EVIDENCE FOR $D^0$–$\bar{D}^0$ MIXING AT BELLE

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(Received October 29, 2007)

The present paper reports on the evidence for $D^0$–$\bar{D}^0$ mixing. We searched for mixing in two processes using 540 fb$^{-1}$ of data recorded by the BELLE detector at the KEKB $e^+e^-$ collider. In the first case, we measured the apparent lifetime for $D^0$ meson decaying to CP eigenstates $K^+K^-$ or $\pi^+\pi^-$, and the lifetime in the $D^0$ decays to $K^-\pi^+$. From the difference of the two lifetimes, the mixing parameter $y_{CP}$ is found to be $y_{CP} = (1.31 \pm 0.32 \text{ (stat.)} \pm 0.25 \text{ (syst.)})\%$, 3.2 standard deviations from zero. In the second measurement, we analyzed the time dependent Dalitz plot for the $D^0 \to K^0\pi^+\pi^-$ decays, from which the mixing parameters $x = (0.80 \pm 0.29^{+0.09+0.15}_{-0.67-0.14})\%$ and $y = (0.33 \pm 0.24^{+0.07+0.08}_{-0.12-0.09})\%$ are determined; the errors are statistical, experimental systematic, and systematic due to the Dalitz decay model, respectively. We also searched for a CP asymmetry between $D^0$ and $\bar{D}^0$ decays, and found no evidence for it.

PACS numbers: 13.25.Ft, 11.30.Er, 12.15.Ff

1. Introduction

The phenomenon of particle and anti-particle mixing has been observed in several systems of neutral mesons [1, 2], $K^0$, $B^0_d$, and most recently $B^0_s$ mesons. Mixing is also possible in the $D$-meson system, but has not been previously observed.

The time evolution of a $D^0$ or $\bar{D}^0$ depends on the mixing parameters $x = (M_1 - M_2)/\Gamma$ and $y = (\Gamma_1 - \Gamma_2)/2\Gamma$, where $M_{1,2}$ and $\Gamma_{1,2}$ are the masses and widths, respectively, of the mass eigenstates, and $\Gamma = (\Gamma_1 + \Gamma_2)/2$. For no mixing, $x = y = 0$. Within the Standard Model (SM), predictions for $x$ and $y$ are dominated by difficult non-perturbative calculations [3, 4]. The largest predictions are $|x|, |y| \sim \mathcal{O}(10^{-2})$ [4]. Loop diagrams including new, as-yet-unobserved particles could significantly affect the values of $x$.

and \(y\) [5]; several predictions are around a few percent. CP-violating effects in \(D\)-mixing in excess of the very small SM prediction would be a clear signal of new physics [6].

Both semileptonic and hadronic \(D^0\) decays have been used to constrain \(x\) and \(y\) [1]. In order to tag the flavor at production, the \(D^0\) meson is usually reconstructed in the decay\(^1\) \(D^+ \rightarrow D^0\pi^+_s\), where the charge of a characteristic slow pion \(\pi_s\) tags the initial \(D^0\) flavor. Usually also the \(D^0\) proper decay time is measured, since the decay time distribution of mixed events depends on the mixing parameters \(x\) and \(y\) and differs from that of not-mixed events. The proper decay time of an event is determined from the distance between the production and the decay vertex. The decay vertex is obtained from \(D^0\) daughter tracks, refitted to originate from a common point. The production vertex is found by constraining the \(D^0\) momentum vector to originate from the \(e^+e^-\) interaction region. The proper decay time resolution is on average equal to one half of the \(D^0\) lifetime.

The present measurements are based on 540 fb\(^{-1}\) of data accumulated with the BELLE detector at the \(\Upsilon(4S)\) resonance.

2. Decays to CP eigenstates \(D^0 \rightarrow K^+K^-\) and \(D^0 \rightarrow \pi^+\pi^-\)

In the measurement of the apparent lifetime of the decays to CP eigenstates \(D^0 \rightarrow K^+K^-\) and \(D^0 \rightarrow \pi^+\pi^-\), the mixing parameter

\[
y_{CP} = \frac{\tau(K^-\pi^+)}{\tau(K^+K^-)} - 1,
\]

is determined, where \(\tau(K^+K^-)\) and \(\tau(K^-\pi^+)\) are the lifetimes of \(D^0 \rightarrow K^+K^-\) (or \(\pi^+\pi^-\)) and \(D^0 \rightarrow K^-\pi^+\) decays, respectively. It can be shown that \(y_{CP} = y \cos \phi - \frac{1}{2}A_Mx \sin \phi\) [7], where \(A_M\) and \(\phi\) are CP violation parameters. If CP is conserved, \(A_M = \phi = 0\) and \(y_{CP} = y\). To date several measurements of \(y_{CP}\) have been reported [8]; the average value is \(\sim 2\) standard deviations (\(\sigma\)) above zero. The new BELLE measurement yields a nonzero value of \(y_{CP}\) with \(> 3\sigma\) significance [9].

In addition, a search for CP violation was carried out by measuring the quantity

\[
A_{\Gamma} = \frac{\tau(D^0 \rightarrow K^-K^+) - \tau(D^0 \rightarrow K^+K^-)}{\tau(D^0 \rightarrow K^-K^+) + \tau(D^0 \rightarrow K^+K^-)},
\]

this observable is equal to \(A_{\Gamma} = \frac{1}{2}A_M y \cos \phi - x \sin \phi\) [7].

The following decay sequence was reconstructed: \(D^{*+} \rightarrow D^0\pi^+_s\), followed by \(D^0 \rightarrow K^+K^-, K^-\pi^+\) or \(\pi^+\pi^-\). A \(D^{*+}\) momentum greater

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\(^1\) Charge conjugate modes are implied unless explicitly stated otherwise.
than 2.5 GeV/c (in the CM) was required to reject D-mesons produced in B-meson decays and to suppress combinatorial background.

The proper decay time of the $D^0$ candidate was then calculated from the projection of the vector joining the two vertices, $\vec{L}$, onto the $D^0$ momentum vector, $t = m_{D^0} \vec{L} \cdot \vec{p}/p^2$, where $m_{D^0}$ is the nominal $D^0$ mass. The decay time uncertainty $\sigma_t$ was evaluated event-by-event from the covariance matrices of the production and decay vertices.

Candidate $D^0$ mesons were selected using two kinematic observables: the invariant mass $M$ of the $D^0$ decay products and the energy $q = (M_{D^*+} - M - m_{\pi^+})c^2$ released in the $D^*+\pi$ decay. Here $M_{D^*}$ is the invariant mass of the $D^0\pi$ combination and $m_{\pi^+}$ is the $\pi^+$ mass.

According to Monte Carlo (MC) simulated distributions of $t$, $M$, and $q$, background events fall into four categories: (1) combinatorial, with zero apparent lifetime; (2) true $D^0$ mesons combined with random slow pions (this has the same apparent lifetime as the signal) (3) $D^0$ decays to three or more particles, and (4) other charm hadron decays. The apparent lifetime of the latter two categories is 10–30% larger than $\tau_{D^0}$, depending on the category and decay channel.

Selection criteria were chosen to minimize the expected statistical error of $\chi_{CP}$, $|M - m_{D^0}|/\sigma_M < 2.3$, $|q - (m_{D^*+} - m_{D^0} - m_{\pi^+})c^2| < 0.80$ MeV, and $\sigma_t < 370$ fs. Here the invariant mass resolution $\sigma_M$ varies from 5.5 MeV/$c^2$ to 6.8 MeV/$c^2$, depending on the decay channel. In the final sample, we found $111 \times 10^3 K^+ K^-$, $1.22 \times 10^6 K^-\pi^+$, and $49 \times 10^3 \pi^+\pi^-$ signal events, with purities of 98%, 99%, and 92%, respectively.

The relative lifetime difference $\chi_{CP}$ was determined from $D^0 \rightarrow K^+K^-$, $K^-\pi^+$, and $\pi^+\pi^-$ decay time distributions by performing a simultaneous binned maximum likelihood fit to the three samples. Each distribution was assumed to be a sum of signal and background contributions, with the signal contribution being a convolution of an exponential and a detector resolution function

$$\frac{dN}{dt} = \frac{N_{\text{sig}}}{\tau} \int e^{-t'/\tau} \times R(t-t') \, dt' + B(t).$$  

The resolution function $R(t-t')$ was constructed from the normalized distribution of the decay time uncertainties $\sigma_t$ (see Fig. 1). The $\sigma_t$ of a reconstructed event ideally represents an uncertainty with a Gaussian probability density: in this case, bin $i$ in the $\sigma_t$ distribution is taken to correspond to a Gaussian resolution term of width $\sigma_i$, with a weight given by the fraction $f_i$ of events in that bin. However, the distribution of “pulls”, i.e. the normalized residuals $(t_{\text{rec}} - t_{\text{gen}})/\sigma_t$ (where $t_{\text{rec}}$ and $t_{\text{gen}}$ are reconstructed and generated MC decay times), is not well-described by a Gaussian. This distribution can be fitted with a sum of three Gaussians of different widths $\sigma_k^{\text{pull}}$ and fractions $w_k$, constrained to the same mean. Therefore, the parameterization
was chosen, with $\sigma_{ik} = s_k \sigma_{k}^{\text{pull}} \sigma_i$, where the $s_k$ are three scale factors introduced to account for differences between the simulated and real $\sigma_k^{\text{pull}}$, and $t_0$ allows for a (common) offset of the Gaussian terms from zero.

![Fig. 1. Normalized distribution of errors $\sigma_t$ of the decay time $t$ for $D^0 \to K^-\pi^+$, showing the construction of the resolution function using the fraction $f_i$ in the bin with $\sigma_t = \sigma_i$.](image)

The background $B(t)$ was parameterized assuming two lifetime components: an exponential and a $\delta$ function, each convolved with corresponding resolution functions as parameterized by Eq. (4). Separate $B(t)$ parameters for each final state were determined by fits to the $t$ distributions of events in $M$ sidebands. MC samples were used to select the sideband region that best reproduces the timing distribution of background events in the signal region.

Fits to the $D^0 \to K^+K^-, K^-\pi^+ \text{ and } \pi^+\pi^-$ data are shown in Fig. 2(a)–(c). The fitted lifetime of $D^0$ mesons in the $K^-\pi^+$ final state, $\tau_{D^0} = (408.7 \pm 0.6 \text{ (stat.)}) \text{ fs}$, is in good agreement with the current world average [1]. The relative apparent lifetime difference between decays to CP-even eigenstates and the $K^-\pi^+$ final state is found to be

$$y_{\text{CP}} = (1.31 \pm 0.32\text{(stat.)} \pm 0.25\text{(syst.)})\%.$$  

Combining the errors in quadrature, this result is 3.2 standard deviations from zero and represents the first experimental evidence for the $D$-mixing, regardless of possible CP violation. The effect is presented visually in Fig. 2(d), which shows the ratio of decay time distributions for $D^0 \to K^+K^-, \pi^+\pi^-$ and $D^0 \to K^-\pi^+$ decays.
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We also searched for CP violation by separately measuring decay times of $D^0$ and $\bar{D}^0$ mesons in CP-even final states. The asymmetry was found to be consistent with zero, $A_\Gamma = (0.01 \pm 0.30(\text{stat.}) \pm 0.15(\text{syst.}))\%$.

3. Time-dependent Dalitz analysis of $D^0 \rightarrow K^0_{s}\pi^+\pi^-$

A measurement of mixing parameters in the self-conjugate decays $D^0 \rightarrow K^0_{s}\pi^+\pi^-$ was performed using a time-dependent Dalitz plot analysis [10]. The time dependence of the $K^0_{s}\pi^+\pi^-$ Dalitz plot distribution allows one to measure $x$ and $y$ directly. This method was developed by CLEO [11] using 9.0 fb$^{-1}$.

Assuming CP conservation, the decay amplitude at time $t$ of an initially produced $D^0$ can be expressed as

$$M(m_-^2, m_+^2, t) = A(m_-^2, m_+^2) \frac{e_1(t) + e_2(t)}{2} + A(m_-^2, m_+^2) \frac{e_1(t) - e_2(t)}{2},$$  (6)
where $A$ is the decay amplitude as a function of the invariant masses squared, $m^2_\pm = m(K_\pm, \pi^\pm)^2$; an analogous expression can be derived for $D^0$.

The time dependence is contained in the terms $e_{1,2}(t) = \exp[-i(m_{1,2} - i\Gamma_{1,2}/2)t]$. Upon squaring $M$, one obtains decay rates containing terms $\exp(-\Gamma t) \cos(x\Gamma t)$, $\exp(-\Gamma t) \sin(x\Gamma t)$ and $\exp[-(1 \pm y)\Gamma t]$.

The overall decay amplitude $A$ can be expressed as a sum of quasi-two-body amplitudes $A_r$ and a constant non-resonant term (subscript NR):

$$A(m^2_-, m^2_+) = \sum_r a_re^{i\phi_r}A_r(m^2_-, m^2_+) + a_{NR}e^{i\phi_{NR}}.$$  \hfill (7)

The functions $A_r$ are products of Blatt–Weisskopf form factors and relativistic Breit–Wigner functions [12].

The $K^0_s$ was reconstructed in the decay to the $\pi^+\pi^-$ final state; an invariant mass within $\pm 10$ MeV of $m_{K^0_s}$ and a common vertex separated from the interaction region were required. The $D^0$ decay point was constructed from charged pion tracks only and the production point was obtained from the intersection of the $D^0$ momentum vector with the $e^+e^-$ interaction region.

The signal and background yields were determined from a two-dimensional fit to the variables $M \equiv m_{K^0_s\pi\pi}$ and $Q \equiv m_{K^0_s\pi\pi\pi} - m_{K^0_s\pi\pi} - m_{\pi}$. The background was classified into two types: random $\pi_s$ background and combinatorial background. In the signal region, defined as $3\sigma$ intervals in $M$ and $Q$, 534 $\times 10^3$ signal events were found, with background fractions of 1% and 4% for the random $\pi_s$ and combinatorial backgrounds, respectively.

For the events in the signal region, a simultaneous un-binned likelihood fit to the Dalitz plot variables $m^2_-$ and $m^2_+$, and the decay time $t$ was performed. The likelihood function is:

$$L = \sum_{i=1}^N \sum_j f_j(M_i, Q_i)P_j(m^2_{-,j}, m^2_{+,j}, t_i),$$ \hfill (8)

where $j = \{\text{sig, rnd, cmb}\}$ denotes the signal or background components, and the index $i$ runs over all events. The event weights $f_j$ are functions of $M$ and $Q$ and were obtained from the $M$–$Q$ fit discussed above.

The probability density function $P_{\text{sig}}$ equals $|M|^2$ convolved with the detector response. The resolution of the decay time $t$ is parameterized by a sum of three Gaussians with a common mean, and with widths $\sigma_k = S_k \cdot \sigma_t$, ($k = 1, 2, 3$), where $\sigma_t$ is the decay time uncertainty calculated event-by-event, and the $S_k$ are scale factors, left as free parameters in the fit.

The random $\pi_s$ background contains real $D^0$ and $\bar{D}^0$ decays uncorrelated to the charge of a slow pion. The probability density function is in this case taken to be $(1 - f_w)|M(m^2_-, m^2_+, t)|^2 + f_w|\bar{M}(m^2_+, m^2_-, t)|^2$, convolved
with the same resolution function as the signal probability density function. The fraction \( f_w \) was determined from fitting events in the \( Q \) sideband. For the combinatorial background, \( P_{\text{comb}} \) is a product of the Dalitz plot and decay time probability density functions. The latter is parameterized as the sum of a delta function and an exponential function convolved with a Gaussian resolution function. The resolution function has a \( \sigma_t \) dependent offset. This and other timing parameters, as well as the Dalitz probability density function, were obtained from the events in the mass sideband.

The free parameters in the fit were \( x, y, \tau_{D^{0}} \), the timing resolution parameters \( S_k \) of the signal, and the Dalitz plot resonance parameters \( a_{r \text{(NR)}} \) and \( \phi_{r \text{(NR)}} \). The resonance model assumed 18 quasi-two-body resonances; masses and widths were taken from world averages. The Dalitz plot and its projections, along with projections of the resulting fit, are shown in Fig. 3.

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![Figure 3](https://example.com/fig3.png)

**Fig. 3.** Dalitz plot distribution and its projections \((m_{+}, m_{-} \text{ and } m_{\pi \eta})\) with superimposed results of the fit.
The decay-time distribution and the fit are shown in Fig. 4. The fitted $D^0$ lifetime $\tau_{D^0} = 409.9 \pm 1.0$ fs is consistent with the world average. The measured mixing parameters are $x = (0.80 \pm 0.29^{+0.09}_{-0.07} + 0.15)\%$ and $y = (0.33 \pm 0.24^{+0.07}_{-0.12} + 0.08)\%$; the errors are statistical, experimental systematic, and systematic due to the Dalitz decay model, respectively. The largest contributions to the systematic uncertainty of the result are found to arise from modeling the Dalitz plot density and from the fit to the decay-time distribution. The result for the mixing parameter $x$ represents the most stringent limit on this parameter obtained up to now. If no assumption is made on CP conservation, a fit of the CP violation parameters yields $|q/p| = 0.86^{+0.30}_{-0.29} + 0.06 \pm 0.08$ and $\arg(q/p) = (−14^{+16}_{-18} + 5^{+2}_{-3})\degree$.

![Decay time distribution with the fit superimposed. The curve below the data points is the time distribution of background. The lower plot shows the residuals.](image)

**4. Summary**

From 540 fb$^{-1}$ of data recorded by the BELLE detector at the KEKB $e^+e^-$ collider, we have found evidence for $D^0\overline{D^0}$ mixing by measuring the apparent lifetime for $D^0$ meson decaying to CP eigenstates. The mixing parameter $y_{CP}$ is found to be $y_{CP} = (1.31 \pm 0.32 \text{(stat.)} \pm 0.25 \text{(syst.)})\%$, 3.2 standard deviations from zero. By analyzing the time dependent Dalitz plot for the $D^0 \rightarrow K^0_s\pi^+\pi^-$ decays, we determined the mixing parameters $x = (0.80 \pm 0.29^{+0.09}_{-0.07} + 0.15)\%$ and $y = (0.33 \pm 0.24^{+0.07}_{-0.12} + 0.08)\%$. We found no evidence for CP asymmetry between $D^0$ and $\overline{D^0}$ decays.
REFERENCES