AN ADVANTAGE OF "UPPER LEVELS"*

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Hadronic atoms allow, in principle, to understand hadron–nucleon interactions just below thresholds. So far, the X-ray atomic transitions have offered atomic level shifts in the "lowest" of accessible atomic states. Level broadenings have been measured directly in these states and indirectly also in higher "upper" levels. Recent experimental progress allows to find level shifts also in some upper states. Such measurements are much easier to analyse as the levels are determined essentially by a single hadron–nucleon collision at a fairly well-determined subthreshold energy. Light anti-protonic and K-mesic atoms are discussed.

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1. Introduction

Hadronic atoms offer a chance to test hadron–nucleon scattering amplitudes below the thresholds as both particles are bound. This energy region is of special interest in cases of quasi-bound states in the hadron–nucleon systems. Two such systems of current interest, the $\bar{p}$ and $\bar{K}^-$ atoms, are similar in this respect.

In both cases, the nuclear absorption is very strong and the orbiting particles are captured at extreme nuclear surface. In this way, studies of capture processes yield information on nuclear surface structure. One point of interest is the possibility to study few nucleon correlations, the other is

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the thickness of the neutron skin in heavy or unstable nuclei. Related question of correlations was put forward [1] long time ago, but so far found no quantitative understanding. Neutron skins have been studied with K-mesic atoms [2, 3] and later with anti-protonic atoms. In particular radiochemical CERN measurements by a Munich–Warsaw [4] collaboration detected neutron haloes in a large number of heavy nuclei. Related studies of neutron haloes via anti-protonic atoms with radioactive nuclei are planned at CERN by the PUMA Collaboration [5]. In all these experiments, the knowledge of $\bar{p}$–N and K^-–N interactions is essential. In this context, studies of "upper" atomic levels, in particular in the lightest nuclei, may turn out to be very profitable. These are briefly discussed below, and to begin with, let us visualise the nuclear scenario of the discussed measurements.

Nuclear regions tested are determined by atomic angular momentum l and the related centrifugal barrier. The atomic–nuclear overlap is roughly given by $\theta(r) = r^{2l}\rho(r)$, where ρ is the nuclear density. The $\theta(r)$ has a sharp maximum indicating that absorption happens in a thin shell of ~ 2 fm radial depth. For anti-protonic atoms, the maxima occur typically at about 7% (lower level) and 1% (upper level) of the central nuclear densities. However, the fate of $\bar{p}$ annihilation products and the chances to leave the nucleus may shift the tested region to even smaller nuclear densities. Such extremely far nuclear surfaces offer several advantages:

— Complex level shifts are determined predominantly by a single hadron–nucleon collisions. If optical potential is used, the form "$t\rho$" is a good approximation and works well in the first order of perturbation.

— The t is a function of energy which is given by separation energies of valence nucleons. The density ρ involved is given mainly by the asymptotic form of the nuclear wave functions.

— Many body effects on t are small and given essentially by the free, albeit off-energy shell, hadron–nucleon interactions.

In this note we discuss the level shifts and widths due to $\bar{p}N$ interactions in the lightest atoms (levels) ^{2}H$(2P)$, ^{3}He$(2P, 3D)$ and ^{4}He$(2P, 3D)$. By "upper" states, we understand states of small atomic-nuclear overlaps characterised by shifts (widths) in the range of few tenths of eV or smaller. All the listed states fall into this category.

2. Relation of level shifts to subthreshold scattering amplitudes

This section presents a formalism used to calculate the related complex shifts $\Delta E - i\Gamma/2$. These are expressed in terms of averages of S- and P-wave

$\bar{N}N$ scattering amplitudes arranged into a sum of multiple scattering series. The basic $\bar{p}N$ interaction is used in the form of

$$V(r) = \frac{2\pi}{\mu_{\bar{p},N}} \left[a_0(E_{\rm cm})\delta(r) + a_1(E_{\rm cm}) \overleftarrow{\partial}\, \delta(r) \overrightarrow{\partial} \right] , \tag{1}$$

where $a_{0,1}$ are scattering lengths and scattering volumes, $\mu_{\bar{p},N}$ is the reduced mass and $\boldsymbol{r}$ is the relative coordinate. Spin and isospin indices are suppressed. The scattering parameters depend on energies available in the $\bar{p}N$ centre-of-mass system. To calculate these, we present an atom of interest as a quasi-three body system composed of: anti-proton, the struck valence nucleon N, and the residual nuclear system R. To quantify it, we need separation energy of the initial nucleus into N and R denoted by $E_{\rm s}$ and the wave function $\varphi_N(\boldsymbol{u})$ in the relative N–R coordinate $\boldsymbol{u}$. For the latter, simple Eckart or Hulthen forms are available and these guarantee proper asymptotic behaviour.

The first order formula for the complex level shifts is

$$\Delta E - i\Gamma/2 = E - \epsilon_c = \Sigma_i \, \langle \varphi_N \, \phi_c | V_i | \phi_c \, \varphi_N \rangle \ , \tag{2}$$

where ϵ_c is the unperturbed electromagnetic atomic energy and ϕ_c is the atomic wave function, both are determined by the QED. In formula (2), the summations over all nucleons and integration over Jacobi coordinates is to be performed. As yet, the fine structures in these levels are not (in general) resolved and the shifts have to be averaged over spin states. In the case of S-wave interaction and $2P$ atomic states-expression (2) yields

$$\Delta E - i\Gamma/2 = \frac{N_p}{B^3 \mu_{N\bar{p}}} \left\langle (au/B)^2 \right\rangle \Sigma_i \bar{a}_0 + O(a_0/B, \langle u \rangle / B) \, , \tag{3}$$

where $\langle (u)^2 \rangle$ is the radius mean squared of the nuclear density expressed in terms of the relative coordinate wave function $\varphi(v)$, B is the Bohr radius, $N_p = 1/24$ is the normalisation factor of ϕ_c and $a = (A-1)/A$ comes from transformation of Jacobi coordinates in the three-body system. Integrals involved in Eq. (2) reduce to a single integral over the spectator R recoil momentum. In this way, some energy averaging is introduced into scattering amplitude $a(E_{\rm cm})$. Since in the $\bar{p}N$ center-of-mass the energy $E_{\rm cm} = -E_{\rm s} - E_{\rm recoil}$, this average is

$$\bar{a}_0 = \int a_0 \left(-E_{\rm s} - \frac{p^2}{2m_{R,\bar{N}N}} \right) |F_L(p)|^2 \, {\rm d}\boldsymbol{p} \Big/ \int |F_L(p)|^2 \, {\rm d}\boldsymbol{p} \, . \tag{4}$$

The extent of the recoil energies is determined by Bessel transforms of the nuclear wave function. For a given atomic L state,

$$F_L(p) = \int \varphi(u) j_L(up) u^2 {\rm d}u \, . \tag{5}$$

Energies involved in Eq. (4) cover some part the unphysical subthreshold region, Table I. The recoil energies given by the Bessel transform depend on the angular momentum L and the spread of $E_{\rm cm}$ becomes smaller with the increasing L. In some cases the, "upper" levels pinpoint $E_{\rm cm}$ quite well.

TABLE I

$E_{\rm cm}$ energies. The entries give averaged separation and recoil energies [MeV]. Numbers in parentheses indicate widths of recoil energy distributions.

Atom	$1S$	$2P$	$3D$
$\bar{p}\,{}^1$H	0	0	0
$\bar{p}\,{}^2$H	−11.1(5)	−7.6(2)	−7.1(1)
$\bar{p}\,{}^3$He	−17.6(7)	−15.5(2)	−13.9(1)
$\bar{p}\,{}^4$He	−34.5(10)	−34.4(10)	−34.4(9)

Interactions in P-waves in $N\bar{N}$ system require similar but longer expressions which involve derivatives of atomic and nuclear wave functions. The latter are not controlled as well as the r.m.s values but are good enough to match the present experimental precision.

Higher order corrections to formula (3) are of two types: the $\langle u\rangle/B$ correction coming from the pure Coulomb wave function (the exponent term) constitutes $\sim 3\%$. It is easy to calculate and expresses Coulomb corrections to scattering parameters a. More difficult are the second order terms in the perturbation series

$$\sum_i \sum_{j\neq i} \langle V_i G_0 V_j\rangle + \sum_i \sum_j \langle V_i G_{NR} V_j\rangle \ . \tag{6}$$

The first term is of the optical potential type, the second involves N–R interaction that takes place between successive collision of anti-protons. We refer to a calculation [6] performed for the deuteron interacting with S-wave meson. Both terms are comparable and might be summed into a quasi-geometric series. In the atomic D states discussed here, these are negligible as well as in the deuteron $2P$ states. In the $2P$ states of ^{3}He, these constitute a 1% correction and become larger in ^{4}He. These are below the precision discussed in the next section and have not been included into results.

2.1. Comparison to the data

Experimental data are compared with calculations based on two versions of $N\bar{N}$ potentials: Paris 09 [9] and Paris 99 [10]. These potentials generate quasi-bound states in ^{33}P-wave with $(E, \Gamma/2)$ equal $(-4.5, 9)$ MeV and

$(-17, 6.5)$ MeV, correspondingly. Effects of those states are seen clearly in the results presented in Table II. The state at -17 MeV is seen to enlarge the widths dramatically in comparison to the effect of state at -4.5 MeV. Quasi-bound states located below $E_{\rm cm}$ generate repulsion, while quasi-bound states located above $E_{\rm cm}$ generate attraction. Again, this effect is seen in the level shifts. Comparison of calculated level shift with the data might suggest some advantage of the Paris 99 solution. Inspection of the deuteron data presented in Table III indicates that level shifts support such a conclusion but level widths do not. It seems that the real position of the quasi-bound state is in between the two Paris model solutions.

TABLE II

Level shifts and widths of $2P$ states in ^{3}He anti-protonic atoms [eV]. Data from Ref. [7]. The experimental level shift is repulsive.

Paris 09	S	P	$S+P$	Exp.
ϵ	9.9	-3.9	6.0	17(4)
Γ	25.9	2.8	28.8	25(9)
Paris 99	S	P	$S+P$	
ϵ	13.7	0.9	14.6	17(4)
Γ	17.1	16.6	33.7	25(9)

TABLE III

Level shifts and widths $2P$ states in ^{2}H anti-protonic atoms [meV]. Data from Ref. [8]. The experimental level shift is repulsive [8].

Paris 09	S	P	$S+P$	Exp.
ϵ	99.0	-58	43	243(26)
Γ	210	312	522	489(30)
Paris 99	S	P	$S+P$	
ϵ	113	58	171	243(33)
Γ	145	206	351	489(30)

The conclusion of our analysis is: the upper level may be instrumental with fixing the properties of quasi-bound states and improving $N\bar{N}$ potentials.

2.2. Light K-mesic atoms

New measurements for helium kaonic atoms yield uncertain but very instructive results [11], see Table IV.

TABLE IV

$2P$ level shifts and widths in kaonic helium atoms [11] [eV].

Atom	ϵ	Γ
$K^-\ ^3$He	$-2 \pm 2 \pm 4$	$6 \pm 6 \pm 7$
$K^-\ ^4$He	$5 \pm 3 \pm 4$	$14 \pm 8 \pm 5$

In analogy to anti-protons, the scenario under the $\bar{K}N$ threshold is determined by a resonant state $\Lambda(1405)$ with a pole close to $E_{\rm cm} = 1410$ MeV that is in the ^{3}He region. On the other side, one has $\Sigma(1385)$ state which exerts maximum repulsive effect in the ^{4}He region. Apparently, these two main agents yield attractive shift in ^{3}He and repulsive in ^{4}He. Now, in order to go above the errors, one has to magnify the shifts and enhance the atomic–nuclear overlaps. The proper targets would be ^{9}Be and ^{6,7}Li. These offer similar values of $E_{\rm cm}$ as ^{4}He and ^{3}He. A simple re-scaling of overlaps generates the level shifts of about 100 eV. One should perhaps consider also studies of $3D$ levels in these atoms. One interesting outcome might be the estimate at what energy the isospin 0 Re $T(\bar{K}N \to \bar{K}N)$ amplitude crosses zero. That will help to settle the controversy as to where is the $\Lambda(1405)$ pole in the complex plane located.

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