ARBITRARY $\omega$–$\phi$ MIXING FORM IN GELL-MANN–OKUBO QUADRATIC MASS RELATION CREATES THE SAME MIXING ANGLE $\theta$ VALUE\(^*\)

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The inverse relations of the four independent couples of physically acceptable $\omega$–$\phi$ mixing forms give expressions for $\omega_8$ and $\omega_0$ as functions of the unknown mixing angle $\theta$ and physical states $\omega$ and $\phi$. Substituting for expressions obtained in such a way for $\omega_8$ repeatedly into Gell-Mann–Okubo quadratic mass relation, which yields quadratic mass of $m_{\omega_8}^2$ as a combination of quadratic masses of $K^*(980)$ and $\rho^0(770)$ vector mesons, always determines the same value of mixing angle $\theta$. Next, the same result is obtained also by using all physically non-acceptable $\omega$–$\phi$ mixing forms.

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1. Introduction

In [1], we have demonstrated that there are generally eight possible $\omega$–$\phi$ mixing forms

\begin{align*}
1. \quad & \omega = \omega_8 \sin \theta + \omega_0 \cos \theta, \quad \phi = -\omega_8 \cos \theta + \omega_0 \sin \theta, \\
2. \quad & \omega = -\omega_8 \sin \theta + \omega_0 \cos \theta, \quad \phi = -\omega_8 \cos \theta - \omega_0 \sin \theta, \\
3. \quad & \omega = \omega_8 \sin \theta - \omega_0 \cos \theta, \quad \phi = \omega_8 \cos \theta + \omega_0 \sin \theta, \\
4. \quad & \omega = -\omega_8 \sin \theta - \omega_0 \cos \theta, \quad \phi = \omega_8 \cos \theta - \omega_0 \sin \theta, \\
5. \quad & \omega = \omega_8 \sin \theta + \omega_0 \cos \theta, \quad \phi = \omega_8 \cos \theta - \omega_0 \sin \theta, \\
\end{align*}

6. $\omega = -\omega_8 \sin \theta + \omega_0 \cos \theta$, \quad $\phi = \omega_8 \cos \theta + \omega_0 \sin \theta$, \\
7. $\omega = \omega_8 \sin \theta - \omega_0 \cos \theta$, \quad $\phi = -\omega_8 \cos \theta - \omega_0 \sin \theta$, \\
8. $\omega = -\omega_8 \sin \theta - \omega_0 \cos \theta$, \quad $\phi = -\omega_8 \cos \theta + \omega_0 \sin \theta$

from which only four, to be denoted by

1. $\omega = \omega_8 \sin \theta + \omega_0 \cos \theta$, \quad $\phi = -\omega_8 \cos \theta + \omega_0 \sin \theta$, \\
4. $\omega = -\omega_8 \sin \theta - \omega_0 \cos \theta$, \quad $\phi = \omega_8 \cos \theta - \omega_0 \sin \theta$, \\
5. $\omega = \omega_8 \sin \theta + \omega_0 \cos \theta$, \quad $\phi = \omega_8 \cos \theta - \omega_0 \sin \theta$, \\
8. $\omega = -\omega_8 \sin \theta - \omega_0 \cos \theta$, \quad $\phi = -\omega_8 \cos \theta + \omega_0 \sin \theta$

are physically acceptable.

That is one considerable result of our investigations.

Another one is demonstrated in this contribution and it concerns of a determination of the $\omega$–$\phi$ mixing angle $\theta$ value by using the Gell-Mann–Okubo quadratic mass relation.

Further, it will be clearly exhibited that arbitrary $\omega$–$\phi$ mixing form, physically acceptable or physically non-acceptable, to be applied in Gell-Mann–Okubo quadratic mass relation of $1^-$ vector mesons, creates for mixing angle $\theta$ the same value.

2. Gell-Mann–Okubo quadratic mass relation

An application of the SU(3) symmetry to a classification of mesons and baryons revealed an existence of the “quarks”, and induced the idea that “mesons” are bound states of quarks and antiquarks, and “baryons” are compound of 3 quarks.

Moreover, experimentally observed hadrons with similar properties are, according to irreducible representations of the SU(3) group, arranged into octuplets, decuplets, 27plets, 35plets etc.

As it is well-known, e.g. the nonet of $1^-$ vector mesons can be represented by $3 \times 3$ octet matrix and a singlet $\omega_0$ of the form of

$$V = \begin{pmatrix} 
\omega_8/\sqrt{6} + \rho^0/\sqrt{2} & \rho^+ & K^{*+} \\
\rho^- & \omega_8/\sqrt{6} - \rho^0/\sqrt{2} & K^{*0} \\
K^{*-} & K^{*0} & -2\omega_8/\sqrt{6} 
\end{pmatrix}, \quad \omega_0. \quad \text{(3)}$$
The mass of every vector meson from matrix (3) can be generally expressed through its quantum numbers, such as strangenes $S$ and isospin $I$, which leads to the following Gell-Mann–Okubo quadratic mass relation:

$$m^2(\omega_8) = \frac{4m^2(K^0) + m^2(\bar{K}^0)}{2} - m^2(\rho^0) = \left(932.14 \text{ MeV}\right)^2,$$

which next is used for a determination of the $\omega-\phi$ mixing angle $\theta$ value.

3. Determination of the $\omega-\phi$ mixing angle value

As the unitary singlet is denoted by $\omega_0$ and the unitary octet by $\omega_8, K^*, \bar{K}^*, \rho$, then physically acceptable mixing forms between $\omega, \phi$ and $\omega_8, \omega_0$ exist as they are presented by (2).

The reversed relations to the mixing forms (2) are obtained by solutions always of the two algebraic equations 1, 4, 5 and 8, with two unknowns, $\omega_8$ and $\omega_0$, and as a result, one gets

1. $\omega_0 = \omega \cos \theta + \phi \sin \theta,$
   $\omega_8 = \omega \sin \theta - \phi \cos \theta,$
2. $\omega_0 = -\omega \cos \theta - \phi \sin \theta,$
   $\omega_8 = -\omega \sin \theta + \phi \cos \theta,$
3. $\omega_0 = \omega \cos \theta - \phi \sin \theta,$
   $\omega_8 = \omega \sin \theta + \phi \cos \theta,$
4. $\omega_0 = -\omega \cos \theta + \phi \sin \theta,$
   $\omega_8 = -\omega \sin \theta - \phi \cos \theta.$

If orthogonal states $|\omega\rangle$ and $|\phi\rangle$ are “eigenfunctions” of the quadratic mass operator $\mathcal{M}^2$, then the non-diagonal matrix elements are equal to zero

$$\langle \omega | \mathcal{M}^2 | \phi \rangle = \langle \phi | \mathcal{M}^2 | \omega \rangle = 0.$$ 

Then utilizing from the first relation of (5) for $|\omega_8\rangle$ the expression $\omega_8$

1. a calculation of the mass squared of the $\omega_8$ particle gives

$$m^2(\omega_8) = \langle \omega_8 | \mathcal{M}^2 | \omega_8 \rangle$$
$$= (\sin \theta |\omega\rangle - \cos \theta |\phi\rangle) \mathcal{M}^2 (|\omega\rangle \sin \theta - |\phi\rangle \cos \theta)$$
$$= \sin^2 \theta \langle \omega | \mathcal{M}^2 | \omega \rangle + \cos^2 \theta \langle \phi | \mathcal{M}^2 | \phi \rangle$$
$$- \sin \theta \cos \theta \langle \omega | \mathcal{M}^2 | \phi \rangle - \cos \theta \sin \theta \langle \phi | \mathcal{M}^2 | \omega \rangle$$

and exploiting (6), finally, one gets

$$m^2(\omega_8) = m^2(\omega) \sin^2 \theta + m^2(\phi) \cos^2 \theta.$$
If from the second relation of (5) for $|\omega_8\rangle$, the expression $\omega_8$ is used

4. a calculation of the mass squared of the $\omega_8$ particle gives

$$m^2(\omega_8) = \langle \omega_8 | M^2 | \omega_8 \rangle$$

$$= (-\sin \theta \langle \omega \rangle + \cos \theta \langle \phi \rangle) M^2 (-|\omega\rangle \sin \theta + |\phi\rangle \cos \theta)$$

$$= \sin^2 \theta \langle \omega | M^2 | \omega \rangle + \cos^2 \theta \langle \phi | M^2 | \phi \rangle$$

$$- \sin \theta \cos \theta \langle \omega | M^2 | \phi \rangle - \cos \theta \sin \theta \langle \phi | M^2 | \omega \rangle$$  

and exploiting (6), finally, one gets

$$m^2(\omega_8) = m^2(\omega) \sin^2 \theta + m^2(\phi) \cos^2 \theta. \quad (10)$$

If from the third relation of (5) for $|\omega_8\rangle$, the expression $\omega_8$ is used

5. a calculation of the mass squared of the $\omega_8$ particle gives

$$m^2(\omega_8) = \langle \omega_8 | M^2 | \omega_8 \rangle$$

$$= (\sin \theta \langle \omega \rangle + \cos \theta \langle \phi \rangle) M^2 (|\omega\rangle \sin \theta + |\phi\rangle \cos \theta)$$

$$= \sin^2 \theta \langle \omega | M^2 | \omega \rangle + \cos^2 \theta \langle \phi | M^2 | \phi \rangle$$

$$+ \sin \theta \cos \theta \langle \omega | M^2 | \phi \rangle + \cos \theta \sin \theta \langle \phi | M^2 | \omega \rangle$$  

and exploiting (6), finally, one gets

$$m^2(\omega_8) = m^2(\omega) \sin^2 \theta + m^2(\phi) \cos^2 \theta. \quad (12)$$

If from the fourth relation of (5) for $|\omega_8\rangle$, the expression $\omega_8$ is used

8. a calculation of the mass squared of the $\omega_8$ particle gives

$$m^2(\omega_8) = \langle \omega_8 | M^2 | \omega_8 \rangle$$

$$= (-\sin \theta \langle \omega \rangle - \cos \theta \langle \phi \rangle) M^2 (-|\omega\rangle \sin \theta - |\phi\rangle \cos \theta)$$

$$= \sin^2 \theta \langle \omega | M^2 | \omega \rangle + \cos^2 \theta \langle \phi | M^2 | \phi \rangle$$

$$+ \sin \theta \cos \theta \langle \omega | M^2 | \phi \rangle + \cos \theta \sin \theta \langle \phi | M^2 | \omega \rangle$$  

and exploiting (6), finally, one gets

$$m^2(\omega_8) = m^2(\omega) \sin^2 \theta + m^2(\Phi) \cos^2 \theta. \quad (14)$$

If the masses of $\omega_8, \omega(782), \phi(1020)$ are taken from [2]

$$m(\omega_8) = 932.14 \text{ MeV} ,$$  

$$m(\omega) = 782.65 \text{ MeV} ,$$  

$$m(\phi) = 1019.462 \text{ MeV} ,$$  

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$$m(\phi) = 1019.462 \text{ MeV} ,$$  

$$m(\omega_8) = 932.14 \text{ MeV} ,$$  

$$m(\omega) = 782.65 \text{ MeV} ,$$  

$$m(\phi) = 1019.462 \text{ MeV} ,$$
then from four previous identical relations, by means of the expression

\[ \sin^2 \theta = \frac{m^2(\phi) - m^2(\omega_8)}{m^2(\phi) - m^2(\omega)} \]  

(18)

one obtains

\[ \theta = \sin^{-1} 0.63192 = 39.19^\circ \]

and, in this way, we have demonstrated that the \( \omega-\phi \) mixing angle \( \theta \) value does not depend on the physically acceptable \( \omega-\phi \) mixing forms.

In a similar way, one can convince himself that the \( \theta \)-value does not even depend on the physically non-acceptable \( \omega-\phi \) mixing forms and is also equal to \( \theta = \sin^{-1} 0.63192 = 39.19^\circ \).

There is a question: In which physical “circumstances” the physically acceptable forms of \( \omega-\phi \) mixing and physically non-acceptable forms of \( \omega-\phi \) mixing will produce different results.

This question will be a subject of our further investigations.

4. Conclusions

Starting from the physically acceptable \( \omega-\phi \) mixing forms, calculating their reversed relations and exploiting the Gell-Mann–Okubo quadratic mass formula for \( 1^- \) octet of vector mesons, the \( \omega-\phi \) mixing angle \( \theta = 39.19^\circ \) has been determined.

However, subsequently it was verified that this result is valid without any specification to physically acceptable, or physically non-acceptable, mixing forms.

The problem is, however, arisen in the calculation of the mixing angle \( \theta' \) for the first excited states of vector mesons, where a substitution of the concerned particle masses into (18) gives \( \sin^2 \theta' = 1.011 > 1 \).

In the evaluation of \( \theta'' \), the following value \( \sin^2 \theta'' = 0.9223 \) is found, from which the value \( \theta'' = \sin^{-1} 0.96 = 73.81^\circ \) is determined.

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REFERENCES