THE $\Sigma_{\pi N}$ TERM, CHIRAL MULTIPLET MIXING AND HIDDEN STRANGENESS IN THE NUCLEON*

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We calculate the $\Sigma_{\pi N}$ term in the chiral mixing approach to baryons, i.e., with SU_L(3) × SU_R(3) chiral multiplets $[(6,3) \oplus (3,6)]$, $[3,\bar{3}] \oplus [(\bar{3},3) \oplus (3,\bar{3})]$, admixed in the baryons, using known constraints on the current quark masses $m_0^u, m_0^d$. We show that the $[(6,3) \oplus (3,6)]$ multiplet’s contribution is enhanced by a factor of $\frac{57}{9} \simeq 6.33$, due to SU_L(2) × SU_R(2) algebra, that leads to $\Sigma_{\pi N} \geq (1 + \frac{48}{9} \sin^2 \theta) \frac{3}{2} (m_0^u + m_0^d) = 60$ MeV, in general accord with “experimental” values of $\Sigma_{\pi N}$. The chiral mixing angle $\theta$ is given by $\sin^2 \theta = \frac{3}{8} (g_{A}^{(0)} + g_{A}^{(3)})$, where $g_{A}^{(0)} = 0.33 \pm 0.08$, or $0.28 \pm 0.16$ is the flavor-singlet axial coupling, and $g_{A}^{(3)} = 1.267$, is the third component of the octet one. These results show that there is no need for $q^4\bar{q}$ components, and in particular, no need for an $s\bar{s}$ component in the nucleon.

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1. Introduction

The nucleon $\Sigma_{\pi N}$ term is a “theoretical measure” of its current quarks’ mass contribution to the total nucleon mass. The difference of the value extracted from the measured $\pi N$ scattering partial wave analyses from 25 MeV has been interpreted as an increase of Zweig-rule-breaking in the nucleon, or equivalently to an increased $s\bar{s}$ content $y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}$ of the nucleon, Refs. [1–3]. As all “measurements” of $\Sigma_{\pi N}$ have yielded values ranging from 55 MeV to 75 MeV [4], that are substantially larger than the expected 25 MeV, it has consequently appeared that the $s\bar{s}$ content of the nucleon must be (very) large.

A number of experiments have measured the $s\bar{s}$ contributions to nucleon observables other than the $\Sigma_{\pi N}$ term [5]. No experiment has found a result larger than a few % of the $u$ (and/or $\bar{u}$) and $d$ (and/or $\bar{d}$) contributions\(^1\), thus making the $s\bar{s}$ content of the nucleon effectively negligible $y \simeq 0$. Thus, the enigma has deepened: how is it possible to have such a large $\Sigma_{\pi N}$ term without any $s\bar{s}$ content in other observables? In the meantime, the nucleon $\Sigma_{\pi N}$ term has been shown as an important ingredient in searches for (supersymmetric) cold dark matter, Ref. [7] and in the QCD phase diagram, thereby only increasing the stakes.

In this report, we show explicitly an alternative mechanism of hadronic $\Sigma_{\pi N}$ term enhancement with strangeness content $y = 0$ and pin-point the source of the enhancement to the $(6, 3) \equiv [(6, 3) \oplus (3, 6)]$, or $(1, \frac{1}{2}) \equiv [(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)]$ chiral component (in the $SU_L(3) \times SU_R(3)$ or $SU_L(2) \times SU_R(2)$ notations, respectively) of the nucleon. This component contributes about three quarters of the enhanced value of $\Sigma_{\pi N} \geq 55$ MeV, which would otherwise be $\geq 14$ MeV, while keeping a vanishing $s\bar{s}$ component in the nucleon. The same $(1, \frac{1}{2})$ chiral component is crucial for the proper description of the nucleon’s isovector axial coupling $g_A^{(3)} = 1.267$.

We show in some detail how the $\Sigma_{\pi N}$ term enhancement emerges from the $SU_L(2) \times SU_R(2)$ chiral algebra. To that end, we use a hadronic two-flavor $SU_L(2) \times SU_R(2)$ chiral mixing model, in which the $s\bar{s}$ content of the nucleon vanishes, $y = 0$, \textit{per definitionem}. Baryons in the spontaneously broken symmetry phase may be effectively described by a few chiral components: it was shown in Refs. [8–10], that several nucleon’s properties can be successfully described by mixing of three chiral multiplet components. Of two historical chiral mixing scenarios [8–10], only the Harari one [9, 10], described by

$$|N\rangle = \sin \theta |(6, 3)\rangle + \cos \theta (\cos \varphi |(3, \bar{3})\rangle + \sin \varphi |(\bar{3}, 3)\rangle),$$

\(^1\) This makes these effects compatible with the (much more) mundane isospin-violating corrections, from which they are indistinguishable [6].
has survived the inclusion of the baryons’ anomalous magnetic moments in the three-flavor case [11]. Here we use the original SU\(_L(3) \times SU_R(3)\) notation to distinguish between the two kinds of \((\frac{1}{2}, 0)\) multiplets in SU\(_L(2) \times SU_R(2)\), though we shall use only the two-flavor multiplets.

2. Calculation

To calculate the nucleon \(\Sigma\pi N = \langle N | \Sigma | N \rangle\) term, we use the \(\Sigma\) operator defined as the double commutator

\[
\Sigma = \frac{1}{2} \epsilon^{c a b} \left[ Q^a_5, \left[ Q^b_5, H_{\chi \text{SB}} \right] \right]
\]

of the axial charges \(Q^a_5\) and the chiral symmetry breaking Hamiltonian \(H_{\chi \text{SB}}\). It was introduced by Dashen [13] as a way of separating out the explicit chiral SU\(_L(2) \times SU_R(2)\) symmetry breaking part \(H_{\chi \text{SB}}\) from the total Hamiltonian. Ensuring that the (spontaneously broken) chiral symmetry is properly implemented is particularly important in a calculation at the hadron level. We have developed in Refs. [11, 15–21] a (linear realization) chiral Lagrangian that reproduces the results of the phenomenological chiral mixing method.

We follow Ref. [12], and use an explicit \(\chi \text{SB} \) “bare” nucleon mass and the corresponding \(\chi \text{SB} \) Hamiltonian density

\[
H^N_{\chi \text{SB}} = \sum_{i=1}^{3} \bar{N}_i M^0_{N,i} N_i + \bar{\Delta}_{(1,\frac{1}{2})} M^0_{\Delta(1,\frac{1}{2})} \Delta_{(1,\frac{1}{2})},
\]

where \(i\) stands for the three chiral multiplets \((1,\frac{1}{2})\), \((\frac{1}{2}, 0)\) and \((0, \frac{1}{2})\). A priori, we do not know the values of the “current” nucleon masses, except for a lower limit: they cannot be smaller than three isospin-averaged current quark masses: \(M^0_{N,i} \geq \bar{m}_q^0 = \frac{3}{2} (m^0_u + m^0_d)\). For simplicity’s sake, we shall assume, as a first approximation, that all three chiral components have the same “current” nucleon mass \(M^0_{N} = M^0_{N(6,3)} = M^0_{N(1,\frac{1}{2})} = M^0_{\Delta(1,\frac{1}{2})} = M^0_{(3,3)} = M^0_{(\frac{1}{2},0)} = M^0_{(3,3)} = M^0_{(0,\frac{1}{2})} = \frac{3}{2} (m^0_u + m^0_d)\).

The chiral SU\(_L(2) \times SU_R(2)\) generators \(Q^a_5\) and their commutators with the nucleon \(N\) and \(\Delta\) fields were worked out in Refs. [16–19]:

\[
\left[ Q^a_5, N_{(1,\frac{1}{2})} \right] = \gamma_5 \left( \frac{5}{3} \frac{T^a}{2} N_{(1,\frac{1}{2})} + \frac{2}{\sqrt{3}} T^a \Delta_{(1,\frac{1}{2})} \right),
\]

\[
\left[ Q^a_5, \Delta_{(1,\frac{1}{2})} \right] = \gamma_5 \left( \frac{2}{\sqrt{3}} T^{\dagger a} N_{(1,\frac{1}{2})} + \frac{1}{3} \tau^a (\frac{1}{2}) \Delta_{(1,\frac{1}{2})} \right),
\]

\(\text{For normalization and notational conventions, see Ref. [12].}\)
\[ [Q^a_5, N(\frac{1}{2}, 0)] = \frac{\gamma^5}{2} \frac{\tau^a}{2} N(\frac{1}{2}, 0), \]
\[ [Q^a_5, N(0, \frac{1}{2})] = -\gamma^5 \frac{\tau^a}{2} N(0, \frac{1}{2}), \]

where \( a = 1, 2, 3 \), \( t_i^{(\frac{3}{2})} \) are the isospin-\( \frac{3}{2} \) generators of the SU(2) group and \( T^i \) are the so-called iso-super-bion (2 \times 4) matrices, that are related to the SU(2) Clebsch–Gordan coefficients \( \langle \frac{3}{2} I_3(\Delta) | I_3(i) \frac{1}{2} I_3(N) \rangle \), with the following properties (see Appendix B of Ref. [18])

\[ T_i^{\dagger} T^k = \frac{3}{4} \delta^{ik} - \frac{1}{6} \left\{ (t_i^{(\frac{3}{2})} , t_j^{(\frac{3}{2})} ) + i \epsilon^{ijk} t_k^{(\frac{3}{2})} \right\}, \]
\[ T_i^a T_i^b = P_i^{\frac{3}{2}}. \] (4)

The chiral SU_L(2) \times SU_R(2) double commutators for the \([(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)]\)

chiral multiplet are

\[ [Q^b_5, [Q^a_5, \bar{N}(1, \frac{1}{2}) N(1, \frac{1}{2})]] = \frac{41}{9} \delta^{ab} \bar{N}(1, \frac{1}{2}) N(1, \frac{1}{2}) \]
\[ + \bar{\Delta}(1, \frac{1}{2}) (2 \delta^{ab} - \frac{4}{9} \left\{ t_i^{(\frac{3}{2})} , t_j^{(\frac{3}{2})} \right\} ) \Delta(1, \frac{1}{2}) + \ldots, \] (5)

where \( \ldots \) stand for the off-diagonal terms, such as \( \bar{N}(1, \frac{1}{2}) \ldots \Delta(1, \frac{1}{2}) \), and their Hermitian conjugates.

We contract Eq. (5) with \( \frac{1}{3} \delta^{ab} \) (where summation over repeated indices is understood) to find

\[ \frac{1}{3} \delta^{ab} [Q^b_5, [Q^a_5, \bar{N}(1, \frac{1}{2}) N(1, \frac{1}{2})]] = \frac{41}{9} \bar{N}(1, \frac{1}{2}) N(1, \frac{1}{2}) + \frac{8}{9} \bar{\Delta}(1, \frac{1}{2}) \Delta(1, \frac{1}{2}) + \ldots, \] (6)

where we have used the identity \( t_i^{(\frac{3}{2})} t_j^{(\frac{3}{2})} = \frac{15}{4} 1_{4 \times 4} \), and similarly for the \( \Delta \)-field contribution

\[ \frac{1}{3} \delta^{ab} [Q^b_5, [Q^a_5, \bar{\Delta}(1, \frac{1}{2}) \Delta(1, \frac{1}{2})]] = \frac{16}{9} \bar{N}(1, \frac{1}{2}) N(1, \frac{1}{2}) + \frac{13}{9} \bar{\Delta}(1, \frac{1}{2}) \Delta(1, \frac{1}{2}) + \ldots \] (7)

This finally leads to

\[ \Sigma_{\pi N} = \sin^2 \theta \left( \frac{41}{9} M^0_{N(1, \frac{1}{2})} + \frac{16}{9} M^0_{\Delta(1, \frac{1}{2})} \right) \]
\[ + \cos^2 \theta \left( \cos^2 \varphi M^0_{N(\frac{1}{2}, 0)} + \sin^2 \varphi M^0_{N(\frac{1}{2}, 0)} \right), \] (8)

which is our basic result here.
3. Result and discussion

Inserting our simplifying assumption that all the “current nucleon” masses are equal, one finds the final result

$$\Sigma_{\pi N} = \left(1 + \frac{16}{3} \sin^2 \theta\right) M_N^0. \ (9)$$

Note that the enhancement term $\frac{16}{3} \sin^2 \theta$ is due to the factor $\frac{41+16}{9} = \frac{19}{3} \approx 6.33$ appearing in Eq. (8) of the $[(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)]$ chiral multiplet which, in turn, is due to the iso-spurion matrices $T_i$. Thus, the enhancement factor $\frac{19}{3}$ in Eq. (8) and consequently also the $\frac{16}{3} \sin^2 \theta$ in Eq. (9), are of SU$_L(2) \times$ SU$_R(2)$ algebraic origin. This leaves ample room for improvement of the $\Sigma_{\pi N}$ predictions, irrespective of the specific value of the chiral mixing angle $\theta$, within the chiral SU$_L(2) \times$ SU$_R(2)$ algebra approach.

The relevant chiral mixing angle $\theta$ has been extracted in Refs. [15–18], as $\frac{8}{3} \sin^2 \theta = g_{A}^{(0)} + g_{A}^{(3)}$, a function of the isovector $g_{A}^{(3)}$, and the flavor-singlet $g_{A}^{(0)}$ axial coupling, where $g_{A}^{(0)} = 0.28 \pm 0.16$, according to Ref. [22], or $g_{A}^{(0)} = 0.33 \pm 0.03 \pm 0.05$, according to Ref. [23]. Here we have taken the values of current quark masses from PDG2012 [24]: $m_{u}^{0} = 2.3 \times 1.35$ MeV and $m_{d}^{0} = 4.8 \times 1.35$ MeV, yielding $\frac{1}{2} \left( m_{u}^{0} + m_{d}^{0} \right) \approx 4.73$ MeV, substantially lower than before (cf. 7.6 MeV in Ref. [25]), and inserted them into the current nucleon mass to find $M_{N}^0 = \frac{3}{2} \left( m_{u}^{0} + m_{d}^{0} \right) \approx 14.2$ MeV and $\Sigma_{\pi N} = 59.5 \pm 2.3$ MeV, with $g_{A}^{(0)} = 0.33 \pm 0.03 \pm 0.05$ [23], or $\Sigma_{\pi N} = 58.0 \pm 4.5$ MeV, with $g_{A}^{(0)} = 0.28 \pm 0.16$ [22], in fair agreement with the “observed” $\Sigma_{\pi N}$ value range (55–75) MeV, see Ref. [4].

The above result of Eq. (9) ought to be viewed as a lower bound on the “true” $\Sigma_{\pi N}$ value, as we have assumed that all current nucleon masses $M_{N_i}^0$ equal three times the isospin-averaged current quark mass $\bar{m}_{q}^{0}$, which is appropriate only when all chiral components of the nucleon correspond to three-quark fields. That condition is not necessary, however, because some $q^4 \bar{q}$ baryon fields belong to the same chiral multiplets [21], and such fields have a larger current mass $M_{N_i}^0 = 5\bar{m}_{q}^{0}$, that consequently leads to a higher value of $\Sigma_{\pi N}$, but merely a sufficient one, as all of these chiral multiplets exist as bi-local three-quark fields [20].

In summary, we have shown that the “observed” values of $\Sigma_{\pi N} \geq 55$ MeV are readily obtained in the chiral-mixing approach without any strangeness content in the nucleon, as a natural consequence of the substantial chiral $(6, 3) = [(6, 3) \oplus (3, 6)] \to (1, \frac{1}{2})$ multiplet component. The precise value of $\Sigma_{\pi N}$ is a linear function, Eq. (9), of the sum of the flavor-singlet $g_{A}^{(0)}$, and the isovector $g_{A}^{(3)}$ axial coupling of the nucleon.
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