PHENIX RESULTS ON LÉVY ANALYSIS OF BOSE–EINSTEIN CORRELATION FUNCTIONS*

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The nature of the quark–hadron phase transition can be investigated through analysing the space-time structure of the hadron emission source. For this, the Bose–Einstein or HBT correlations of identified charged particles are among the best observables. In this paper, we present the latest results from the RHIC PHENIX experiment on such measurements.

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1. Introduction

The PHENIX experiment at the BNL Relativistic Heavy Ion Collider (RHIC) has collected comprehensive data in multiple different collision systems from $p+p$, $p+A$ through $A+A$ up to $U+U$ collisions, at energies that are varied in the region where the change from the first order to crossover phase transition is expected to occur. The importance of the RHIC beam energy scan program lies in the possibility of investigating the phase diagram of QCD matter, and the quark–hadron phase transition. One of the best tools to gain information about the particle-emitting source is the measurement of Bose–Einstein or HBT correlations of identical bosons. In our latest measurements, we utilize Lévy-type sources [4,5] to describe the measured correlation functions. In the case of a second order QCD phase transition, one of the source parameters, the index of stability $\alpha$, is related to one of the critical exponents (the so-called correlation exponent $\eta$). Thus, the Bose–Einstein correlation data may yield information on the nature of the quark–hadron phase transition, particularly, it may shed light on the location of the critical endpoint (CEP) on the phase diagram.

2. Beam energy dependence of HBT radii

Today, high-energy physics experiments measure the scale parameter of HBT correlation functions (often called HBT radii) as a function of particle type, transverse momentum, azimuthal angle, collision energy, and collision geometry. Recently, PHENIX measured the Gaussian HBT radii of two-pion Bose–Einstein correlation functions in Au+Au collisions at several beam energies [1]. The extracted radii, which were compared to recent STAR [2] and ALICE [3] data, show characteristic scaling patterns as a function of the transverse mass of the emitted pion pairs, consistent with hydrodynamic-like expansion. In Fig. 1, we show specific combinations of the three-dimensional radii [1–3] that are sensitive to the medium expansion velocity and lifetime, and the pion emission duration. These show non-monotonic $\sqrt{s_{NN}}$ dependencies, which may be an indication of the critical endpoint in the phase diagram of hot and dense nuclear matter.

![Fig. 1. The $\sqrt{s_{NN}}$ dependencies of combinations of HBT radii sensitive to expansion velocity (a) and emission duration (b) from [1–3].](image)

3. A possible way of finding the critical point

If we leave behind the Gaussian assumption for the correlation shape, there may be another way to search for the place of the critical point on the phase diagram, connected to the non-Gaussian shape of the correlation functions. The generalization of the Gaussian is the so-called Lévy-distribution [4, 5]

$$L(\alpha, R, r) = \frac{1}{(2\pi)^3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha},$$

where the $\alpha = 2$ case corresponds to the Gaussian, and the $\alpha = 1$ to the Cauchy distribution. With this assumption for the shape of the source, the shape of the correlation functions becomes
where \( k \) is the absolute value of half of the relative momentum. From statistical physics we know that the definition of critical exponent \( \eta \) comes from the spatial correlations being proportional to \( r^{-(d-2+\eta)} \), where \( d \) represents the number of dimensions. In the case of symmetrical Lévy-stable distributions, the spatial correlations are proportional to \( r^{-1-\alpha} \). So it turns out that the Lévy exponent \( \alpha \) is identical to the critical exponent \( \eta \). The value of \( \eta \) at the CEP can be predicted if we assume that the QCD universality class is the same as the universality class of the 3-dimensional Ising model \([6, 7]\): \( \eta = 0.03631(3) \) \([8]\). In the case of random field 3D Ising model, this value is somewhat different: \( \eta = 0.50 \pm 0.05 \) \([9]\). If we measure the \( \alpha \) parameter at different beam energies, we may gain insight into the phase diagram of QCD matter, as the change in its behaviour is connected to the proximity of the CEP.

4. Lévy analysis of HBT correlation functions

The data sample used in the latest analysis consists of \( \sim 7 \) billion minimum bias \( \text{Au}+\text{Au} \) events at \( \sqrt{s_{NN}} = 200 \) GeV recorded by PHENIX during Run-10. We measured two-pion correlation functions for \( \pi^+\pi^+ \) and \( \pi^-\pi^- \) pairs with average \( p_T \) between 0.2 and 0.22 GeV/c measured in the longitudinal co-moving frame. The fit shows the measured correlation function and the complete fit function, while a “Coulomb-corrected” fit function \( C^{(0)}/k \) is also shown, with the data multiplied by \( C^{(0)}/C^{\text{Coul}} \). In this analysis, we measured 62 such correlation functions (for ++ and −− pairs, in 31 \( p_T \) bins).
pairs in 31 $p_T$ bins (ranging from 180 MeV/c to 850 MeV/c), where $p_T$ is the average transverse momentum of the pair. If one uses a spherically symmetric Lévy-type source and couples it with the core–halo model, then the shape of the two-particle correlation function takes the form seen in Eq. (2). However, fitting this functional form does not yield meaningful physics results since it does not take into account the final state Coulomb interaction. To handle this, we have incorporated the effect of the Coulomb repulsion of the identically charged pion pairs into the fit function. An example fit is shown in Fig. 2.

In Fig. 3, we present the resulting physical fit parameters ($\lambda, R, \alpha$) versus pair $m_T$ (corresponding to the given $p_T$ bin), as well as a newly found scale parameter, $1/\hat{R} = (\lambda(1 + \alpha))/R$. In Fig. 3 (a), we can see that the $m_T$ dependence of $R$ shows the usual decreasing trend predicted by hydrodynamics for an expanding source. Figure 3 (b) shows that the strength parameter $\lambda$ seems to saturate at high $m_T$, and a decreasing trend is clearly visible with decreasing $m_T$. The value of $\alpha$ (shown in Fig. 3 (c)) is approximately in-

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**Fig. 3.** Fit parameters versus average $m_T$ of the pair with statistical and symmetric systematic uncertainties shown as bars and boxes, respectively. (a) Lévy scale parameter $R$. (b) Correlation strength parameter $\lambda$. (c) Lévy exponent $\alpha$. (d) Newly found scale parameter $1/\hat{R}$.
dependent of $m_T$ and it is far from the Gaussian limit ($\alpha = 2$) and the 3D Ising limit ($\alpha \leq 0.5$) corresponding to the conjectured value at the CEP. Regarding $U_A(1)$ symmetry restoration [10], our new preliminary results are consistent with earlier preliminary results [11] within statistical uncertainties. The three physical fit parameters are strongly correlated (the value of the correlation coefficients between $(\lambda, R)$, $(R, \alpha)$ are above 90%), so we searched for parameter combinations that represent the fit function in a more unambiguous way. We indeed found such a parameter: $\hat{R} = R/\lambda(1 + \alpha)$. If we calculate $R$ as $R = \hat{R}\lambda(1 + \alpha)$ and use this new parameter for the fitting, the correlation coefficients between $(\lambda, \hat{R})$, $(\hat{R}, \alpha)$ drop to 20–30%. Figure 3 (d) indicates that $1/\hat{R}$ is approximately linear in $m_T$. The modified Lévy scale parameter $\hat{R}$ was found experimentally, but, as for now, the physical interpretation remains an open question. However, now as the method of this measurement is well-established, the next step is the analysis of the lower energy data sets — this may lead us forward in exploring the phase diagram of hot and dense, strongly interactive matter.

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