

BOUNDS ON PLANCK-SCALE DEFORMATIONS OF
CPT FROM LIFETIMES AND INTERFERENCE*

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Deformed relativistic kinematics, expected to emerge in a flat-spacetime limit of quantum gravity, predicts the Planck-scale violation of discrete symmetries. Momentum-dependent deformations of CPT are derived from the κ -deformed Poincaré algebra. Deformation of CPT symmetry leads to a subtle violation of Lorentz symmetry. This entails small but measurable phenomenological consequences as corrections to characteristics of time evolution: particle lifetimes or oscillations in two-particle states at high energy. We argue that using current experimental precisions on the muon lifetime, one can bound $\kappa > 10^{14}$ GeV at the LHC energy and move this limit to 10^{16} GeV at the Future Circular Collider. Weaker limits on deformation can be also obtained from interference of neutral mesons. In the case of B^0 s from Υ decay, it amounts to $\kappa > 10^8$ GeV at confidence level 99%.

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1. Introduction

Invariance under CPT, the combined transformation of space inversion P, charge conjugation C and time reversal T is believed to be strictly obeyed due to theorems based on premises constituting natural axioms of the local quantum field theory [1]. Among phenomenological consequences of the CPT theorem, there are equality of masses and lifetimes of particles and antiparticles. These claims were experimentally tested. The most precise constraint comes from neutral kaon physics, where masses of the K^0 and \bar{K}^0 are equal with accuracy of 4×10^{-19} GeV at 95% confidence level [2]¹.

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¹ The number given in Ref. [2] represents the world average, where the main contribution comes from the CPLEAR experiment at CERN [3].

Together with the anti-CPT theorem [4] (*cf.* discussion in Ref. [5]), stating that CPT violation entails violation of the Lorentz invariance, these theoretical results may suggest that any experimental hint of CPT non-conservation might deeper affect the quantum field theory.

Effects of quantum gravity have long been suggested as a possible source of CPT violation. As noted in Ref. [6], there is no fundamental arrow of time in its own right but only with a choice of matter or antimatter. In addition, in presence of an inherent quantum-gravitational background, CPT operator is no longer well-defined. Scattering operator cannot map pure in- into out-states, and *vice versa*, due to destruction of information in presence of micro black holes [7]. Because of that, any system propagating in a quantum-gravitational background exhibits irreversibility, analogously to dissipative processes, but here connected to CPT violation. It has, therefore, inspired approaches based on dissipative quantum dynamics (for historical account *cf.* Ref. [8]) and resulting in a number of models with CPT or Lorentz violation ascribed to gravity-induced quantum decoherence [9]. Alternatively, another approach inspired by quantum gravity was considered [10] using the usual framework of quantum field theory but with an explicitly CPT non-invariant term added to the Lagrangian. These models, either based on decoherence or Standard Model extension, were considerably, but not definitely, constrained by experimental data on neutral kaons [11, 12].

2. Planck-scale deformation and discrete symmetries

By analogy to the Heisenberg uncertainty relation, the concept of minimal energy and length scales leads to non-commutative geometry, defined by the commutation relations

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}, \quad \mu, \nu = 0, \dots, 3, \quad (1)$$

where $\theta^{\mu\nu}$ can be related to geometric properties of the space-time [13]. The concept has been further developed to the geometry generated by the κ -deformed Poincaré algebra — generating Lorentz boosts, momenta and rotations — and defined the commutation relations

$$[t, x^j] = ix^j/\kappa, \quad [t, k^j] = -ik^j/\kappa, \quad (2)$$

where κ is expected to be of the order of Planck's mass $m_P \simeq 2 \times 10^{18} \text{ GeV}/c^2$ [14]. The κ -deformed momentum space is a submanifold of the four-dimensional de Sitter space, defined in the five-dimensional Minkowski space by constraint

$$-p_0^2 + p_1^2 + p_2^2 + p_3^2 + p_4^2 = \kappa^2; \quad p_0 + p_4 > 0, \quad (3)$$

where κ is related to the curvature of the momentum manifold.

Essential to understand the action of discrete symmetries in deformed space is a proper definition of deformed rule of the four-momentum composition and thus of momentum inversion. As elaborated in Ref. [15] and using the Hopf algebra, inverse of three-momentum is given by its *antipode* \ominus as

$$S(p_i) \equiv \ominus p_i = -p_i \frac{\kappa}{E + p_4}, \quad i = 1, 2, 3 \quad (4)$$

and for energy by requiring the mass-shell relation or preservation of the Casimir operator

$$\begin{aligned} m^2 &= E^2 - \mathbf{p}^2 \\ &= S(E)^2 - S(\mathbf{p})^2, \end{aligned} \quad (5)$$

by

$$S(E) = \frac{\kappa^2}{E + p_4} - p_4. \quad (6)$$

Using these findings, action of the κ -deformed $\Theta_\kappa = \text{CPT}_\kappa$ on the four-momentum can be written in leading order of $1/\kappa$ as

$$\begin{aligned} \Theta_\kappa p_0 &= p_0 - \frac{\mathbf{p}^2}{\kappa} + \mathcal{O}(1/\kappa^2), \\ \Theta_\kappa \mathbf{p} &= \mathbf{p} - \frac{p_0 \mathbf{p}}{\kappa} + \mathcal{O}(1/\kappa^2) \end{aligned} \quad (7)$$

($p_0 = E$) and charges are always multiplied by -1 . In particular, the particle's Lorentz boost factor $\gamma = E/m$, after Θ_κ deformation becomes

$$\gamma_\kappa = \frac{1}{m} (E - \mathbf{p}^2/\kappa) \quad (8)$$

for antiparticle.

3. Measurement of the κ -deformation

Consider unstable particle at rest described by the wave function depending on its proper time

$$\psi = \sqrt{\Gamma} e^{-\Gamma t/2 + imt}, \quad (9)$$

where its mass m and lifetime $\tau = 1/\Gamma$ are CPT-invariant. In the particle's rest frame, the CPT is undeformed and the particle's (_p) and antiparticle's (_a) masses and lifetimes are equal due to the CPT theorem. Their decay probabilities obey the same decay laws

$$\mathcal{P}_p = \mathcal{P}_a = \Gamma e^{-\Gamma t} \quad (10)$$

but they differ after Lorentz transformation to a moving system

$$\begin{aligned}\mathcal{P}_p &= \frac{\Gamma E}{m} e^{-\Gamma E/mt}, \\ \mathcal{P}_a &= \Gamma \left(\frac{E}{m} - \frac{\mathbf{p}^2}{\kappa m} \right) e^{-\Gamma \left(\frac{E}{m} - \frac{\mathbf{p}^2}{\kappa m} \right) t}.\end{aligned}\quad (11)$$

Consequences of deformation could thus be examined experimentally by precisely measuring the particle and antiparticle lifetimes. These are equally dilated due to Lorentz boost but only one of them is deformed under CPT transformation. To be measurable, the effective correction $\mathbf{p}^2/(\kappa m)$ has to be comparable to the experimental accuracy of the measurement σ_τ/τ .

In the scheme described above and developed in Ref. [16], violation of the CPT is momentum-dependent and thus depends on the Lorentz frame. It thus explicitly relates the CPT and Lorentz noninvariance, only suggested in general terms by the CPT theorems.

For known unstable particles, the accuracy of measurements of their lifetimes amounts typically to 10^{-4} for mesons π^\pm and K^0 and 10^{-6} for leptons μ^\pm [2]. Any measurement of the lifetime requires experimentally measured momenta to be Lorentz-transformed to the particle's rest frame. Inaccuracies of laboratory momenta and energies thus propagate to the rest frame and, if large, can strongly affect σ_τ . This is particularly discouraging in nonaccelerator experiments where energies of cosmic particles are occasionally very high, exceeding even 10^6 GeV and thus $\mathbf{p}^2 \sim 10^{12}$ GeV² but, at the same time, experimental uncertainties usually tend to be large and hard to control.

In order to quantify our findings, in Fig. 1 (upper left), we plot the correction $\mathbf{p}^2/(\kappa m)$ for the muon. If deformation κ is close to the Planck mass 10^{19} GeV, as expected, any detectable correction requires momenta of the order of 10^6 GeV, unattainable at today's accelerating facilities. Such energies are available in cosmic-ray experiments. However, using them for our purposes would require a measurement of their lifetimes and reach very challenging accuracy of their energy determination. We can also estimate a limit on the deformation κ that can be set for the present energies at the LHC and those planned at FCC [17], both at CERN. Using experimental accuracies of the lifetimes and requiring $\mathbf{p}^2/(\kappa m) = \sigma_\tau/\tau$ for $p = 6.5$ TeV (LHC) and 50 TeV (FCC), one obtains the values of κ labeling curves in Fig. 1. As can be seen there, the limiting value of $\kappa = 4 \times 10^{14}$ GeV can be obtained using muons at the LHC and, in future, $\kappa = 2 \times 10^{16}$ GeV at FCC. Further improvement at these energies requires progress in accuracy of the lifetime measurement.

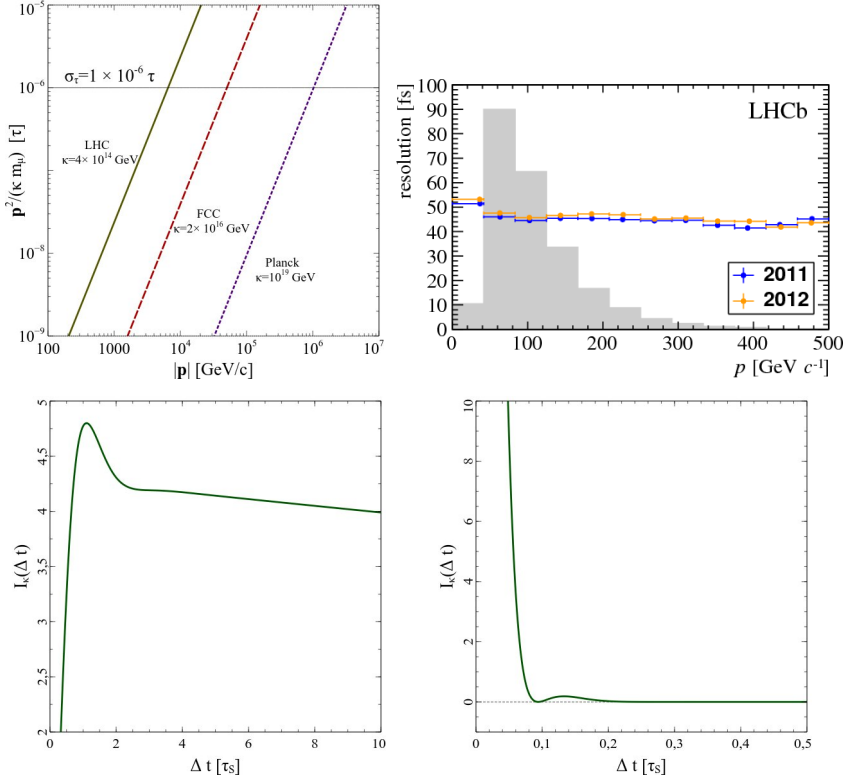


Fig. 1. (Color online) Upper left: The correction $\mathbf{p}^2/(\kappa m)$ to the muon lifetime. Two curves, labeled LHC and FCC, are for the deformation parameters κ corresponding to corrections equal to experimental accuracies for maximal momenta at the Large Hadron Collider (continuous green, LHC) and the Future Circular Collider (dashed red, FCC). The dotted line (violet) corresponds to the Planck mass $\kappa = 10^{19} \text{ GeV}$. Upper right: The momentum-dependent time resolution of the LHCb detector. The superimposed histogram represents the momentum distribution of μ^\pm and K^\pm used for this measurement. Lower left: Lorentz-boosted spectra ($\gamma = 4.3$) of decay time difference for pairs of kaons from decay $\phi(1020) \rightarrow K_L K_S$, in units of τ_S . Lower right: Lorentz-boosted spectra ($\gamma = 44$) of decay time difference for pairs of B mesons from decay $\Upsilon(10580) \rightarrow B_H B_L$, in units of τ_L .

Interesting possibilities of search for CPT violation are given by interferometry of neutral mesons produced in coherent, two-particle states in decays of pseudoscalar mesons, *e.g.* $\phi(1020) \rightarrow K_L K_S$ and $\Upsilon(10580) \rightarrow B_H B_L$ (*cf.* Ref. [9]). Interference patterns are usually studied in difference of decay times, $\Delta t = |t_2 - t_1|$

$$I(\Delta t) \sim e^{-\Gamma_L \Delta t} + e^{-\Gamma_S \Delta t} - 2e^{\bar{\Gamma} \Delta t} \cos(\Delta m \Delta t), \quad (12)$$

where $t_{1(2)}$ stands for decay time of the first (second) meson in its rest frame and Δm for difference of masses of the heavier and lighter of the two neutral mesons and $\bar{\Gamma} = (\Gamma_L + \Gamma_S)/2$. Applying deformed CPT, Eq. (5), transforms decay spectrum given by Eq. (12) to

$$\begin{aligned} I(\Delta t) \sim & (\gamma - \mathbf{p}^2 / (m\kappa)) (e^{-\gamma \Gamma_L \Delta t} + e^{-\gamma \Gamma_S \Delta t}) \\ & + \gamma \Delta t \mathbf{p}^2 / (m\kappa) (\Gamma_L e^{-\gamma \Gamma_L \Delta t} + \Gamma_S e^{-\gamma \Gamma_S \Delta t}) \\ & - 2\gamma e^{-\gamma \bar{\Gamma} \Delta t} [(1 + \bar{\Gamma} \Delta t \mathbf{p}^2 / (m\kappa)) \cos(\gamma \Delta m \Delta t) \\ & + \Delta m \Delta t \mathbf{p}^2 / (m\kappa) \sin(\gamma \Delta m \Delta t)]. \end{aligned} \quad (13)$$

Lorentz boost, by increasing masses of mesons, effectively amplifies oscillation frequency of the oscillatory term. In order to have interference pattern experimentally measurable, the amplified frequency cannot exceed the inverse time resolution of an apparatus $\frac{1}{\gamma \Delta m} > \sigma_t$. Time resolution of the most precise spectrometer at the LHC amounts to 45 fs and belongs to the LHCb (*cf.* Fig. 1, upper right). It gives constraints to the maximum Lorentz boost for pairs of K^0 and B^0 mesons to be $\gamma = 4.3$ and $\gamma = 44$, respectively. Decay time spectra (13) for these boosts are presented in Fig. 1 (lower left and right). Limits on κ deformation can be estimated by the Monte Carlo simulation using the log-likelihood method and are found to be 2×10^5 GeV and 1.2×10^8 GeV, at 99.9% confidence level, for pairs of K^0 s and B^0 s, respectively.

4. Conclusions

Deformed CPT transformation can be used to estimate deformation parameter κ from experiment. The CPT-violating corrections to energy-momentum are of the order of $\mathbf{p}^2 / (m\kappa)$. This kind of CPT violation automatically violates Lorentz invariance. Numerical estimates show that by using precisely known lifetimes of μ^\pm , one could expect to limit κ at 10^{16} GeV, *i.e.* only three orders of magnitude lower than the Planck mass, by incorporating the FCC at $\sqrt{s} = 100$ TeV. Limitations from the neutral-meson interferometry are less stringent.

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