LEADING AND SUB-LEADING FLOWS
AT THE LHC FROM THE CMS*

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The initial state fluctuations of the colliding heavy-ion nuclei play a major role in understanding the anisotropic flow of final state particles. Furthermore, an important signature of these fluctuations is the flow (event-plane) angle dependence from $p_T$ that induces a measurable effect of factorization breaking in a pure relativistic hydrodynamic picture. Here, the effect of factorization breaking is described using a new method based on principal component analysis (PCA) and two-particle correlations. The method exposes leading and sub-leading mode, the leading corresponding to the standard elliptic and triangular flow and the sub-leading representing a new variable that is a direct response to initial state fluctuations. In this study, first measurements of the subflow are presented, as a function of transverse momentum in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV and high-multiplicity $p$Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV with CMS data.

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1. Introduction

Relativistic heavy-ion collisions are an ideal terrain for studies of strongly interacting matter. Energies at RHIC (Relativistic Heavy Ion Collider) gave rise to a new state of matter, the quark–gluon plasma (QGP) [1], as a cross over medium transitioning from a parton bound interaction following the QCD phase diagram. The strongest evidence of QGP are high values of elliptic flow and the presence of jet quenching (suppression) of high $p_T$ hadrons. Relativistic hydrodynamics [2,3] has proven to be a powerful tool in predicting the flow values, regarding the QGP as a strongly coupled quark–gluon liquid exhibiting strong partonic collectivity. One of the assumptions that was used for a long time in the hydrodynamical simulation of a collision

event is a smooth and symmetrical initial density profile of the overlapping region. However, it was realised that a more realistic picture of this would be a non-uniform (lumpy-like) initial density profile which is caused by quantum fluctuations of nucleons and partons within the nucleus. As an example, the existence of these initial fluctuations unveiled the full nature of higher flow harmonics in symmetrical collisions [4].

2. Factorization breaking effect

As mentioned previously, a strong signature of initial state fluctuations is the presence of higher flow harmonics \( n > 2 \). The harmonics are a manifestation of spatial anisotropy in the per-event emission of final state particles in the collision. The single particle distribution in the azimuthal phase space is expanded as

\[
\frac{dN}{d\phi} = \frac{N_0}{2\pi} \sum_{n=-\infty}^{n=\infty} V_n(p_T, \eta) e^{-in\phi},
\]

with \( V_n = \upsilon_n e^{in\Psi_n} \), where \( \upsilon_n \) is the anisotropic flow coefficient and \( \Psi_n \) the flow (phase) angle. Now, using \( V_{-n} = V_n^* \) in the previous equation, it follows

\[
\frac{dN}{d\phi} = \frac{N_0}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} \upsilon_n(p_T, \eta) \cos(n(\phi - \Psi_n(p_T, \eta))) \right).
\]

The single particle harmonic \( \upsilon_n \) is a clear function of \( p_T \) and rapidity but the flow angle was considered for a long time as a global phase that fluctuates randomly from event to event. Recent hydrodynamical studies showed [5–7] that local hotspots perturb the event plane of a smooth medium which can, in effect, give high \( p_T \) particles different values of the flow angle \( i.e. \Psi_n = \Psi_n(p_T) \). In order to probe this flow angle \( p_T \) dependence, the so-called factorization breaking effect is investigated. Referring to the complex plane definition of azimuthal distribution (1), the two-particle correlation harmonic will be equal to the averaged product of single particle flow coefficients

\[
V_n\Delta \left( p_T^a, p_T^b \right) = \left\langle V_n(p_T^a) V_n^* \left( p_T^b \right) \right\rangle = \left\langle \upsilon_n^a \upsilon_n^b e^{in(\Psi_n^a - \Psi_n^b)} \right\rangle.
\]

Because of parity symmetry, only the cosine term remains. One also concludes that only when the flow angle is a global phase do we have factorization. To investigate the factorization effect, one can use a Pearson-like coefficient

\[
r_n = \frac{V_n\Delta \left( p_T^a, p_T^b \right)}{\sqrt{V_n\Delta \left( p_T^a, p_T^a \right) V_n\Delta \left( p_T^b, p_T^b \right)}} \sim \left\langle \cos \left( n \left( \Psi_n \left( p_T^a \right) - \Psi_n \left( p_T^b \right) \right) \right) \right\rangle
\]
which is proportional to the underlined cosine term. If the ratio is unity, factorization holds, if it is below unity, this is evidence of factorization breaking. CMS results [8] show a clear sign of factorization breaking, especially for Pb–Pb data in the case of the elliptic flow and ultra-central collisions, where initial state fluctuations are dominant.

3. Principal component analysis and event-by-event fluctuations

Recognizing the strong implications of initial state fluctuations on final state effects, a new model was introduced [9] that extracts the flow fluctuations directly from data. The method combines the statistical tool called principal component analysis and standard two-particle correlations which, in effect, give a new measurable flow observable. The building block of the method is constructing a real, symmetrical matrix $\hat{V}_{n\Delta}(p^a_T, p^b_T)$ of two-particle harmonics for $N_b$ number of $p_T$ differential bins. In a pure hydro-picture, this is a covariance matrix which is, by definition, positive semidefinite, hence positive eigenvalues. In practice, PCA procedure makes a spectral decomposition of the covariance matrix where the principal components refer to the orthogonal eigenvectors. The components are ordered by size (data variance) and isolate the most important fluctuations. The calculated components (modes) $V^{(1)}_n(p_T), V^{(2)}_n(p_T), \ldots$ can be used as a basis for the event-by-event expansion of harmonic flow

$$V_n(p_T) = \xi^{(1)}_n V^{(1)}_n(p_T) + \xi^{(2)}_n V^{(2)}_n(p_T) + \ldots,$$

where $\xi^{(\alpha)}_n$ are random complex uncorrelated variables with zero mean i.e. $\langle \xi^{(\alpha)}_n \xi^{(\beta)}_n \rangle = \delta_{\alpha\beta}$ and $\langle \xi^{(\alpha)}_n \rangle = 0$. Now, calculating the two-particle harmonic from equation (3) by means of expansion (5), we have

$$V_{n\Delta}(p^a_T, p^b_T) = \sum_{\alpha=1}^{N_b} V^{(\alpha)}_n(p^a_T) V^{(\alpha)*}_n(p^b_T).$$

In order to calculate the modes, we rewrite the spectral decomposition as $V_{n\Delta}(p^a_T, p^b_T) = \sum_{\alpha} \lambda^{(\alpha)} e^{(\alpha)}(p^a_T) e^{(\alpha)*}(p^b_T)$ which gives

$$V^{(\alpha)}_n(p_T) = \sqrt{\lambda^{(\alpha)}} e^{(\alpha)}(p_T),$$

where $e^{(\alpha)}(p_T)$ are normed eigenvectors and eigenvalues $\lambda^{(\alpha)}$ follow strict descending ordering $\lambda^{(1)} > \lambda^{(2)} \ldots > \lambda^{(n)}$. Equation (6) directly shows that only in the case of one mode does factorization hold. Finally, connecting the modes with anisotropy per particle $\nu_n$ an additional norming step is needed as

$$\nu^{(\alpha)}_n(p_T) = \frac{V^{(\alpha)}_n(p_T)}{\langle M(p_T) \rangle}.$$
The observable $v_{n}^{(1)}$ is designated as the leading flow which corresponds to the standard single particle anisotropy. The observable $v_{n}^{(2)}$ stemming from the second mode is the sub-leading flow which is a direct response to initial state fluctuations. The two-particle harmonics $V_{n\Delta}(p_{T}^{a}, p_{T}^{b})$ of the covariance matrix are calculated as in [8] with the exception that no norming per event is done by number of pairs \textit{i.e.},

$$V_{n\Delta}(p_{T}^{a}, p_{T}^{b}) = \langle \cos n\Delta\phi \rangle_{S} - \langle \cos n\Delta\phi \rangle_{B} , \quad (9)$$

where $\langle \cdot \rangle$ is averaging by all the events and the last term referring to the non-uniform acceptance of the detector. In addition, the correlations between the tracks are done with a rapidity cut of $|\Delta\eta| > 2$ that suppresses jet contributions which are short-ranged. Since the initial norming definition (8) is only flow driven (Monte Carlo study [9]), the rapidity cut implies an additional factor that needs to be added as

$$\tilde{V}_{n\Delta}(p_{T}^{a}, p_{T}^{b}) = \left\langle \frac{N_{p}^{\text{all}}(a,b)}{N_{p}^{\text{cut}}(a,b)} \right\rangle V_{n\Delta}(p_{T}^{a}, p_{T}^{b}) , \quad (10)$$

where the harmonic is multiplied by the ratio of the total number of pairs and number of pairs with the rapidity cut. Using this new harmonic the covariance matrix was built.

4. Results

In this study, measurements of the leading and sub-leading flow are conducted using PbPb data at $\sqrt{s_{NN}} = 2.76$ TeV and high-multiplicity pPb data at $\sqrt{s_{NN}} = 5.02$ TeV from the CMS experiment. Figure 1 shows the elliptic leading and sub-leading flow for PbPb collisions as a function of $p_{T}$ for eight centrality regions. For the purpose of comparison, ALICE data [10] is also plotted. Observing the leading flow values, we see a very good agreement with ALICE data, corresponding to the standard single particle anisotropy as expected. Here, the new information is the sub-leading flow that has a clear signal for all centralities with stronger systematic errors for more central regions. We observe a rise of the sub-leading flow signal with $p_{T}$ which is in correspondence with the factorization breaking effect. This trend is the same for all centralities but is not linear going from ultra-central to very peripheral. Figure 2 shows the elliptic leading and sub-leading flow in the case of high-multiplicity pPb data. The calculations are done for four high-multiplicity intervals. Here, the leading flow is compared to data from [11], again being in good agreement with standard single particle anisotropy. As in the case of Pb–Pb, the sub-leading flow rises with $p_{T}$ where we observe a fable multiplicity dependence. As it was discussed in
the previous section, the existence of the sub-leading flow infers the effect of factorization breaking.

Fig. 1. Elliptic leading and sub-leading flow as a function of $p_T$ for Pb–Pb data at nucleon–nucleon center-of-mass energy of 2.76 TeV. The results span eight centrality regions.

Fig. 2. Elliptic leading and sub-leading flow as a function of $p_T$ for pPb data at nucleon–nucleon center-of-mass energy of 5.02 TeV. The results span four multiplicity intervals.
We can demonstrate this in an explicit way but expressing the ratio $r_n$ from equation (4) in terms of principal components (modes)

$$r_n \approx 1 - \frac{1}{2} \left[ \frac{V_n^{(2)}(p_{T}^a)}{V_n^{(1)}(p_{T}^a)} - \frac{V_n^{(2)}(p_{T}^b)}{V_n^{(1)}(p_{T}^b)} \right]^2,$$

(11)

which gives $r_n = 1$ when the sub-leading flow is zero. The last equation also shows that the effect of factorization breaking is induced by the ratio of the sub-leading and leading flow and not by the sub-leading flow itself.

5. Conclusion

Initial state fluctuations prove to be an important factor in understanding the final state correlations in relativistic heavy-ion collisions. One such signature is the effect of factorization breaking induced by the flow angle $p_T$ dependence. This effect can also be quantified by a new observable called the sub-leading flow, following a method based on principal component analysis applied to the two-particle correlation matrix. Here, first measurements of the sub-leading flow are conducted with lead–lead and proton–lead CMS data at $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.02$ TeV, respectively. The results offer a new insight in the behaviour of factorization breaking and can be used to further constrain the plasma response to the initial geometry.

REFERENCES