavy baryons (Ω_{cc}), the minimum and maximum percentage of relative error values are 0.03% and 3.662% between our calculations and the masses reported by Shah *et al.* [38].

Our estimated orbital excited masses of Ω_{bb}, $1P$ state have difference in the range of ≈ 300 MeV with other predictions. Our $2P$ state $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ are 135 and 136 MeV higher than in Ref. [28] and $J^P = \frac{5}{2}^-$ is 16 MeV lower than Ref. [53], respectively. For $3P$ state $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ show only 1 and 8 MeV difference with Ref. [38]. The reported mass of Ref. [53] for $J^P = \frac{5}{2}^-$ state is 52 MeV lower than our prediction. Our $4P$–$5P$,

TABLE VI

The masses of orbital excited states of Ω_{bc} (in GeV).

State	Our calc.	[38]	State	Our calc.	[38]
$(1^2P_{1/2})$	7.476	7.386	$(2^2D_{1/2})$	7.806	7.843
$(1^2P_{3/2})$	7.470	7.373	$(2^2D_{3/2})$	7.768	7.829
$(1^4P_{1/2})$	7.490	7.392	$(2^4D_{3/2})$	7.792	7.834
$(1^4P_{3/2})$	7.486	7.379	$(2^2D_{5/2})$	7.744	7.816
$(1^4P_{5/2})$	7.451	7.363	$(2^4D_{5/2})$	7.767	7.821
$(2^2P_{1/2})$	7.619	7.674	$(2^4D_{7/2})$	7.724	7.805
$(2^2P_{3/2})$	7.595	7.664	$(3^2D_{1/2})$	8.017	8.088
$(2^4P_{1/2})$	7.633	7.679	$(3^2D_{3/2})$	7.978	8.075
$(2^4P_{3/2})$	7.610	7.669	$(3^4D_{3/2})$	8.002	8.079
$(2^4P_{5/2})$	7.594	7.655	$(3^2D_{5/2})$	7.954	8.063
$(3^2P_{1/2})$	7.782	7.935	$(3^4D_{5/2})$	7.976	8.068
$(3^2P_{3/2})$	7.777	7.925	$(3^4D_{7/2})$	7.935	8.054
$(3^4P_{1/2})$	7.796	7.939	$(4^2D_{1/2})$	8.266	8.317
$(3^4P_{3/2})$	7.793	7.930	$(4^2D_{3/2})$	8.228	8.305
$(3^4P_{5/2})$	7.758	7.918	$(4^4D_{3/2})$	8.252	8.309
$(4^2P_{1/2})$	8.002	8.175	$(4^2D_{5/2})$	8.204	8.294
$(4^2P_{3/2})$	7.978	8.167	$(4^4D_{5/2})$	8.229	8.298
$(4^4P_{1/2})$	8.017	8.179	$(4^4D_{7/2})$	8.185	8.285
$(4^4P_{3/2})$	7.994	8.171	$(1^4F_{3/2})$	7.792	7.742
$(4^4P_{5/2})$	7.979	8.160	$(1^2F_{5/2})$	7.744	7.723
$(5^2P_{1/2})$	8.242	8.400	$(1^4F_{5/2})$	7.768	7.728
$(5^2P_{3/2})$	8.237	8.393	$(1^4F_{7/2})$	7.723	7.711
$(5^4P_{1/2})$	8.256	8.404	$(1^2F_{7/2})$	7.720	7.705
$(5^4P_{3/2})$	8.252	8.396	$(1^4F_{9/2})$	7.681	7.690
$(5^4P_{5/2})$	8.218	8.386	$(2^4F_{3/2})$	8.001	7.965
$(1^2D_{1/2})$	7.634	7.577	$(2^2F_{5/2})$	7.956	7.949
$(1^2D_{3/2})$	7.597	7.561	$(2^4F_{5/2})$	7.978	7.953
$(1^4D_{3/2})$	7.618	7.566	$(2^4F_{7/2})$	7.935	7.938
$(1^2D_{5/2})$	7.571	7.547	$(2^2F_{7/2})$	7.931	7.934
$(1^4D_{5/2})$	7.595	7.552	$(2^4F_{9/2})$	7.892	7.921
$(1^4D_{7/2})$	7.552	7.534			

states respectively, have ≈ 100 MeV and ≈ 170 MeV difference with Ref. [38]. Our outcomes for $1D$ state $J^P = \frac{5}{2}^+$ and $J^P = \frac{7}{2}^+$ show difference of 159 and 43 MeV with Ref. [62]. Our obtained masses for $2D$ states are different from Ref. [38] predictions in the range of ≈ 80 MeV.

Our calculated masses for $3D$ and $4D$ states have difference in the range of ≈ 20 and ≈ 100 MeV lower than in Ref. [38]. Moving to $1F$ state $J^P = \frac{7}{2}^-$ and $J^P = \frac{9}{2}^-$, the values are 36 and 148 MeV higher than in Ref. [62].

Comparing our findings with the masses reported by Shah *et al.* [38], the minimum and maximum percentage of relative error values are 0.001% and 10.87% for the ground and excited states of doubly heavy baryons Ω_{bb}.

The orbital mass spectrum of the third doubly heavy baryon, Ω_{bc}, is predicted by Shah *et al.* [38] for the first time, as they believed. We also have not considered the diquark mechanism in our model and calculated orbital mass spectrum for the Ω_{bc} baryon. In the case of the doubly heavy baryon, Ω_{bc}, our outcomes for the orbital excited masses are compared and discussed with Shah *et al.* [38] predictions in the following paragraph.

Our results for $1P$ states are different from their predictions in the range of $\approx$ 90 MeV. We can easily observe that our calculations for $2P$, $1D$–$2D$ and $1F$–$2F$ states match with Shah's predictions. Our calculated masses for $3P$–$5P$ states are approximately 150, 170 and 150 MeV lover than their findings, respectively. Our results for $3D$ and $4D$ states are higher than their calculations, respectively, in the range of $\approx$ 90 and 70 MeV. Comparing our calculations with the masses reported by Shah *et al.* [38], the minimum and maximum percentage of relative error values are 0.027% and 7.329% for the ground and excited states of doubly heavy baryons Ω_{bc}.

5. Conclusion

In this study, we have computed the mass spectra of ground and excited states for doubly heavy Ω baryons by using a hypercentral constituent quark model. For this purpose, we have analytically solved the radial Schrödinger equation for three identical interacting particles under the effective hypercentral potential by using the Ansatz approach. Our proposed potential is regarded as a combination of the Coulomb-like term plus a linear confining term and the harmonic oscillator potential. We also added the first order correction and the spin-dependent part to the potential. Our model has succeeded to assign the J^P values to the exited states of doubly heavy baryons (Ω_{cc}, Ω_{bb}, Ω_{bc}). Comparison of the results with other predictions revealed that they are in agreement and our proposed model can be useful for investigating the doubly heavy baryons states masses. For example, for the ground, radial and orbital excited states masses of doubly heavy Ω baryons, the minimum and the maximum percentage of relative error values are 1% and 6% between our calculations and the masses reported by Shah *et al.* [38]. As the final point, it should be clearly stated that the approach, despite its valuable predictions and results, does have its limitations including using the quasi-exact solutions and the fit process.

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