SCALAR MESONS IN $\tau \to K\pi\nu$ AND $\tau \to \eta\pi\nu$ DECAYS

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The $\tau$ decay modes $\tau \to P_1 P_2 \nu$ provide clean probes of the couplings of the flavoured scalar mesons to the $\bar{u}s$ or $\bar{u}d$ scalar currents. We review the theoretical constraints which relate the $P_1 P_2$ scalar form factors with $P_1 P_2$ scattering for $P_1 P_2 = K\pi, \eta\pi$ and their applications.

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1. Introduction

The lightest glueball in QCD is expected to be a scalar resonance and it is still an open problem to properly identify the corresponding state(s) in the physical spectrum (see e.g. [1] for a review). There has been significant progress, recently, in clarifying the status of the lightest physical scalar resonances. In particular, two “broad” resonances, the $\sigma$ (or $f_0(500)$) and $\kappa$ (or $K_0^*(800)$) have now been accepted into the PDG. The most reliable determinations concern the $\sigma$ [2, 3]; they make use of theoretical tools combined with accurate experimental data on low energy $\pi\pi$ scattering (in particular, the recent results from the NA48/2 [4] and DIRAC [5] collaborations). Reaching a similar level of confidence for the $\kappa$ would be important since its existence would disfavour the interpretation proposed in [6] of the $\sigma$ as a glueball-related state. New experimental measurements related to $\pi K$ scattering have been performed, notably a determination of $S$-wave phase-shifts from $D_{s4}$ decays [7] and a first measurement of the $\pi K$ atom lifetime [8], but better precision is required.

In this paper, I discuss the relevance of $\tau$ decays, in particular the decay modes into two pseudo-scalar mesons, $P_1 P_2$, for probing the scalar resonances which couple to $P_1 P_2$. In particular, these amplitudes provide a
means of measuring experimentally the coupling of scalar resonances to the scalar operators $\bar{u}s$ (for the $S = 1$ scalar resonances) and $\bar{u}d$ for the $I = 1$ resonances. These couplings are important for confirming the nonet assignments of resonances [9] and provide a quantitative measure of their degree of exoticity.

2. $\tau \to K\pi\nu$ decays and $\pi K$ scattering

The decay amplitudes of the $\tau$ lepton into two pseudo-scalar mesons, $\tau \to P_1 P_2 \nu$, are particularly simple objects (depending on a single variable) which are strongly tied to $P_1 P_2$ scattering. Let us first recall some results on $P_1 P_2 = K\pi$ and we will next consider $P_1 P_2 = \eta\pi$. The decay amplitude involves the matrix element of the vector current which is expressed in terms of two form factors, e.g. for $K^+\pi^0$

$$\langle K^+(p_K)\pi^0(p_\pi)|\bar{u}\gamma^\mu s|0\rangle = \frac{1}{\sqrt{2}} \left[ f_+^{K\pi}(t)(p_K - p_\pi)^\mu + f_-^{K\pi}(t)(p_K + p_\pi)^\mu \right]$$

(with $t = (p_K + p_\pi)^2$). The scalar form factor is defined as the combination $f^0_{K\pi}(t) = f_+^{K\pi}(t) + t/(m_K^2 - m_\pi^2)f_-^{K\pi}(t)$. Making use of the Ward identity for the vector current, $i\partial_\mu\bar{u}\gamma^\mu s = (m_s - m_u)\bar{u}s$, one sees that the scalar form factor describes the matrix element of the scalar current $\bar{u}s$

$$(m_s - m_u)\langle K^+(p_K)\pi^0(p_\pi)|\bar{u}s|0\rangle = -\frac{1}{\sqrt{2}} \left( m_K^2 - m_\pi^2 \right) f^0_{K\pi}(t).$$

Watson’s theorem implies that the phases of the form factors of $f_+^{K\pi}$ and $f^0_{K\pi}$ must be identical to the $I = 1/2$ phase shifts with $J = 1$ and $J = 0$ respectively, in the energy region where $\pi K$ scattering is elastic.

One can actually evaluate the form factor phases in a somewhat larger energy region, because the onset of inelasticity for $P_1 P_2$ scattering is strongly dominated by two-body channels. This was first used for the $\pi\pi$ scalar form factors in Ref. [10]. In the case of $K\pi$ with $J = 0$, the main inelastic channel is $K\eta'$ [11] and neglecting further inelastic channels, the Cauchy representation takes the form of a set of coupled singular integral equations

$$\left( \begin{array}{c} f^0_{K\pi}(t) \\ f^0_{K\eta'}(t) \end{array} \right) = \frac{1}{\pi} \int_0^\infty dt' \frac{dt'}{t' - t} \Sigma(t') \left( f^0_{K\pi}(t') \right) \left( f^0_{K\eta'}(t') \right),$$

where $\Sigma$ is a diagonal matrix of kinematical factors and $T_0$ is the $2 \times 2$ partial-wave T-matrix. Equations (3) were first studied in Ref. [12]. Solving

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1. In the case of $P_1 P_2 = \pi\pi$, the form factors are related to $I = 1$, $J = 1$ and the non-resonant $I = 2$, $J = 0$ scattering.
the equations\textsuperscript{2} gives, in particular, the phase of $f_0^{K\pi}$, which should be reliable in the region where inelasticity is effectively dominated by $K\eta'$. Figure 1 shows the result and illustrates the generic feature that the sharp onset of inelasticity is associated with a sharp dip of the form-factor phase. This dip corresponds to a minimum in the modulus of the form factor which implies some suppression of the coupling of the $K_0^*(1430)$ resonance to the $\bar{u}s$ operator.

Fig. 1. Phase of the $K\pi$ scalar form factor generated by solving Eqs. (3).

Fig. 2. The $\tau \to K\pi\nu$ decay width as a function of the $K\pi$ energy. The contributions from the vector (scalar) form factors correspond to the dotted/red (dash-dotted/blue) curves.

\textsuperscript{2} Proper asymptotic conditions must be imposed on the T-matrix which ensure a unique solution compatible with QCD asymptotic behaviour.
An analogous description can be developed for the vector form factor \( f^+_{K\pi} \) [13]. The onset of inelasticity, in this case, is dominated by the quasi-two-body channels: \( K^*\pi, K\rho \). The experimental measurement of the energy distribution \( d\Gamma_{\tau\to K\pi\nu}/dE_{K\pi} \) [14] may be used to improve the determination of the \( J = 1 \pi K \) scattering phase shift (see Ref. [15]). Figure 2 illustrates the vector and scalar form factor contributions to the energy distribution. The scalar contribution dominates over the vector one in the region \( \sqrt{t} < 0.8 \text{ GeV} \) and its size is confirmed by experiment. It displays a low energy enhancement which may be interpreted as reflecting the effect of the \( K^*_0(800) \) resonance and favours a non-exotic nature.

### 3. The \( \eta\pi \) scalar form factor and the isospin violating \( \tau \to \eta\pi\nu \) amplitude

The amplitude for \( \tau \to \eta\pi\nu \) involves two form factors \( f^\eta_{\pi} \), \( f^0_{\eta\pi} \) exactly as in Eq. (1). Weinberg’s G-parity argument [16] indicates that these form factors are isospin violating. This is also seen in the case of the scalar form factor from the Ward identity: \( i\partial_\mu \bar{u}\gamma^\mu d = (m_d - m_u)\bar{u}d - eA_\mu \bar{u}\gamma^\mu d \). We will revisit here the estimate of the scalar form factor making use of analyticity, chiral symmetry constraints, and analogies with the \( K\pi \) scalar form factor.

Concerning analyticity, one might be concerned about anomalous thresholds, since the \( \eta \) meson is not stable in QCD. This was investigated in detail in Ref. [17] based on Mandelstam’s approach: one starts from an unphysical situation where the mass of the \( \eta \) is small \( m_\eta < 3m_\pi \) and one follows the motion of the endpoint singularities, while varying the mass towards its physical value. The result is that no anomalous threshold appears.

Near \( t = 0 \), the form factors are constrained by chiral symmetry: they were computed up to NLO in ChPT in Refs. [18, 19]. At this order, the value at \( t = 0 \) obeys a parameter-free relation with an isospin violating combination of \( K\pi \) form factors which can be evaluated rather precisely using experimental inputs on \( Kl_3 \) decays from \( K \) factories (see [20]),

\[
f^\eta_{\pi}(0) = f^\eta_{0\pi}(0) = \frac{1}{\sqrt{3}} \left( \frac{f^{K^+\pi^0}_{\pi}(0)}{f^{K^0\pi^+}_{\pi}(0)} - 1 \right) = (1.49 \pm 0.23) \times 10^{-2}. \tag{4}
\]

A further constraint is associated with the derivative of the form factor

\[
f^\eta_{0\pi}(0) = f^\eta_{0\pi}(0) \left\{ \frac{1}{12F^2_\pi} \left[ 48L_5^\pi - \frac{1}{16\pi^2} \left( 9 \log \frac{m_K^2}{\mu^2} + 11 \right) + 4m^2_\pi \tilde{J}_{\eta\pi}(0) \right] \right. \\
+ \frac{\sqrt{3}e^2}{18\Delta_{\eta\pi}} \left[ -2(2S_2 + S_3) + \frac{11}{16\pi^2}Z \right] \right\}. \tag{5}
\]
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Numerically, this corresponds to a value for the $\eta$–$\pi$ scalar radius

$$\langle r^2 \rangle_{\eta\pi}^S \equiv \frac{6f_{0}^{\eta\pi}(0)}{\bar{f}_{0}^{\eta\pi}(0)} = (0.083 \pm 0.006) \text{ fm}^2$$  \hspace{1cm} (6)

which is remarkably small: it is twice smaller than the analogous $K\pi$ scalar radius (interestingly, the two radii should be equal in the large $N_c$ limit).

We can now estimate $f_{0}^{\eta\pi}$, writing a phase dispersive representation,

$$f_{0}^{\eta\pi}(t) = f_{0}^{\eta\pi}(0) \exp \left\{ \zeta t + \frac{t^2}{\pi} \int_0^\infty dt' \frac{\phi^{\eta\pi}(t')}{(t')^2 (t' - t)} \right\},$$  \hspace{1cm} (7)

where $\zeta = \langle r^2 \rangle_{\eta\pi}^S / 6$. Finiteness when $t \rightarrow \infty$ yields the sum rule,

$$\zeta = \frac{1}{\pi} \int_0^\infty dt' \frac{\phi^{\eta\pi}(t')}{(t')^2}.$$  \hspace{1cm} (8)

Further properties of $\phi^{\eta\pi}$ is that it should obey Watson’s theorem below the $KK$ threshold and that it should display a dip at some point $t_1$ above. The sum rule (8) proves rather constraining. Using the $\eta\pi$ phase-shift model of Ref. [21], the position of the dip is restricted to the range of $2m_K < \sqrt{t_1} < 1.2$ GeV. In the dispersive approach, the exotic nature of one of the $a_0$ resonances corresponds to the dip lying close to its mass. Figure 3 illustrates the phase $\phi^{\eta\pi}$ generated in this approach. The corresponding

![Figure 3](image-url)
shape of scalar component of the energy distribution of the $\tau \to \eta \pi \nu$ decay as a function of the dip position $t_1$ is shown in Fig. 4 (the vector component, computed in Ref. [17] is also shown). This could be probed experimentally at the future super-B or charm-tau factory.

Fig. 4. Contribution to the energy distribution of the $\tau \to \eta \pi \nu$ branching fraction from the scalar form factor for several values of the dip position.

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