IRRELEVANCE OF $f_0(500)$ IN BULK THERMAL PROPERTIES

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We discuss why the scalar–isoscalar resonance $f_0(500)$ should, in practice, not be included in thermal models describing the freeze-out of heavy-ion collisions. Its contribution to pion multiplicities is, in principle, relevant since it is light and that it decays only to pions. However, it is cancelled to a very good numerical precision by the non-resonant scalar–isotensor repulsion among pions. Our approach is an application of a well-known theorem relating spectral function to phase shifts. The numerical results are solely based on pion–pion scattering data and thus model independent.

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1. Introduction

The scalar–isoscalar resonance $f_0(500)$ is now firmly established [1]. The Particle Data Group (PDG) reports the position of its pole in the range of $(400–550)−i(200–350)$ [2]. Investigations based on dispersive analysis show even smaller errors: $(400 ± 6^{+31}_{-13}) − i(278 ± 6^{+34}_{-43})$ in Ref. [3] and $(457^{+14}_{-13}) − i(279^{+11}_{-7})$ in Ref. [4].

The resonance $f_0(500)$ is the lightest scalar state; moreover, it decays only into pions. Then, one is lead to think that $f_0(500)$ is important for the determination of pion multiplicities in thermal models for relativistic heavy-ion collisions (see e.g. Refs. [6,7] and references therein). Indeed, a simple inclusion of $f_0(500)$ as a Breit–Wigner resonance would lead to a sizable increase (about 3–5% [8]) of pions. However, in these proceedings (based on the findings of Ref. [5]), we show that this conclusion is not correct. In fact,
when the repulsion of pion–pion interaction in the scalar–isotensor channel is properly taken into account using the formalism developed in Refs. [9–11], the effect of $f_0(500)$ cancels to a very good numerical accuracy. We show this cancellation in a model-independent way, since the only input is given by the well-known pion–pion scattering data in these two scalar channels.

As a net result, one can neglect in thermal models both the scalar–isoscalar attraction due to $f_0(500)$ and the non-resonant scalar–isotensor repulsion.

2. Cancellation of $f_0(500)$

A successful description of hadron emissions at the freeze-out of relativistic heavy-ion collisions is achieved with the help of thermal models. For simplicity, we restrict our presentation to a gas which includes stable pions ($I = 0, J_{PC} = 0^{-+}$, where $I$ stays for isospin, $J$ for the total spin, and $P$ and $C$ for parity and charge-conjugation), the $\rho$-resonance ($I = 1, J_{PC} = 1^{--}$), the resonance $f_0(500)$ ($I = 0, J_{PC} = 0^{++}$), and the non-resonant contribution of the repulsion in the $I = 2, J_{PC} = 0^{++}$ channel. (Other contributions with different $I$ and $J_{PC}$ correspond to heavier mesons and are neglected here.)

The logarithm of the partition function $Z$ is given by the sum of contributions of all channels

$$\ln Z = \ln Z_\pi + \ln Z_{(1,1^{--})} + \ln Z_{(0,0^{++})} + \ln Z_{(2,0^{++})}.$$ 

All other thermodynamic quantities follow: $P = -T \ln Z/V$, $\varepsilon = -\partial_\beta \ln Z/V$, etc. For what concerns stable pions (we do not include chemical potentials), one has

$$\ln Z_\pi = 3V \int_\mathcal{B} \ln \left[ 1 - e^{-\sqrt{p^2 + M_\pi^2}/T} \right]^{-1}, \quad \int_\mathcal{B} = \int \frac{d^3p}{(2\pi)^3},$$

where $V$ is the volume, $\mathcal{B}$ the pion momentum, $M_\pi$ the pion mass, and the factor 3 the isospin degeneracy. Following Refs. [9,10], we can express the contribution in the channel $I = 1, J_{PC} = 1^{--}$ as

$$\ln Z_{(1,1^{--})} = 3 \times 3 \int_0^{\Lambda_1} \frac{d\delta_{(1,1)}(m)}{\pi dm} \int \ln \left[ 1 - e^{-\sqrt{p^2 + m^2}/T} \right]^{-1}, \quad (1)$$

where $\delta_{(1,1)}$ is the measured $\pi\pi$ phase shift as a function of $m = \sqrt{s}$. We set $\Lambda_1 = 1$ GeV as maximal energy in the integral, then only the $\rho$-meson is present in this range. The spectral function of the $\rho$-meson can be approximated as
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\[ d_\rho(m) = \frac{d\delta_{(1,1)}(m)}{\pi dm}. \]  

(2)

Thus, one can take into account the $\rho$-meson in a thermal gas in a model-independent way by introducing the well-known scattering data in Eq. (1). For a small width, $d_\rho(m)$ can be well-approximated by a Breit–Wigner function, $d_\rho(m) \approx \frac{\Gamma}{2\pi} \left[ (m - M_\rho)^2 + \Gamma^2/4 \right]^{-1}$, and, in the limit of zero width, one correctly obtains $d_\rho(m) = \delta(m - M_\rho)$. Thus, the example of the $\rho$-meson shows quite general features of a thermal gas. The approximation of using a Breit–Wigner expression — typically used in practice — emerges.

We now turn to the main topic of the present work. For $I = J = 0$, the contribution of $f_{0}(500)$ is included in the integral

\[ \ln Z_{(0,0++)} = \int_0^{A_0} dm \frac{d\delta_{(0,0)}}{\pi dm} \int_p \ln \left[ 1 - e^{-\sqrt{p^2 + m^2}/T} \right]^{-1}, \]  

(3)

where $A_0 \simeq 0.8$ GeV (far above the average mass of $f_{0}(500)$ but below $f_{0}(980)$, which is not considered here). The spectral function of the $f_{0}(500)$ is approximated by $d_f_{0}(500)(m) = \frac{1}{\pi} d\delta_{(0,0)}/dm$. The form of $d_f_{0}(500)(m)$ is far from being a Breit–Wigner [5] and is even not normalized to unity. This is in agreement with the fact that the resonances $f_{0}(500)$ is not the chiral partner of the pion and is not a quark–antiquark field [1] (the chiral partner of $\pi$, the ‘true’ $\sigma$ of linear Sigma Models, should be identified with the heavier scalar resonance $f_{0}(1370)$ [12]).

As a last step, we consider the joint contribution of both $I = 0$ and $I = 2$ channels

\[ \ln Z_{(0,0++)} + \ln Z_{(2,0++)} = \int_0^{A_0} dm \left[ \frac{d\delta_{(0,0)}}{\pi dm} + 5 \frac{d\delta_{(2,0)}}{\pi dm} \right] \int_p \ln \left[ 1 - e^{-\sqrt{p^2 + m^2}/T} \right]^{-1}, \]  

(4)

where the factor 5 in front of $d\delta_{(2,0)}/dm$ is the degeneracy $2I + 1$. Data on pion–pion scattering show the following peculiar fact [5]:

\[ \frac{d\delta_{(0,0)}}{\pi dm} + 5 \frac{d\delta_{(2,0)}}{\pi dm} \simeq 0 \quad \text{for} \quad 2M_\pi \leq m \lesssim 0.8 \ \text{GeV}, \]  

(5)

which is valid to a very good numerical accuracy. Then, $\ln Z_{(0,0++)} + \ln Z_{(2,0++)} \simeq 0$! The contribution of $f_{0}(500)$ cancels.

3. Conclusions

In these proceedings, we have shown that the resonance $f_{0}(500)$ can, in practice, be neglected in all isospin-averaged quantities of a thermal hadronic
gas, e.g. for pion multiplicities. Then, the proton-to-pion puzzle becomes even stronger, leaving other explanations open [13]. On the other hand, in correlation studies of pion-pair production, the cancellation does not occur, hence $f_0(500)$ may play a relevant role [14].

A similar cancellation occurs for the not yet confirmed $K^*_0(800)$ ($I = 1/2, J^{PC} = 0^{++}$, e.g. Ref. [15] and references therein), whose contribution is (only partly) compensated by the $I = 3/2, J^{PC} = 0^{++}$ channel [5, 16].

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