MODELLING THE INVERSE ZENO EFFECT FOR THE NEUTRON DECAY*

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Beam- and trap-based methods find incompatible results for the lifetime of the neutron: the former delivers a value which is about 8.7 ± 2.1 s longer than the latter. Very recently (`arXiv:1906.10024 [hep-ph]`), it has been proposed that the inverse Zeno effect (IZE) could be responsible for the shorter lifetime in trap experiments. Here, we compare two different models of measurement, one obtained by ideal measurements at equal time intervals (sometimes called 'bang–bang') and by a continuous measurement: the IZE turns out to be in both cases very similar, showing that the results do not depend on the details of the measurement process.

1. Introductory remarks

The lifetime of the neutron represents an unsolved puzzle: different measurement types find incompatible results [1]. On the one hand, experiments based on the so-called beam method — in which protons emitted by a neutrons beam are detected — lead to the average $\tau_n^{\mathrm{beam}} = 888.1 \pm 2.0$ s; on the other hand, trap or cavity experiments, in which surviving neutrons confined in a magnetic or gravitational trap are counted, deliver the result $\tau_n^{\mathrm{trap}} = 879.37 \pm 0.58$ s. The deviation 8.7 ± 2.1 s corresponds to a 4σ discrepancy.

In recent works, this mismatch has been interpreted [2, 3] in the following way: an invisible dark decay of the neutron causes the beam method (which detects the emitted protons) to erroneously measure a longer lifetime. In this context, the correct lifetime is given by τ_n^{trap}. This approach involves beyond-Standard Model physics and could be problematic for what concerns the stability of neutron stars [4] as well as the present knowledge of neutron decay parameters [5].

Also very recently, an alternative idea was discussed in Ref. [6]. Here, the inverse Zeno effect (IZE) increases the decay rate in trap experiments, where about 10^8 neutrons are kept in a cold environment. Namely, the dephasing/decoherence of the neutron state may occur sufficiently fast to reach the regime in which the IZE is realized. Thus, within this idea, the correct lifetime is given by τ_n^{beam}.

In this work, we compare two different models of measurements leading to the IZE. Besides the model based on continuous measurement of Ref. [6], we also test the model based on instantaneous bang–bang measurements: qualitatively similar results are obtained, thus confirming that the IZE is not dependent on the details of the model.

2. Different realizations of the IZE

The decay law in quantum mechanics is not exactly an exponential [7] (see Ref. [8] for the analogous result in quantum field theory). If intermediate measurements are performed, the decay width can change [9], and eventually be sizably reduced (QZE, [10]). In some conditions, however, also the IZE (larger measured decay width) is possible [11–13].

Let us consider a certain decay width, parametrized by the function $\Gamma(\omega)$, where ω is the energy of the final state minus the energy in the initial state (it then formally ranges between 0 and ∞). The usual decay width is obtained by setting $\omega = \omega_{\text{onshell}}$, $\Gamma_{\text{onshell}} = \Gamma(\omega_{\text{onshell}})$. The form of $\Gamma(\omega)$ depends on the details of the unstable state, but is zero for $\omega < 0$ and for $\omega \to \infty$. As shown in Refs. [11, 12] (see also the review [13]) the measured decay width reads

$$\Gamma^{\text{meas}}(\tau, \omega_C) = \int_0^{\omega_C} f(\tau, \omega)\Gamma(\omega)\mathrm{d}\omega\,, \tag{1}$$

where ω_C is the maximal off-shellness, typically linked to the formation process of the unstable state. The response function $f(\tau, \omega)$ models the measurement, which averagely takes place at τ, 2τ, etc. The three general properties of f are: $\int_{-\infty}^{\infty} f(\tau, \omega)\mathrm{d}\omega = 1$, $f(\tau \to \infty, \omega) = \delta(\omega - \omega_{\text{onshell}})$, and $f(\tau \to 0, \omega) = \text{small const.}$ The first is the normalization, the second implies that, for an undisturbed system, the 'on-shell' decay is obtained, the third implies that $\Gamma^{\text{meas}}(\tau \to 0) = 0$ (QZE). The details of the function $f(\tau, \omega)$ depend on the type of the measurement. For instantaneous ideal measurements performed at times τ, 2τ, ... and for a continuous measurement of the final state, it reads respectively [11, 12],

$$f_1(\tau,\omega) = \frac{\tau}{2\pi}\text{sinc}^2\left[(\omega - \omega_{\text{onshell}})\frac{\tau}{2}\right],$$

$$f_2(\tau,\omega) = \frac{1}{\pi\tau}\left[(\omega - \omega_{\text{onshell}})^2 + \tau^{-2}\right]^{-1}. \qquad (2)$$

For each choice of f, one has a different measured decay width, $\Gamma_k^{\text{meas}}(\tau,\omega_C) = \int_0^{\omega_C} f_k(\tau,\omega)\Gamma(\omega)\mathrm{d}\omega$. More complicated measurement models would lead to different response function (for the possible effect of imperfect measurements, see Ref. [14]). The QZE is realized when $\Gamma^{\text{meas}} < \Gamma_{\text{onshell}}$, while the IZE when $\Gamma^{\text{meas}} > \Gamma_{\text{onshell}}$. Both the QZE and the IZE have been experimentally verified on a genuinely unstable quantum system [15].

Next, we turn to the specific case that we are interested in: the weak decay of the neutron, for which we use the simplified decay width $\Gamma(\omega) = g_n^2\omega^5$ (valid for $\omega \lesssim \omega_{\text{onshell}} + m_\pi$; for the full formula, see Ref. [3]). The on-shell values are $\omega_{\text{onshell}} = m_n - m_p - m_e = 0.782333$ MeV and $\Gamma_{\text{onshell}} = g_n^2\omega_{\text{onshell}}^5 = \hbar/\tau_n^{\text{beam}} = \hbar/888.1\ \text{sec}^{-1} = 7.41146 \times 10^{-25}$ MeV (out of which $g_n = 1.59028 \times 10^{-12}$ MeV^{-2}).

The fact that the function $\Gamma(\omega)$ is rising around ω_{onshell} implies that the IZE is possible if the time interval τ between subsequent measurements is sufficiently short. In fact, $\Gamma_k^{\text{meas}}(\tau,\omega_C) > \Gamma_{\text{onshell}}$ for any value of τ (and by using reasonable values of the off-shellness ω_C; in the following, we shall use $\omega_C = 5\omega_{\text{onshell}}$).

Unfortunately, we do not know which is the correct model for measurement in the case of trap experiments. In Ref. [6], the function f_2 was used for illustrative purposes. Here, we compare both models mentioned above to see to which extent the results are comparable. Note, for both response functions, the numerical value of the 'highest energy' ω_C is an important parameter, since the measured decay width would diverge without this cut-off. The value of τ for which the width in the trap experiment is $\Gamma_k^{\text{meas}}(\tau,\omega_C)/\Gamma_{\text{onshell}} = 1.0098$ is $\tau = 12569.4$ MeV^{-1} when using f_1 and $\tau = 12569.9$ MeV^{-1} when using f_2; these two values are extremely similar (for the discussion about why this value of τ is reasonable for trap experiments, see Ref. [6]). In Fig. 1 we also show the dependence on τ (we use slightly smaller values of τ to see better the effect). The left panel shows that the IZE occurs and that both response functions generate very similar results. This is confirmed by the right panel, where the ratio is shown to be very close to unity. Varying ω_C does not change the outcome as long as, of course, the same value is used for both models.
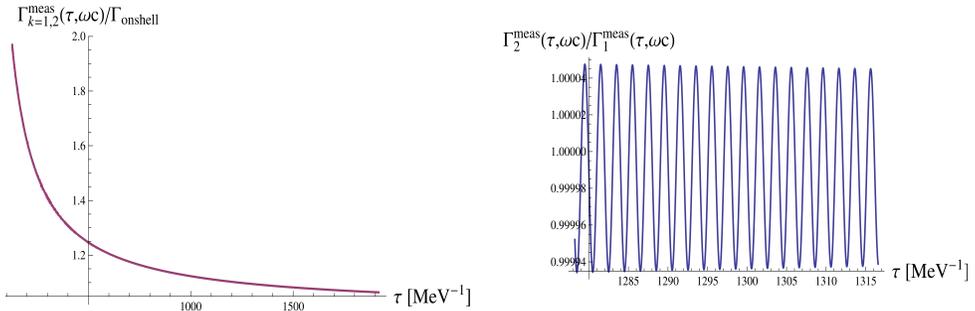
Fig. 1. Left panel: the functions $\Gamma_{1,2}^{\mathrm{meas}}(\tau, \omega_C = 5\omega_{\mathrm{onshell}})/\Gamma_{\mathrm{onshell}}$ are plotted as a function of τ. They are larger than unity, hence the IZE occurs. Both curves are so similar that it is almost impossible to distinguish them. Right panel: ratio $\Gamma_2(\tau, \omega_C = 5\omega_{\mathrm{onshell}})/\Gamma_1(\tau, \omega_C = 5\omega_{\mathrm{onshell}})$ as a function of τ in a small time interval: the ratio is very close to one. This is true for other values of τ as well.

3. Concluding remarks

In this work, we have tested two different models of measurement that lead to IZE effect for the decay of the neutron in trap experiments. Both of them lead to very similar results, thus showing that the IZE is not dependent on the details of the measurement process. In the future, one should repeat the previous study by using more advanced measurement models which go beyond the simple bang–bang measurement (leading to f_1) or the continuous measurement of the final state (leading to f_2). It would be particularly interesting to model the continuous measurement of the initial state.

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